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# ADAPTIVE EQUIVALENT CONSUMPTION MINIMIZATION STRATEGY FOR HYBRID ELECTRIC VEHICLES

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## ABSTRACT

This paper proposes a new method for solving the energy management problem for hybrid electric vehicles (HEVs) based on the equivalent consumption minimization strategy (ECMS). After discussing the main features of ECMS, an adaptation law of the equivalence factor used by ECMS is presented, which, using feedback of state of charge, ensures optimality of the strategy proposed. The performance of the A-ECMS is shown in simulation and compared to the optimal solution obtained with dynamic programming.

## 1 INTRODUCTION

Energy management of hybrid electric vehicles is now a mature field, with many papers devoted to optimal or quasi-optimal control strategies (see the overview in [1]). Since its introduction [2], the equivalent consumption minimization strategy (ECMS) has been successful as an implementable algorithm that achieves good results in terms of fuel consumption. Initially, the strategy was conceptually justified by energy balance considerations, while successive studies increased its theoretical bases by proving that the instantaneous minimization is in fact connected to the Hamiltonian function of optimal control theory [3, 4]. Its equivalence with Pontryagin's minimum principle [5] proves that ECMS can generate the optimal energy management for a given cycle, provided that it is properly tuned by choosing the appropriate value of *equivalence factor*. Since perfect tuning is possible only with a-priori knowledge of the cycle, the current efforts are directed towards online adaptation of ECMS, in order to achieve quasi-optimal results even without a-priori tuning of the strategy. Several versions of *adaptive ECMS* have been presented: [6] uses driving cycle prediction and performs receding-horizon optimization, [7] uses pattern recognition to identify the driving conditions, while the most recent and interesting approaches are based on the feedback of the current state of charge value [8, 9], and try to change dynamically the value of the equivalence factor in order to contrast the SOC variation (and thus maintain its value around the reference level). The approach proposed here is also based on feedback of the state of charge, but it is conceptually different because it relies on the concept of charge-sustaining horizon, imposing charge-sustainability over a finite time horizon. The paper is organized as follows: Section 2 introduces and formalizes the optimal control problem for a generic HEV; Section 3 reviews the basic concepts of the ECMS, pointing out its intrinsic limitations, essentially due to the need for calibration. Section 4 proposes an adaptive ECMS strategy (A-ECMS), i.e a method to automatically tune the ECMS based on state-of-charge feedback. Section 5, finally, shows simulation results demonstrating the effectiveness of the proposed strategy.

## 2 PROBLEM FORMULATION

The optimal control problem in a HEV consists in finding the minimum fuel consumption during vehicle operation, while respecting the design limitations of each component and the drivability/performance specifications. The aim is to minimize a cost function defined as an integral over a finite horizon. The finite horizon typically corresponds to a complete regulatory driving cycle or a short real-world trip. The optimization objective considered in this work is the fuel consumption during a trip, and the constraints are:

- charge-sustainability: the battery SOC at the beginning and the end of the trip should be equal;
- satisfaction of driver's demand: at each instant, the total torque output of the powertrain should be equal to the driver's demand;
- actuator limitations: at each instant, the output of each machine in the powertrain (engine, motor, and generator) cannot exceed its maximum torque/power rating; similarly, the total battery power must remain within the acceptable limits in both charge and discharge operation.

The specific expression of the constraints and the choice of the control variables depends on the vehicle architecture considered. In general, the objective is to find the function $u(t)$, $t \in [t_0, t_f]$, $u(t) \in R^m$ such that

$$J\left(x(t_0), u(t), x(t_f)\right) = \int_{t_0}^{t_f} \dot{m}_f\left(u(t)\right) dt \tag{1}$$

is minimized.

The system state $x(t)$ is the battery state of charge; $\dot{m}_f\left(u(t)\right)$ is the fuel consumption, and $u(t)$ is the control variable, which depends on the powertrain architecture. For example, it may be the engine torque in the case of a parallel HEV, or the engine torque and speed for a series HEV; it may also include binary variables, such as the engine status (on/off) if the powertrain architecture allows more operating modes. Note that the fuel consumption $\dot{m}_f(u(t))$ does not depend explicitly on the state of charge.

The nonlinear state equation is:

$$\dot{x}(t) = f(x, P_{batt}) \tag{2}$$

where $P_{batt}(t)$ is the electrical power delivered by the battery. An explicit form of Eq. (2) is obtained using a simple circuit model composed of a voltage source and a resistance:

The battery is normally very complex to represent. In this case, in order to obtain a reasonably simple simulator, no temperature dependency is considered, hysteresis and dynamics are neglected, and a simple circuit model (represented in Fig. 1) is used to compute the state of charge variation as a function of the power at the terminals and of the circuit parameters.

The state of charge variation represents the state equation of the energy management problem, and can be written as

$$\dot{x}(t) = -\frac{I(t)}{Q_{max}} \tag{3}$$

where $I(t)$ is the current flowing through the battery (positive during discharge) and $Q_{max}$ is the battery charge capacity. In order to make this relation implementable in the framework of the model described here, it is necessary to express the current in terms the battery power. The first step is to write the balance equation for the equivalent circuit:

$$V_L = V_{oc} - I \cdot R_{eq}, \tag{4}$$

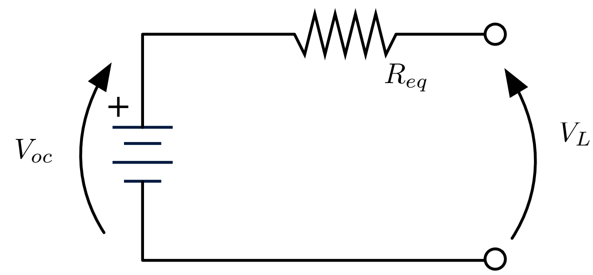


Figure 1. Battery circuit model.

where $V_{oc} = V_{oc}(SOC)$ is the open circuit voltage, $V_L$ the voltage at the battery terminals and $R_{eq}$ is the equivalent internal resistance. Multiplying each side of the eq. 4 by the terminal current it is possible to find the battery power:

$$P_{batt} = V_L \cdot I = V_{oc} \cdot I + R_{eq} \cdot I^2. \tag{5}$$

Solving Eq. 5 with respect to I yields:

$$I = \frac{V_{oc} + \sqrt{V_{oc}^2 - 4 \cdot P_{batt} \cdot R_{eq}}}{2 \cdot R_{eq}} \tag{6}$$

with $I > 0$ when discharging and $I < 0$ charging. The SOC dynamics is therefore:

$$\dot{x}(t) = -\frac{1}{Q_{max}} \frac{V_{oc}(x) + \sqrt{V_{oc}^2(x) - 4R_{eq}(x)P_{batt}(t)}}{2R_{eq}(x)} = f(x, P_{batt}). \tag{7}$$

The optimization is also subject to the following constraints:

$$P_{pwt}(t) = P_{dem}(t) \;\forall t \in [t_0, t_f] \tag{8}$$

$$0 \leq P_{ICE}(t) \leq P_{ICE,max}(t) \;\forall t \in [t_0, t_f] \tag{9}$$

$$P_{em,i,min}(t) \leq P_{em,i}(t) \leq P_{em,i,max}(t) \;\forall t \in [t_0, t_f] \tag{10}$$

$$P_{batt,min}(x) \leq P_{batt}(t) \leq P_{batt,max}(x) \;\forall t \in [t_0, t_f] \tag{11}$$

$$x_{min} \leq x(t) \leq x_{max} \;\forall t \in [t_0, t_f] \tag{12}$$

$$x(t_0) = x_0, \qquad x(t_f) = x_0 \tag{13}$$

where $P_{pwt}$ is the total power delivered by the powertrain, $P_{dem}$ is the power demand (e.g. generated by the driver), $P_{ICE}$ is the engine mechanical power, and $P_{em,i}$ is the power of the $i^{th}$ electric machine (there may be more than one machine). The subscripts *max* and *min* refer to the maximum and minimum limits of each variable. Equation (8) means that the range of operating conditions considered at each instant must be such that the powertrain delivers the amount of power (or torque) that is demanded by the driver. Therefore, the sequence of power demand $P_{dem}(t)$ is an essential information for the solving the optimization problem.

## 3 THE EQUIVALENT CONSUMPTION MINIMIZATION STRATEGY

A global optimal solution can only be found if $P_{dem}(t)$ is known in advance for all the optimization horizon. A method often used to this purpose is dynamic programming (DP) [10, 11], which is capable of determining the optimal solution to the discretized problem. This solution is generally sub-optimal for the continuous problem, because of the approximation introduced with the discretization; however, if the grid is fine enough, the approximation is negligible. The DP algorithm can only be solved off-line, because it requires the driving cycle to be completely known a-priori, and therefore it is not implementable online (unless perfect prediction of the cycle is available). Furthermore, the high computational load makes the DP optimization prohibitive on typical onboard micro controllers.

An alternative method to solve the optimal control problem is to apply Pontryagin's minimum principle [4, 5, 12]. This theorem states that, if the control law $u^*(t)$ is optimal, the following conditions are satisfied:

1. $u^*(t)$ minimizes at each instant the Hamiltonian of the optimal control problem:

$$H(x,u,\lambda) \geq H(x,u^*,\lambda), \ \forall u(t) \neq u^*(t)$$

where the Hamiltonian is defined as

$$H(x,u,\lambda) = \lambda(t) \cdot f(x,u) + \dot{m}_f(u) \tag{14}$$

with $\lambda(t)$ an auxiliary variable called *co-state* of the system.

2. the co-state variable satisfies the following dynamic equation:

$$\dot{\lambda}(t) = -\frac{\partial H}{\partial x} = -\frac{\partial f(x,u)}{\partial x} \tag{15}$$

The conditions given by the minimum principle are *necessary*, not sufficient. Every solution that satisfies the necessary conditions is called an *extremal* solution. If the optimal solution exists, then it is also extremal. The opposite, however, is not true: a solution may be extremal without being optimal. However, if the problem has a unique optimal solution, and the application of the minimum principle gives only one extremal solution, then this is the optimal solution.

The minimum principle can be used to find solution candidates by computing and minimizing the Hamiltonian function at each instant. This generates, by construction, extremal controls. If the Hamiltonian is a convex function of the control, then there is only one extremal solution, which is therefore optimal.

The Hamiltonian function (14) has the physical meaning of an *equivalent fuel consumption*: in fact, it is the sum of the actual fuel consumption and of a term that is dimensionally the same as a fuel consumption, but is proportional to the battery SOC variation and therefore accounts for the virtual fuel consumption associated to the use of the battery. The co-state $\lambda(t)$ defines the equivalence between fuel use and battery use. For a more intuitive formulation, an a-dimensional *equivalence factor* can be introduced, defined as $s(t) = -\lambda(t) \cdot \frac{Q_{lhv}}{E_{batt}}$, where $E_{batt}$ is the energy capacity of the battery and $Q_{lhv}$ the fuel lower heating value, i.e. its energy density (per unit of mass). With this definition, the Hamiltonian – or equivalent fuel consumption – can be rewritten as

$$H(x,u,\lambda) = \dot{m}_{eqv}(x,u,s) = s(t) \cdot \frac{E_{batt}}{Q_{lhv}} \cdot f(x,u) + \dot{m}_f(u) \tag{16}$$

As pointed out in [5], this formulation is fundamentally similar to the original ECMS formulation [2], in which the authors proposed to minimize an equivalent fuel consumption defined in the same way as Eq. (16), with the exception that the battery power appeared instead of the product $E_{batt} \cdot f(x,u)$. This term, in fact, has the physical dimension of a power, but the presence of the function $\dot{x} = f(x,u)$ accounts implicitly for the losses and nonlinearities in the battery, thus giving a quantity proportional the net variation of the state of charge (*electrochemical power*).

The original formulation of ECMS derived from impressive engineering intuition and was proved to work well, even without formal proof of optimality. The derivation from the minimum principle adds to the intuition in two ways: it provides a theoretical background and proof of optimality, and introduces a slight reformulation that makes the ECMS more effective and easy to tune. In fact, in the original ECMS formulation the equivalence factor represents the chain of efficiencies through which fuel is transformed into electrical power and vice-versa, and it changes for each operating condition of the powertrain. In Eq. (16), instead, the equivalence factor is an optimization variable that acts as a single tuning parameter, as it is going to be explained now.

The control obtained by minimization of the Hamiltonian function (16) depends, obviously, on the co-state variable $s(t)$, which is unknown, and whose initial condition $s(t_0)$ is free. On the other hand, the state of charge $x(t)$ must satisfy an initial and a final constraint, given by Eq. (13). Since the evolution of $x(t)$ depends on the value of $s(t)$, it is possible to find an appropriate initial value $s(t_0)$ of the equivalence factor, such that the final state value $x(t_f)$ reaches the prescribed condition. In other words, there is a value of $s(t_0)$ for which the solution is perfectly charge-sustaining [5, 9]. This is defined as the optimum equivalence factor for the driving cycle, and is effectively a tuning parameter for the strategy.

The optimal value of $s(t_0)$ depends on the driving cycle; this is obvious looking at the original, intuitive derivation of $s$ as the fuel-equivalent cost of battery discharge, because the amount of fuel needed to recharge the battery depends strongly on the amount of energy available for regeneration (or *free energy*, as defined in [13]).

The biggest problem from an implementation point of view is that the optimality of the ECMS solution is extremely sensitive to the value of the equivalence factor, which can be tuned appropriately only if the driving cycle is known a priori. In real-world conditions, this is not feasible because driving conditions are not repetitive nor completely predictable; therefore, some kind of

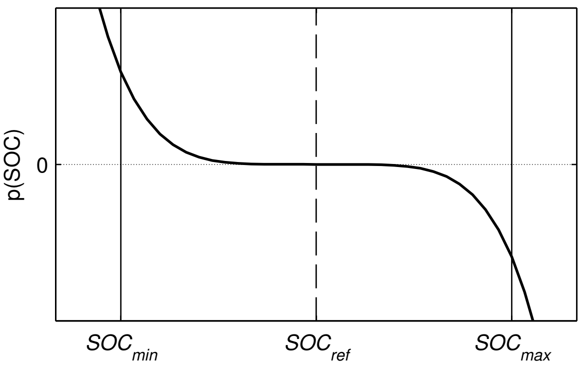

Figure 2. An example of ECMS penalty weight

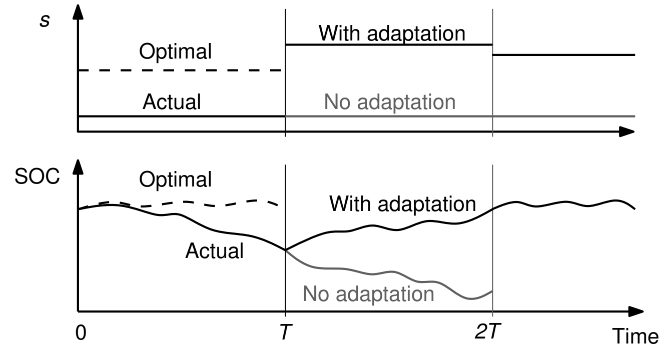

Figure 3. Adaptive ECMS concepts

online adaptation or auto-tuning of the equivalence factor is necessary to provide robustness and, ideally, to obtain results close to the optimum.

A method, proposed already in [2], consist to introduce a *penalty weight* $p(x)$ based on the current value of the state of charge. This correction function, an example of which is shown in Fig. 2, modifies the value of the equivalence factor when the SOC is close to its maximum or minimum acceptable levels, in order to avoid over-charging or over-discharging the battery. The cost of discharging the battery is increased at low SOC to prevent further discharge, and it is decreased at high SOC to encourage discharge. This method improves the robustness of the strategy, but does not actually adapt the base value of the equivalence factor (the value at nominal SOC). In other words, the long-term value of $s$ which remains a constant for all driving conditions, and therefore is not optimal for any of them, taking necessarily a compromise value.

## 4 ADAPTIVE ECMS

Adaptive ECMS strategies, in which the equivalence factor is changed during the driving cycle, were proposed by several authors. In [6], a Kalman filter was used to predict future values of vehicle speed, and the optimal equivalence factor was found by online optimization over a receding horizon; the method gave good results but was computationally demanding. In [7], instead, it was proposed to categorize each driving pattern in one of four categories: urban, suburban, extraurban, and highway. For each of these categories, an optimal equivalence factor is found a-priori; a pattern recognition algorithm then detects at each instant the category of the current driving conditions, and applies the appropriate equivalence factor. This provides a better baseline value of $s$, while the penalty function $p(x)$ is present to provide practical robustness, and also includes an integral term that corrects prolonged SOC deviations from the reference value. In [8] and [9], instead, a feedback controller is used, and the value of the equivalence factor is updated at each instant to account for the deviation of the state of charge from its reference value $x_0$. The idea behind this is that the instantaneous correction of the equivalence factor allows to counteract the deviation of SOC from its reference. In fact, the proportional part of this correction has the same effect as the penalty weight of Fig. 2, and is equivalent to a linear function $p(x)$.

The adaptive ECMS method proposed in this paper is also based on SOC feedback, which is useful to ensure robustness of the strategy; however, it is conceptually different from the previous work because the adaptation is not done continuously at each time step, but only at regular intervals of duration $T$. In fact, it is normal that the SOC deviates from the reference value during the vehicle operation, but the charge sustainability constraint requires that the value of SOC at the end of the cycle is equal to the reference value. Since in real-world conditions the duration of the driving cycle is not known a priori, the charge-sustainability condition is enforced on shorter time frames. The horizon during which there should be no SOC variation is chosen in such a way that at least two charge/discharge cycles are allowed, to let the vehicle make use of the available energy buffer.

The algorithm can be explained with the help of Fig. 3. During the first section of the cycle (0–T), an initial guess is made for the equivalence factor, which is obviously different from the optimal value (the one that would give charge-sustainability at the end of the section). If equivalence factor is too low, as in the example, the battery tends to be discharged, and it would be depleted if $s$ were kept at the same level (dashed lines). If, instead, at time $t = T$ the value of $s$ is modified to account for this fact, the trend can be reversed and the overall solution results charge-sustaining.

The adaptation law is implemented by setting a new value of equivalence factor every $T$ seconds:

$$s_{k+1} = s_k + c_P \cdot (x_0 - x(t)),\ t = k \cdot T,\ k = 1, 2, ... \quad (17)$$

where $s_{k+1}$ represents the new value of the equivalence factor, which will be used in the time interval $t \in [kT,\ (k+1)T]$; $s_k$ is the previous value, and $(x_0 - x(t))$ is the difference between the reference SOC $x_0$ and the actual value, at the instant of the adaptation. $c_P$ is the proportional gain of feedback controller and is a tuning parameter for the strategy.

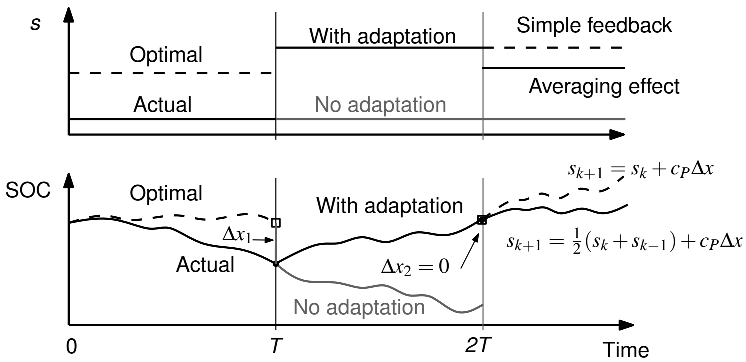


Figure 4. Adaptive ECMS concepts

This basic formulation is still imperfect. In fact, it is not correct to look at the SOC differences between two generic points separated by a fixed time $T$, because they may be two completely different driving conditions: for example, the SOC should be lower during acceleration transients and higher after regenerative braking; if the comparison in (17) is made between these two points, it leads to wrong behavior. The solution is to make $T$ "stretchable", i.e., apply the adaptation only when the vehicle has reached the same conditions at which $s$ was previous adapted (for example, at vehicle stopped). This can work if there is the certainty that the same conditions will be verified very often, implying a limited stretching of $T$. Since this strategy has been developed for an urban bus, it is very reasonable to wait for the vehicle to stop before adapting $s$.

Furthermore, to improve the stability of the solution, the adaptation law implemented is modified as:

$$s_{k+1} = \frac{1}{2}(s_k + s_{k-1}) + c_P \cdot (x_0 - x(t))). \qquad (18)$$

The reason of this modification comes from an engineering intuition and can be explained considering the situation in Fig. 4. When a first-tentative equivalence factor is used in the first adaptation interval ($k = 1$), the battery SOC decreases; therefore, at the beginning of the following interval ($k = 2$), the adaptation law (17) corrects the value of the equivalence factor by increasing it, thus generating a SOC profile with an increasing trend, which will need to be corrected in the opposite direction at the next interval. If the driving cycle were the same in each interval, and the correction (17) brought $\Delta x_2$ to zero at the end of interval 2, while $\Delta x_1 < 0$, then the optimal value of $s_0$ for the following section ($k = 3$) would be the average between the previous two. Eq. (17), instead, would not apply any correction and let $s_0$ remain the same. In order to improve the performance of the adaptation, then, the feedback law is modified as in (18) so that, if the SOC is increasing and then increasing in two successive intervals, the value of equivalence factor in the third interval will tend to generate a charge-sustaining solution.

Table 1. Main characteristics of the vehicle

| | |
|---|---|
| Vehicle mass | 1800 kg |
| Engine max. power | 100 kW |
| Motor max. power | 25 kW |
| Battery energy capacity | 1.5 kWh (5400 kJ) |

## 5 SIMULATION RESULTS

The adaptive ECMS strategy described in Section 4 is tested in simulation using a simple case study, a pre-transmission parallel HEV with the characteristics shown in Tab. 1. The vehicle is represented using a backward, quasi-static model, which accounts for the efficiency of the motor and the engine using their maps; the battery model is an electrical circuit composed of a voltage source and a resistance. For comparison, the optimal solution obtained with dynamic programming (DP) is also included. The DP solution was obtained using the code described in [14].

Three driving cycles are presented in the following sections, starting from an ideal case and then moving to more realistic examples.

### 5.1 Ideal conditions

If the driving cycle is composed of repetitions of identical sections and the adaptation period $T$ is chosen to coincide with the duration of the sections, the adaptation strategy (18) converges to the optimal value of $s$ for the section. As an example, consider the short section shown in Fig. 5, of duration $T = 120$ s. Several curves are shown. The charge-sustaining solution is obtained by applying DP or the *optimal ECMS*, which is the ECMS with constant equivalence factor, tuned for the specific cycle using an iterative method, which is only possible offline. The optimal ECMS and DP are essentially coincident, which means that the ECMS would be capable of providing the optimal solution, if optimally tuned. Both these strategies, however, rely on complete knowledge of the driving cycle. The non charge-sustaining solutions, instead, are generated by ECMS with equivalence factors different than the optimal value $s_{opt}$. In particular, the solution is charge-increasing solution if $s > s_{opt}$, and charge-depleting if $s < s_{opt}$.

To demonstrate the behavior of proposed adaptation strategy, the same section is repeated 20 times. The initial value of $s$ is set to one of the "wrong" values of Fig. 5, but then it is corrected every $T$ seconds by applying the adaptation law (18). $T$ is equal to the section duration. The SOC value and the evolution of the equivalence factor are shown in Fig. 6, demonstrating the convergence towards the optimal value.

### 5.2 Real-world driving cycle, uniform

In real-world cases, the driving cycle sections are not identical to each other. However, it can be assumed that the differences between subsequent sections are not too large, i.e. that the

 

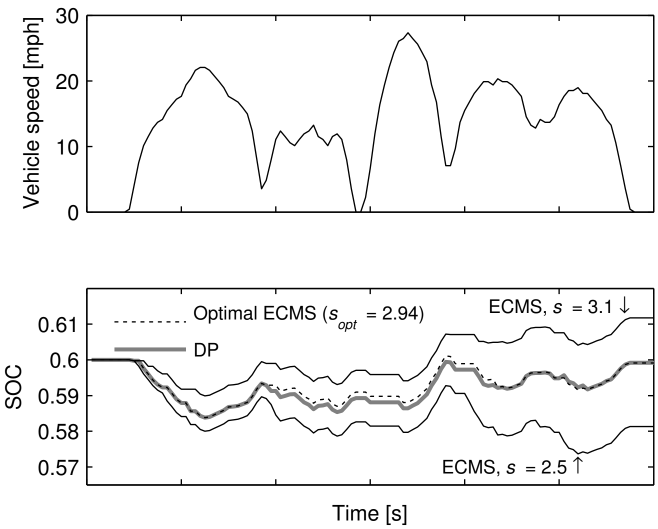


Figure 5. Behavior of ECMS with constant equivalence factor during a short section of driving cycle

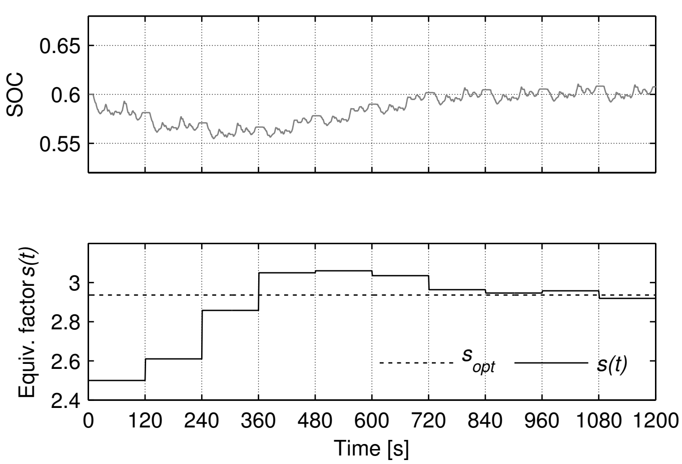


Figure 6. Adaptive ECMS applied to 10 repetitions of the driving cycle section shown in Fig. 5.

optimal equivalence factor does not change substantially from on section to the next, except for some isolated events. Radical changes of driving pattern do, in fact, occur in real-world: for example, the vehicle can move from urban areas to the highway. These isolated changes typically separate two relatively large and uniform portions. The scope of the adaptation is react to these changes and converge, within a few adaptation periods, to the optimum value for the current driving conditions. In this section, simulation results relative to a long, uniform urban cycle are presented. The cycle is not the repetition of identical sections, but it presents similar characteristics for its entire duration. The results are shown in Fig. 7, which shows the A-ECMS solution compared to the optimal reference given by DP (or the optimally tuned ECMS). Unlike the other two strategies, The A-ECMS has no a-priori knowledge of the driving cycle, and only relies on the SOC feedback implemented with Eq. (18). Despite this, it maintains the SOC in the acceptable range and remains close to the optimal solution.

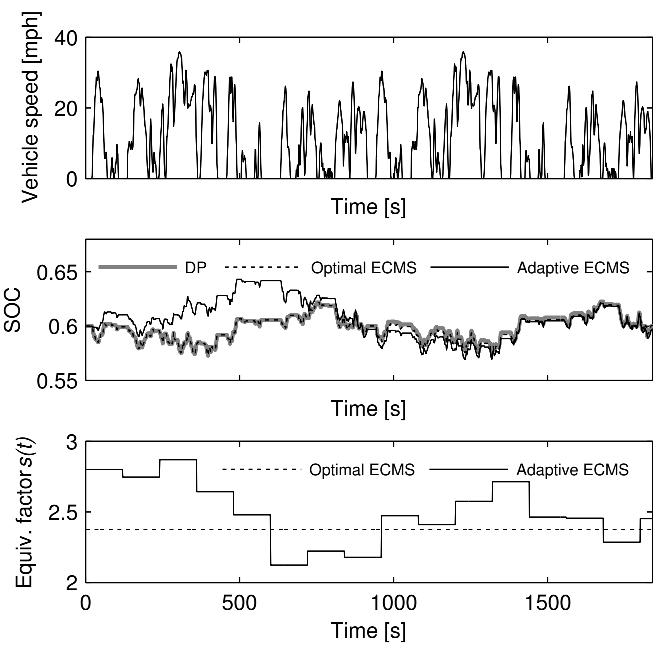


Figure 7. Comparison of the A-ECMS with the optimal solution, for an urban driving cycle. Top: speed profile; middle: SOC; bottom: equivalence factor

### 5.3 Real-world driving cycle, non-uniform

To test the ability of the proposed strategy to deal with varying driving conditions, a test composed by urban, highway and suburban conditions is used. The results are shown in Fig. 8, and demonstrate that the adaptive strategy behaves correctly in this case as well.

A comparison of fuel consumption values for the two cycles shown is reported in Tab. 2.

## 6 CONCLUSION

This paper has presented a new energy management strategy for HEVs based on the on-line adaptation of the equivalence factor used by the ECMS. The adaptation is performed via feedback of state of charge and it is implementable on-line at low computational burden. The solution proposed achieves results close to the optimal global solution obtained from dynamic programming on different driving cycles.

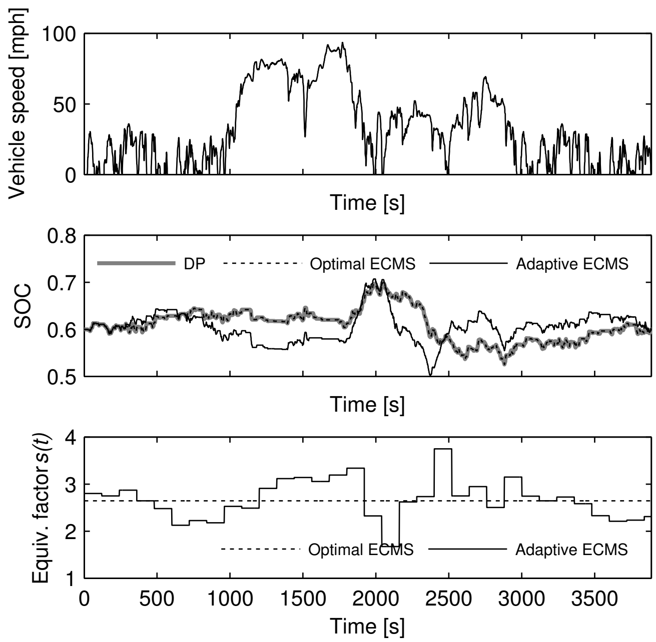


Figure 8. Comparison of the A-ECMS with the optimal solution, for a mixed driving cycle (urban-highway-suburban). Top: speed profile; middle: SOC; bottom: equivalence factor

Table 2. Comparison of fuel consumption. All values are corrected for final SOC variation and are normalized to the DP results

| Driving cycle | DP | optimal ECMS | A-ECMS |
|---|---|---|---|
| 2 Artemis urban | 100% | 100% | 101% |
| Urban - Highway - Suburb. | 100% | 100% | 101.5% |
| 3 FUDS | 100% | 100% | 101.3% |
| 3 NEDC | 100% | 100% | 107.1% |

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