2009 American Control Conference
Hyatt Regency Riverfront, St. Louis, MO, USA
June 10-12, 2009

ECMS as a realization of Pontryagin's minimum principle for HEV control

Lorenzo Serrao, Simona Onori, Giorgio Rizzoni

Abstract— **An analytical derivation of the Equivalent Consumption Minimization Strategy (ECMS) for energy management of hybrid electric vehicles (HEVs) is presented, based on Pontryagin's minimum principle. The derivation is obtained using a generic formulation of the energy management problem in HEVs and is valid for any powertrain architecture. Simulation results obtained for a series HEV are also provided.**

I. Introduction

Hybrid electric vehicles are characterized by the presence of two energy sources on board, the fuel tank powering the engine and a battery; at each instant, the sum of the power flows from each of the two sources equals the total power delivered at the vehicle wheels. Energy management in hybrid vehicles [1] consists in deciding the power repartition between the two sources, with the objective to minimize the fuel consumption, pollutant emissions, or a compromise among them. The energy management of hybrid vehicles is an optimal control problem, in which the minimization objective is defined by an integral over an extended period of time (typically, a regulatory driving cycle). However, it poses some specific challenges, related to the fact that the driving cycle is not known in advance.

If the cycle were known, anlytical optimal control techniques [2], [3], [1], [4] or Dynamic Programming [5], [6] could be used to find the optimal solution to the problem; however, the solution obtained in this way cannot be implemented in real-time, and can only be used as a benchmark for other implementable strategies. These include rule-based control [7], [8], model predictive control[9], [10], stochastic dynamic programming [11], [12], and the equivalent consumption minimization strategy (ECMS) [13], [14]. The basic idea behind ECMS is to associate the use of the electrical energy buffer to a future increase or decrease of fuel consumption, and to minimize at each instant the equivalent fuel consumption, i.e. the sum of the actual fuel consumption and of virtual fuel consumption associated to the use of the electrical energy. The main objective of this paper is to show the analytical optimality of the ECMS, based on Pontryagin's minimum principle.

II. The Optimal Control Problem in Hybrid Electric Vehicles

A generic representation of a hybrid electric powertrain is given in Fig. 1. In this figure, only the primary power flows are shown, i.e. those that correspond to energy moving from a storage device into the driveline, or vice-versa (when possible). The schematics is independent from the powertrain architecture (i.e., series, parallel, series/parallel) and is valid for all vehicles in which the fuel tank and a reversible energy storage device (such as a battery) are present. The details of the powertrain architecture are essential to determine how the chemical energy stored in the fuel or the electro-chemical energy stored in the battery are transformed into electrical or mechanical energy to drive the vehicle, but in this work the point of view is more general and these details are not considered.

All the authors are with the Center for Automotive Research at the Ohio State University, Columbus, OH. Lorenzo Serrao (serrao.4@osu.edu) is the corresponding author.

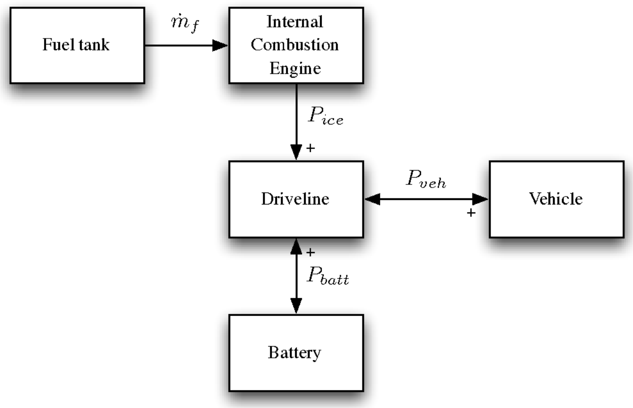

Fig. 1. Main energy flows in a hybrid electric powertrain

Given a prescribed driving cycle defined by the velocity history $v(t),\ t \in [t_0, t_f]$, it is possible to determine the corresponding power $P_{veh}(t)$ that is necessary to follow the cycle as

$$P_{veh} = \left(m\dot{v}(t) + r_0 + r_1 v(t) + r_2 v^2(t) + mgsin\gamma(t)\right) v(t) \quad (1)$$

where: m is the vehicle mass; r_0, r_1, r_2 are constant coefficients that model rolling resistance and aerodynamic drag; g is the acceleration of gravity and $\gamma(t)$ the road slope angle. The history of $\gamma(t)$ is another driving cycle parameter.

The objective of the energy management strategy in hybrid electric vehicles is to determine how $P_{veh}(t)$ is shared between the on-board energy transformers (engine, batteries, electric machines) in order to minimize an integral cost defined over the entire optimization horizon $[t_0, t_f]$. Typically, the integral cost is the fuel consumption over the driving cycle; in some instances, a weighted average of fuel consumption and pollutant emissions can also be minimized. In general, then, the cost function to be minimized can be expressed as:

978-1-4244-4524-0/09/$25.00 ©2009 AACC

$$J = \int_{t_0}^{t_f} \left[\dot{m}_f + \alpha_{NOx}\dot{m}_{NOx} + \alpha_{PM}\dot{m}_{PM} + \alpha_{HC}\dot{m}_{HC}\right] dt$$

where the α factors are weighting coefficients, $\dot{m}_f$ is the fuel mass flow rate (instantaneous fuel consumption), $\dot{m}_{NOx}$ is the rate of nitrogen oxide emission, $\dot{m}_{PM}$ is the rate of particulate matter emission, and $\dot{m}_{HC}$ is the rate of unburned hydrocarbon emission. All these quantities depend on the engine operating conditions and can be modeled (at least in a first-approximation approach) as static functions of engine power and speed:

$$\dot{m}_i = \dot{m}_i(P_{ice}, \omega_{ice}),\ i = f, NOx, PM, HC$$

The control action is the repartition of the load, and can be defined in different ways, depending on the specific powertrain architecture taken into consideration and on the number of degrees of freedom. The choice of the control, instant by instant, must be such that the following constraints are satisfied:

$$0 \le P_{ice}(t) \le P_{ice,max}\ \forall t \in [t_0, t_f] \quad (2)$$

$$P_{em,e,min} \le P_{em,e}(t) \le P_{em,e,max}\ \forall t \in [t_0, t_f] \quad (3)$$

$$P_{batt,min} \le P_{batt}(t) \le P_{batt,max}\ \forall t \in [t_0, t_f] \quad (4)$$

$$SOE_{min} \le SOE(t) \le SOE_{max}\ \forall t \in [t_0, t_f] \quad (5)$$

where P_{ice} is the engine mechanical power, $P_{em,e}$ is the electric machine electrical power (there can be more than one electric machine, in which case there is a constraint equation for each), P_{batt} is the battery power, SOE is the battery state of energy, defined as the ratio of the energy currently stored in the battery to the maximum amount of energy that can be stored:

$$SOE = \frac{E_{batt}(t)}{E_{max}}$$

The subscripts max and min refer to the maximum and minimum limits of each variable.

This article focuses on single degree of freedom systems, either series or parallel, in which the battery power can be used as the only control variable for the power split problem. In more complex architectures, such as the systems using EVTs (electrically variable transmissions), the battery power is one of two (or possibly more) control variables.

The state of the system is represented by the battery state of energy. Its derivative is a function of the battery electrical power P_{batt}:

$$\dot{SOE} = \begin{cases} -\frac{P_{batt}(t)}{\eta_{batt}(SOE, P_{batt})E_{max}} & \text{if } P_{batt}(t) \ge 0 \\ -\frac{\eta_{batt}(SOE, P_{batt})P_{batt}(t)}{E_{max}} & \text{if } P_{batt}(t) < 0 \end{cases} \quad (6)$$

$P_{batt}(t) \ge 0$ indicates discharging and $P_{batt}(t) < 0$ indicates charging.

Indicating the battery power with u and the battery state of energy with x, the system dynamic equation can be written as

$$\dot{x} = \begin{cases} -\frac{u(t)}{\eta_{batt}(x,u)E_{max}} & \text{if } u(t) \ge 0 \text{ (discharge)} \\ -\frac{\eta_{batt}(x,u)u(t)}{E_{max}} & \text{if } u(t) < 0 \text{ (charge)} \end{cases} \quad (7)$$

If only the fuel consumption is considered, the cost function is:

$$J = \int_{t_0}^{t_f} \dot{m}_f(u(t), t)dt \quad (8)$$

Note that $\dot{m}_f$ is an explicit function of the time because it depends on the driving cycle, which can be considered as a time-varying parameter in this context. $\dot{m}_f$ is not an explicit function of the state of the system SOE, but depends only on the battery power and the vehicle power demand.

Additional algebraic equations that depend on the specific powertrain architecture relate the engine power P_{ice}, the electric machine power P_{em} and the battery power P_{batt}, and create new constraints for the control variable in addition to (4). The overall constraints on the control are synthetically represented as

$$u(t) \in \mathcal{U}(t) \quad (9)$$

where $\mathcal{U}(t)$ is the set of admissible controls.

The constraints on the state of energy of the battery can be expressed using the auxiliary function $G(x) = [G_1(x)\ G_2(x)]^T$ as:

$$G(x) = \begin{cases} G_1(x) = (x(t) - x_{max}) \le 0 \\ G_2(x) = (x_{min} - x(t)) \le 0 \end{cases} \quad (10)$$

The terminal conditions are given by the state of energy at the initial and final time, as:

$$\begin{cases} x(t_0) = x_0 \\ x(t_f) = x_f \end{cases} \quad (11)$$

In general, in charge-sustaining hybrid vehicles, the initial and final state of energy should be the same, hence $x_f = x_0$. However, it is possible to specify that the final state of energy is within a (small) distance from the initial value: $x(t_f) \in S = x_0 \pm \delta$.

The energy management problem can be formally defined as follows.

Problem 1: Find the control law $u(t)$ that minimizes the cost

$$J = \int_{t_0}^{t_f} \dot{m}_f(u(t), t)dt$$

subject to constraints

$$\dot{x} = \begin{cases} -\frac{u(t)}{\eta_{batt}(x,u)E_{max}} & \text{if } u(t) \ge 0 \text{ (discharge)} \\ -\frac{\eta_{batt}(x,u)u(t)}{E_{max}} & \text{if } u(t) < 0 \text{ (charge)} \end{cases}$$

$$\begin{cases} x(t_0) = x_0 \\ x(t_f) \in S \end{cases}$$

$$G(x) = \begin{cases} G_1(x) = (x(t) - x_{max}) \leq 0 \\ G_2(x) = (x_{min} - x(t)) \leq 0 \end{cases}$$

$$u(t) \in \mathcal{U}(t)$$

III. Application of Pontryagin's minimum principle

Pontryagin's minimum principle (PMP) provides a set of necessary conditions that the optimal solution $u^*(t)$ must satisfy. Since the minimum principle provides necessary conditions for optimality, it can be used to identify solution candidates. Because of the constraints on the state variable, the formulation of the principle depends on whether the state constraints are active (i.e., the state assumes a boundary value) or not (i.e., it changes during the optimization horizon). The formulation of the minimum principle used here is provided in [15, §2.5].

Assume that the state constraints are active only in a subset $B = [t_1, t_2] \cup [t_3, t_4] \cup ... \subseteq [t_0, t_f]$ of the optimization interval $[t_0, t_f]$. In order to introduce these constraints in the formulation of the principle, the total time derivatives of G_1 and G_2 are used, up to the order ℓ in which u appears explicitly for the first time, which in this case is $\ell = 1$, since $\dot{x} = f(x, u)$ according to (7):

$$\begin{cases} G_1^{(1)}(x, u) = \frac{dG_1}{dt} = \dot{x}(t) = f(x, u) \\ G_2^{(1)}(x, u) = \frac{dG_2}{dt} = -\dot{x}(t) = -f(x, u) \end{cases} \tag{12}$$

Define the Hamiltonian function of the system as:

$$H(x, u, t) = \begin{cases} \dot{m}_f(t, u) + \lambda f(x, u) & t \notin B \\ \dot{m}_f(t, u) + \lambda f(x, u) + \mu_1 G^{(1)}(x, u) & t \in B \end{cases} \tag{13}$$

with $\mu_1 \geq 0$. In other words, the Hamiltonian function during the intervals in which the constraints are active is augmented by a term that depends on the derivative of the constraint function. The term μ_1 is unknown a-priori, the principle only states that it exists; it can be found by trial-and-error.

The minimum principle for the HEV power split problem can be formulated as follows [15].

Theorem 1: If $u^*(t)$ is the optimal control for Problem 1, and $x^*(t)$ is the corresponding state trajectory, then the following conditions are satisfied for all $t \in [t_0, t_f]$:

1) u^* minimizes the Hamiltonian H for all $t \in [t_0, t_f]$;
2) $G(x) < 0 \ \forall t \notin B$ and $G(x) = 0 \ \forall t \in B$ (i.e., $x = x_{max}$ or $x = x_{min} \ \forall t \in B$); moreover, for all $t \in B$, $G^{(1)}(x, u) = 0$, that is, $\dot{x}^*(t) = 0 \ \forall t \in B$ (this follows from (12)).
3) The terminal conditions are satisfied: $x^*(t_0) = x_0$ and $x^*(t_f) = x_0$.
4) The optimal co-state trajectory $\lambda^*(t)$ satisfies the following dynamic equations:

$$\dot{\lambda}^*(t) = -\nabla_x \left. H \right|_* = -\frac{\partial \dot{m}_f(t, u)}{\partial x} - \lambda^* \frac{\partial f(x^*, u^*)}{\partial x}$$

for $t \notin B$, and:

$$\dot{\lambda}^*(t) = -\frac{\partial \dot{m}_f(t, u)}{\partial x} - \lambda^* \frac{\partial f(x^*, u^*)}{\partial x} - \mu_1 \nabla_x G^{(1)}(x, u) \tag{14}$$

for $t \in B$.

The co-state equations can be simplified by considering that $\dot{m}_f$ is not an explicit function of the state of energy x; using the expression of $f(x, u)$ given in (7), they become:

$$\begin{cases} \dot{\lambda}^*(t) = \frac{\lambda^* u(t)}{\eta_{batt}^2(x,u) E_{max}} \frac{\partial \eta_{batt}(x,u)}{\partial x} & \text{if } u(t) \geq 0 \\ \dot{\lambda}^*(t) = \frac{\lambda^* u(t)}{E_{max}} \frac{\partial \eta_{batt}(x,u)}{\partial x} & \text{if } u(t) < 0 \end{cases} \tag{15}$$

for $t \notin B$ and, considering that $\nabla_x G^{(1)} = \pm \frac{\partial f(x,u)}{\partial x}$,

$$\begin{cases} \dot{\lambda}^*(t) = (\lambda^*(t) \pm \mu_1) \frac{u(t)}{\eta_{batt}^2(x,u) E_{max}} \frac{\partial \eta_{batt}(x,u)}{\partial x} & \text{if } u(t) \geq 0 \\ \dot{\lambda}^*(t) = (\lambda^*(t) \pm \mu_1) \frac{u(t)}{E_{max}} \frac{\partial \eta_{batt}(x,u)}{\partial x} & \text{if } u(t) < 0 \end{cases} \tag{16}$$

for $t \in B$.

The sign in front of μ_1 depends on which of the constraints (10) is active: it is negative if $x(t) \geq x_{max}$ and positive if $x(t) \leq x_{min}$).

IV. Derivation of the Equivalent Consumption Minimization Strategy

For a more compact notation in the following, define the auxiliary function[1]:

$$p(x) = \begin{cases} \mu_1 & \text{if } G_1(x) \geq 0 \\ -\mu_1 & \text{if } G_2(x) \geq 0 \\ 0 & \text{if } G_1(x) < 0 \text{ and } G_2(x) < 0 \end{cases} \tag{17}$$

Replacing the expression of the dynamic equation (7) into the Hamiltonian function H, this becomes

$$H(x, u, t) = \\ = \begin{cases} \dot{m}_f(t, u(t)) - (\lambda(t) + p(x)) \frac{u(t)}{\eta_{batt}(x,u) E_{max}} & \text{if } u(t) \geq 0 \\ \dot{m}_f(t, u(t)) - (\lambda(t) + p(x)) \frac{\eta_{batt}(x,u) u(t)}{E_{max}} & \text{if } u(t) < 0 \end{cases}$$

Let us now define a charge and discharge equivalence factors as

$$s_{dis}(t) = -\frac{Q_{lhv}}{\eta_{batt} E_{max}} \lambda(t) \tag{18}$$

and

$$s_{ch}(t) = -\eta_{batt} \frac{Q_{lhv}}{E_{max}} \lambda(t) = \eta_{batt}^2 s_{dis}(t) \tag{19}$$

where Q_{lhv} is the lower heating value of the fuel (representing its energy content per unit of mass, or power content per unit of mass flow rate).

Using the definitions (17), (18), and (19), the Hamiltonian of the system H can be represented as:

[1] Note that, in principle, $G_1(x)$ and $G_2(x)$ cannot be positive, only negative or zero; the definition of $p(x)$ reported here is complete with the case in which the boundaries are not only reached, but also exceeded, which could happen if the constant μ_1 is not chosen properly

$$H(t,x(t),u(t)) = \begin{cases} \dot{m}_f(t,u) + s_{dis}(t)\left(1+k(t)p(x)\right)\frac{u(t)}{Q_{lhv}} & \text{if } u(t)\geq 0 \\ \dot{m}_f(t,u) + s_{ch}(t)\left(1+k(t)p(x)\right)\frac{u(t)}{Q_{lhv}} & \text{if } u(t)<0 \end{cases} \quad (20)$$

where the term $u(t)/Q_{lhv}$ has units of power/heating value and therefore represents a mass flow rate, and $s_{dis}(t)$ and $s_{ch}(t)$ are a-dimensional equivalence factors. The factor $(1+k(t)p(x))$ is a penalty function that takes a value different than 1 only when the state boundaries are reached.

The Hamiltonian can thus be interpreted as the sum of the actual fuel consumption in the engine, $\dot{m}_f$, and of a term that has the same units and is related to the use of the battery power $u(t)$. This additional term represents the virtual fuel consumption associated to the battery use, and, as intuitively explained in the first papers on ECMS [16], [13], it is related to the future fuel consumption due to the use of the battery at the present time. In particular, using the battery to supply part of the power demand means that the present fuel consumption becomes lower, but an additional fuel quantity will be needed later to recharge the battery. Vice versa, using the engine to charge the battery increases the instantaneous fuel consumption but will allow for future savings when the stored energy is used.

Therefore, the Hamiltonian of the system, object of instantaneous minimization, can be regarded as an instantaneous equivalent fuel consumption: $H = \dot{m}_f + \dot{m}_{elec} = \dot{m}_{eqv}$. Thus, as an implication of the Pontryagin's principle, the optimal solution to the problem of minimizing the total fuel consumption over a driving cycle must also minimize the instantaneous equivalent fuel consumption, defined using the opportune equivalence factors.

V. Implementation Example

A. Model description

A control strategy obtained by solving Problem 1 with Pontryagin's minimum principle is implemented in simulation on a series hybrid electric vehicle configuration, represented in Fig. 2. A quasi-static, backward representation of the vehicle longitudinal dynamics is considered in the simulation model [17]. The driving cycle is therefore considered an input to the simulator and is assumed to be met, while the torque/power/speed of each machine are calculated backwards.

The mechanical power of the traction motor is the same as the vehicle power (1): $P_{em,m} = P_{veh}$.

The corresponding electrical power is derived using the efficiency map of the motor:

$$P_{em,e} = \begin{cases} \frac{P_{em,m}}{\eta_{em}(v(t),P_{em,m})} & \text{if } P_{em,m}\geq 0 \\ \eta_{em}(v(t),P_{em,m})P_{em,m} & \text{if } P_{em,m}<0 \end{cases}$$

and must be satisfied by the sum of the battery power and the generator power:

$$P_{batt} + P_{gen} = P_{em,e}$$

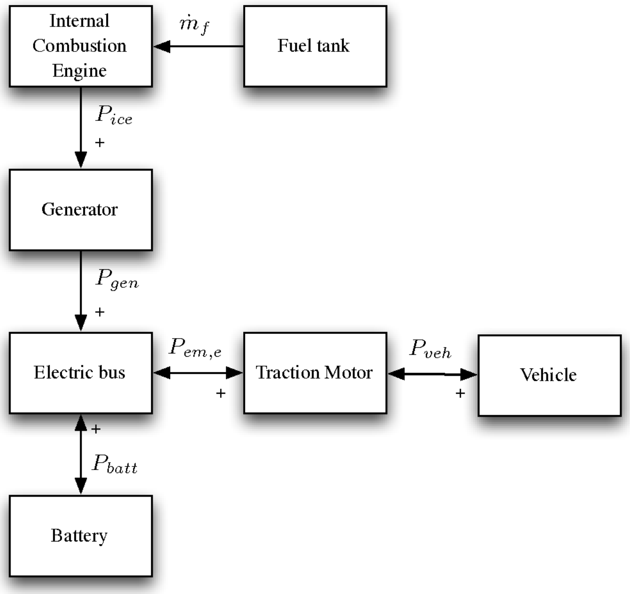

Fig. 2. Series hybrid electric powertrain

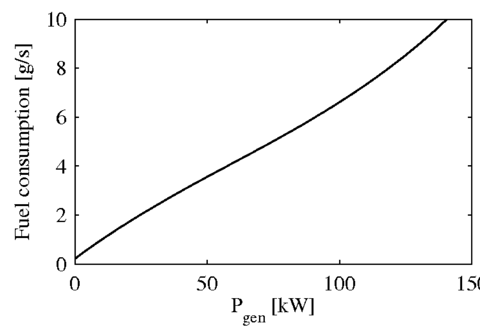

Fig. 3. Fuel consumption map of the engine and generator combined

Therefore, the fuel consumption can be expressed as

$$\dot{m}_f = \dot{m}_f(P_{gen}) = \dot{m}_f(P_{em,e}(t) - u(t)) \quad (21)$$

and can be represented using the efficiency maps of the engine and the generator as shown in Fig. 3 (this map implies the fact that the engine speed is chosen automatically as the speed that minimizes the fuel consumption for each level of electrical power P_{gen}, which is a typical approach used in series hybrid vehicles).

The battery model is given by the state of energy dynamic equation (6):

$$\dot{x} = \begin{cases} -\frac{u(t)}{\eta_{batt}E_{max}} & \text{if } u(t)\geq 0 \text{ (discharge)} \\ -\frac{\eta_{batt}u(t)}{E_{max}} & \text{if } u(t)<0 \text{ (charge)} \end{cases}$$

assuming a constant battery efficiency η_{batt}. The fact that the battery efficiency is constant simplifies the numerical solution of the optimization problem because it implies $\dot{\lambda}^*(t) = 0$ for all t, i.e. $\lambda(t) = \lambda_0 \; \forall t \in [t_0, t_f]$.

B. Pontryagin's minimum principle implementation

In order to identify a solution candidate, the conditions listed in Theorem 1 must be satisfied. This implies that the dynamic equation of the system (7) and the co-state equations (15)-(16) need to be solved together, while imposing the condition that $u(t) = \arg\min H(x, u, t)$.

In general, the solution of the two point boundary value problem is carried out numerically, using an iterative procedure in which the free initial (in this case, constant) value of the co-state variable λ_0 is set to an arbitrary value and the equations are solved numerically, obtaining a corresponding value of the final state $x^*(t_f)$. The value of λ_0 is then changed (using an iterative procedure such as the variation of extremals algorithm [18]) until the terminal condition on the state is met, i.e. until $|x^*(t_f) - x_0| < \delta$ (where δ is the acceptable tolerance on the terminal condition).

In every iteration, the control is implemented using the following steps at each time instant:

1) Given the present power demand, determine the maximum and minimum battery power $u_{min}(t)$ and $u_{max}(t)$:

$$\begin{cases} u_{max}(t) = \min\{P_{em,e}(t) - P_{gen,min}, P_{batt,max}\} \\ u_{min}(t) = \max\{P_{em,e}(t) - P_{gen,max}, P_{batt,min}\} \end{cases} \tag{22}$$

2) Divide the interval $[u_{min}(t), u_{max}(t)]$ into a finite number of control candidates $u_i,\ i = 1, 2, \ldots, N_u$;
3) Using the definition (20), calculate the Hamiltonian function corresponding to each control candidate: $\hat{H}_i = \hat{H}(u_i)$;
4) Select the control that minimizes the equivalent fuel consumption: $u^* = \arg\min_u(\hat{H}(u))$
5) Apply the control, computing the successive value of the state and co-state variables using the corresponding dynamic equations; take into account the state boundaries and apply the necessary jumps to the value of the co-state if the boundaries are reached.

After running these steps for the entire driving cycle, the terminal value of the state of energy $x^*(t_f)$ is computed; if this is close enough to the desired value (i.e. the initial value x_0), the iterations are terminated; otherwise, a new initial value for the co-state is selected and the procedure repeated. The value of the tuning parameter μ_1 is set to 1 at all time (the solution is not so sensitive to the value of μ_1 as it is to λ_0; any value of μ_1 high enough gives the same results, but if it is too small, the solution does not respect the state boundaries).

C. Traditional ECMS implementation

The traditional ECMS strategy differs from the implementation of the minimum principle because it uses the approximation that the equivalence factors are constant during the cycle and neglects their jumps in the instants at which the state boundaries are reached (in other words, the term $p(x)$ is always zero). Because of this, in order to keep the state of energy within its desired values, the maximum and minimum battery power defined by (22) are forced to zero whenever the corresponding limit of state of energy is being reached (i.e., the maximum, or discharge, battery power is zero if the state of energy is at its lowest boundary value; viceversa the minimum, or charge, power is zero if the battery is completely charged). The ECMS algorithm minimizes the equivalent fuel consumption defined by:

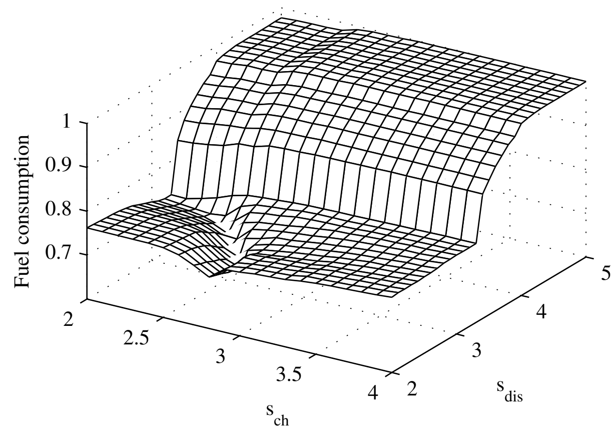

Fig. 4. Effect of equivalence factors on fuel consumption (values corresponding to the cycle of Fig. 5, normalized with respect to the maximum).

$$\dot{m}_{eqv} = \begin{cases} \dot{m}_f(t, u(t)) - s_{dis}\frac{u(t)}{Q_{lhv}} & \text{if } u(t) \geq 0 \\ \dot{m}_f(t, u(t)) - s_{ch}\frac{u(t)}{Q_{lhv}} & \text{if } u(t) < 0 \end{cases}$$

which is minimized considering a set of discrete values at each time, within the minimum and maximum admissible control, as described for the Hamiltonian function.

The charge and discharge equivalence factors are not known a priori; their value influences the results of the strategy and the best solution in terms of fuel consumption is only obtained with a specific couple of values, which are found by trial-and-error or numerical optimization. For example, the fuel consumption during the driving cycle of Fig. 5 corresponding to a range of values for s_{ch} and s_{dis} is shown in Fig. 4.

D. Comparison between Pontryagin's minimum principle and ECMS implementation

As noted earlier, the two approaches are equivalent, but the solution using conventional ECMS implements a more straightforward algorithm.

Simulation results obtained after choosing the most appropriate values of the equivalence factors (for ECMS) and initial co-state value (for Pontryagin's minimum principle) are shown in the figures 5 and 6.

VI. Conclusion

In this paper we presented a formal statement of the equivalence of Pontryagin's Minimum Principle and the Equivalent Consumption Minimization Strategy. In both cases, the solution reduces the global optimization problem

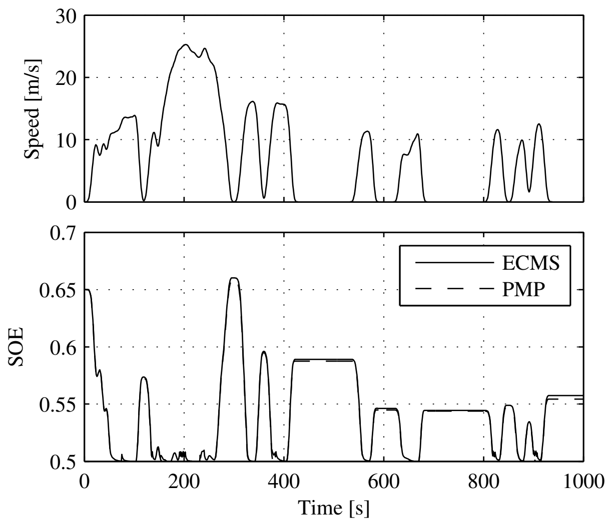

Fig. 5. Velocity profile and battery state of energy

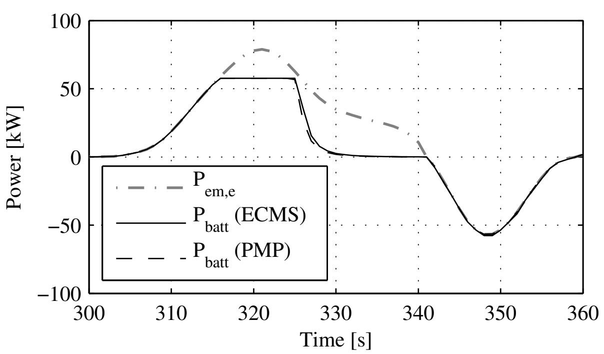

Fig. 6. Power split during the simulation (detail of 60 s)

to an instantaneous optimization problem. While the solution of Pontryagin's minimum principle needs, in principle, the knowledge of the driving cycle, the implementation of the ECMS does not, and therefore can be achieved on-line, under proper calibration of its parameters (which must be done off-line). If the assumption of a convex functional cost holds, the analytical solution given by the minimum principle is not only necessary but also sufficient and therefore, a global and unique solution to the optimization problem is guaranteed. Moreover, due to the equivalence of the minimum principle and the ECMS, the global optimal solution can be found by directly implementing the ECMS.

REFERENCES

[1] A. Sciarretta and L. Guzzella, "Control of hybrid electric vehicles," *IEEE Control Systems Magazine*, pp. 60–70, April 2007.

[2] M. Anatone, R. Cipollone, and A. Sciarretta, "Control-oriented modeling and fuel optimal control of a series hybrid bus," *SAE paper 2005-01-1163*, 2005.

[3] X. Wei, L. Guzzella, V. Utkin, and G. Rizzoni, "Model-based fuel optimal control of hybrid electric vehicle using variable structure control systems," *Journal of Dynamic Systems, Measurement, and Control*, vol. 129, p. 13, 2007.

[4] L. Serrao and G. Rizzoni, "Optimal control of power split for a hybrid electric refuse vehicle," *Proceedings of the 2008 American Control Conference*, 2008.

[5] A. Brahma, Y. Guezennec, and G. Rizzoni, "Optimal energy management in series hybrid electric vehicles," *Proceedings of the 2000 American Control Conference*, vol. 1, no. 6, pp. 60–64, 2000.

[6] C. Lin, H. Peng, J. Grizzle, and J. Kang, "Power management strategy for a parallel hybrid electric truck," *IEEE Transactions on Control Systems Technology*, vol. 11, no. 6, pp. 839–849, 2003.

[7] N. Schouten, M. Salman, and N. Kheir, "Fuzzy logic control for parallel hybrid vehicles," *IEEE Transactions on Control Systems Technology*, vol. 10, no. 3, pp. 460–468, 2002.

[8] J. Won and R. Langari, "Intelligent energy management agent for a parallel hybrid vehicle - part II: torque distribution, charge sustenance strategies, and performance results," *IEEE Transactions on Vehicular Technology*, vol. 54, no. 3, pp. 935–953, 2005.

[9] R. Beck, F. Richert, A. Bollig, D. Abel, S. Saenger, K. Neil, T. Scholt, and K. Noreikat, "Model predictive control of a parallel hybrid vehicle drivetrain," *Proceedings of the 44th IEEE Conference on Decision and Control, 2005 and 2005 European Control Conference (CDC-ECC'05)*, pp. 2670–2675, 2005.

[10] L. Johannesson, M. Åsbogård, and B. Egardt, "Assessing the potential of predictive control for hybrid vehicle powertrains using stochastic dynamic programming," *IEEE Transactions on Intelligent Transportation Systems*, vol. 8, no. 1, pp. 71–83, March 2007.

[11] C. Lin, H. Peng, and J. Grizzle, "A stochastic control strategy for hybrid electric vehicles," *American Control Conference*, 2004.

[12] E. Tate, J. Grizzle, and H. Peng, "Shortest Path Stochastic Control for Hybrid Electric Vehicles," *International Journal of Robust and Nonlinear Control*, vol. 18, no. 14, pp. 1409 – 1429, 2008.

[13] G. Paganelli, G. Ercole, A. Brahma, Y. Guezennec, and G. Rizzoni, "General supervisory control policy for the energy optimization of charge-sustaining hybrid electric vehicles," *JSAE Review*, vol. 22, no. 4, pp. 511–518, 2001.

[14] P. Pisu, C. J. Hubert, N. Dembski, G. Rizzoni, J. R. Josephson, J. Russell, and M. Carroll, "Modeling and design of heavy duty hybrid electric vehicles," *Proceedings of the 2005 ASME International Mechanical Engineering Congress and Exposition*, 2005.

[15] H. P. Geering, *Optimal Control with Engineering Applications*. Berlin Heidelberg: Springer, 2007.

[16] G. Paganelli, T. Guerra, S. Delprat, J. Santin, M. Delhom, and E. Combes, "Simulation and assessment of power control strategies for a parallel hybrid car," *Proceedings of the Institution of Mechanical Engineers, Part D: Journal of Automobile Engineering*, vol. 214, no. 7, pp. 705–717, 2000.

[17] L. Guzzella and A. Sciarretta, *Vehicle Propulsion Systems: Introduction to Modeling and Optimization*. Springer, 2007.

[18] D. Kirk, *Optimal control theory*. Prentice-Hall Englewood Cliffs, N. J, 1970.