Proceedings of the ASME 2009 Dynamic Systems and Control Conference
DSCC2009
October 12-14, 2009, Hollywood, California, USA

DSCC2009-2725

AUTOMOTIVE BATTERY PROGNOSTICS USING DUAL EXTENDED KALMAN FILTER

Matteo Rubagotti
Dept. of Computer Engineering
and Systems Science
University of Pavia
Email: matteo.rubagotti@unipv.it
(Visiting Scholar at the
Center for Automotive Research)

Simona Onori *
Center for Automotive Research
The Ohio State University
Columbus, OH
Email: onori.1@osu.edu

Giorgio Rizzoni
Center for Automotive Research and
Department of Mechanical Engineering
The Ohio State University
Columbus, OH
Email: rizzoni.1@osu.edu

ABSTRACT

This paper proposes a strategy for estimating the remaining useful life of automotive batteries based on dual Extended Kalman Filter. A nonlinear model of the battery is exploited for the on-line estimation of the State of Charge, and this information is used to evaluate the actual capacity and predict its future evolution, from which an estimate of the remaining useful life is obtained with suitable margins of uncertainty. Simulation results using experimental data from lead-acid batteries show the effectiveness of the approach.

Introduction

A lot of efforts have been done over the years in order to find efficient models of electrochemical batteries for automotive applications, with the main purpose of estimating the State of Charge (SoC), defined as the percentage of stored electric charge [1]. This information is often used in applications related to electric and hybrid-electric vehicles, where the knowledge of the SoC is crucial for the energy management optimization [2]. A lot of work has been done also in characterizing the aging phenomena in batteries, in order to estimate the so-called State of Health (SoH) of the system. This latter is defined as a variable that accounts for the level of damage accumulated by the battery, and is used to understand the remaining life span. Nonetheless, it is extremely difficult to give reliable models for the aging process: simple models may not take into account important phenomena and may give wrong predictions in real applications, while model based on physical-chemical understanding of the system can be too complicated for real-time automotive applications (for instance, requiring the measurement of internal variables). Examples of aging models of both PbA and Li-Ion batteries can be found in [3], [4], and [5].

In general, estimating the age of the battery can be very useful to avoid replacing the battery when it is still working well, as well as to be able to promptly substitute it when the battery reaches its end-of-life. Variables that can be related to the SoH are, for example, the internal resistance, which increases with the age, and the capacity (that is, the maximum amount of charge that can be stored in the battery) which decreases with age. While the internal resistance remains almost constant for a long time and then suddenly increases when the end-of-life is close, the capacity decreases in time with a more linear trend, which can be then directly related to the SoH. Of course, the capacity cannot be directly measured, but it is known that the variation of the SoC of the battery, given a certain input current, depends on the value of the capacity. The estimation of the SoC can then allow to properly estimate the capacity, hence, the battery SoH. Hence, it is possible to give an estimate of the Remaining Useful Life (RUL) of the system, and eventually design a prognostics strategy. For an overview on prognostics the reader can refer to [6], while a recent approach to battery prognostics can be found in [7].

Exploiting this relationship between the state-of-charge (the value of which can be estimated very precisely, according to recent papers([8], [9], [10], [11])) and capacity dynamics is the key to estimate the capacity itself, using a state observer.

In this paper, a dual observer based on the Extended Kalman Filter (EKF) is used to estimate the SoC of a PbA battery and track its capacity evolution. Once the value of the capacity at

*Corresponding author.

the present moment is obtained, the EKF is used to predict, with suitable uncertainty margins, its future evolution. In this work the capacity is considered as the only damage variable related to the residual life of the system, hence, its future evolution is directly related to the estimation of the RUL.

The paper is organized as follows: Section 1 introduces the model of the system, and Section 2 describes the proposed prognostics strategy. Simulation results are shown in Section 3, while some conclusions are gathered in Section 4.

1 Battery modeling

In order to model the battery to estimate SoC and track capacity evolution, the system is described by equations that take into account both the fast dynamics (SoC estimation) and the slow dynamics (capacity estimation). These two variables have different time scales, the first being related to the charges and discharges of the battery which can take place in minutes or hours, and the second being related to the aging phenomena that take a long time to become evident. The 'fast dynamics' (see [10]) is modelled using a second order equivalent circuit, to which some hysteresis modeling is added. The hysteresis effect is due to the fact that following a discharge, the battery voltage relaxes to a value less than the nominal Open Circuit Voltage (OCV) for that SoC, while following a charge the battery voltage relaxes to a value that is larger than the nominal OCV for that SoC [12]. The 'fast dynamics' is given by

$$\begin{cases} \dot{V}_1(t) = -\frac{1}{R_1C_1}V_1(t) + \frac{1}{C_1}i(t) \\ \dot{V}_2(t) = -\frac{1}{R_1C_1}V_2(t) + \frac{1}{C_2}i(t) \\ \dot{h}(t) = -\left|\frac{\eta\gamma i(t)}{S(t)}\right| h(t) + M\left|\frac{\eta\gamma i(t)}{S(t)}\right| \\ \dot{z}(t) = -\frac{\eta}{S(t)}i(t) \end{cases} \tag{1}$$

where $z(t)$ is the SoC, $S(t)$ is the capacity, $i(t)$ is the input current, η is the battery efficiency, here assumed equal to 1 for simplicity, $V_1(t)$ and $V_2(t)$ are the voltages across the capacitances C_1 and C_2, respectively, of the equivalent circuit shown in Fig. 1. R_0, R_1, R_2, C_1 and C_2 are parameters (resistances and capacitances) of the circuit which are considered constant in this work. The OCV is obtained as a static function of the SoC. Considered that in the region of normal operation of a PbA battery (that is, $0.6 \leq z \leq 0.9$) this relation can be approximated as a linear one, it is assumed: $OCV = \alpha + \beta z$, where α and β are constant values obtained from experimental data. The term $h(t)$ represents the hysteresis effect on the measured external voltage, while γ and M are other parameters related to the hysteresis phenomenon: in particular, the value of M is considered constant in this work, but some refinements can be made, in order to represent it as a function of the SoC and its derivative [10]. The corresponding output equation is

$$V(t) = OCV(z(t)) - V_1(t) - V_2(t) + h(t) - R_0 i(t) \tag{2}$$

where $V(t)$ is the measured external voltage.

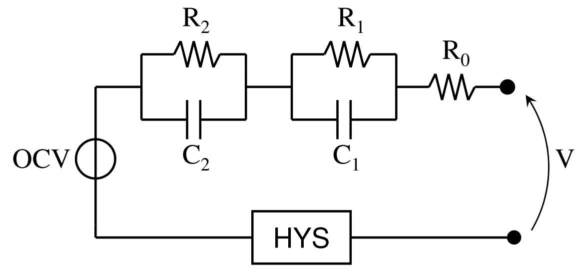

Figure 1. An equivalent circuit representation of the used model for SOC estimation

In order to approach the observer design, the observability properties of the system are investigated, [13]. The system state is expressed by the vector

$$x(t) = \begin{bmatrix} V_1(t) \; V_2(t) \; h(t) \; z(t) \end{bmatrix}^T$$

and the nonlinear equations (1)-(2) can be expressed as

$$\begin{aligned} \dot{x}(t) &= f(x(t), i(t)) \\ V(t) &= g(x(t), i(t)) \end{aligned}$$

The Lie derivatives of the output equation g with respect to f [13] are defined as

$$\begin{aligned} L_f^0 g &\triangleq g \\ L_f^1 g &\triangleq \nabla g \cdot f = \tfrac{\partial g}{\partial x} \cdot f \\ L_f^2 g &\triangleq \tfrac{\partial}{\partial x}(L_f^1 g) \\ L_f^3 g &\triangleq \tfrac{\partial}{\partial x}(L_f^2 g) \end{aligned}$$

which, by defining the vector $l(x,i) \triangleq \begin{bmatrix} L_f^0 g & L_f^1 g & L_f^2 g & L_f^3 g \end{bmatrix}^T$, can be written as

$$l(x,i) = \begin{bmatrix} -V_1 - V_2 + h + \alpha + \beta z \\ \frac{V_1}{R_1C_1} + \frac{V_2}{R_2C_2} - ah - \left(\frac{1}{C_1} + \frac{1}{C_2} + \frac{\eta\beta}{S}\right) i + Ma \\ -\frac{V_1}{(R_1C_1)^2} - \frac{V_2}{(R_2C_2)^2} + a^2 h + \left(\frac{1}{C_1} + \frac{1}{C_2}\right) i - Ma^2 \\ \frac{V_1}{(R_1C_1^3} + \frac{V_2}{(R_2C_2)^3} - ah^3 - \left(\frac{1}{C_1} + \frac{1}{C_2}\right) i + Ma^3 \end{bmatrix}$$

where $a \triangleq \left|\frac{\eta\gamma i}{S}\right|$. Now, the local observability of the system depends in general on the rank of the matrix

$$O \triangleq \left.\frac{\partial l(x,i)}{\partial x}\right|_{x=x_0,\; i=i_0}$$

where x_0 and i_0 are fixed values of the state and the input. In our case, it yields

$$O = \begin{bmatrix} -1 & -1 & 1 & \beta \\ \frac{1}{R_1C_1} & \frac{1}{R_2C_2} & -\left|\frac{\eta\gamma i_0}{S(t)}\right| & 0 \\ -\frac{1}{(R_1C_1)^2} & -\frac{1}{(R_2C_2)^2} & \left|\frac{\eta\gamma i_0}{S(t)}\right|^2 & 0 \\ \frac{1}{(R_1C_1)^3} & \frac{1}{(R_2C_2)^3} & -\left|\frac{\eta\gamma i_0}{S(t)}\right|^3 & 0 \end{bmatrix}$$

Note that, for our system, the observability does not depend on the state value, but only on the input and the parameters. Considering that $R_1C_1 \gg R_2C_2$, in practice there are two conditions that make the rank of the matrix O diminish:

1. if $\beta \sim 0$, that is the case when a change of SoC does not influence the output voltage V, and then it is impossible to track the value of the SoC;
2. if no current is given or drained from the battery, that is $i(t) \sim 0$.

The system is then locally observable everywhere under mild conditions.

As for the model of the 'slow dynamics', that is, the capacity dynamics, it is known that it is influenced by different factors (usually called 'severity factors'), which could be, for instance (see [14] and the references therein)

1. the State of Charge σ_1;
2. the C-rate[1] of the current σ_2;
3. the external temperature σ_3.

How to find a precise model of the aging process relying on these parameters is still an open issue. In this paper we formulate a general model, the focus being on the proposed strategy rather than on the aging modelling. The 'slow dynamics' can be expressed as

$$\dot{S}(t) = \varepsilon\phi\left(S, \sigma_1, \sigma_2, \sigma_3, ...\right) \quad (3)$$

where the presence of the term $\varepsilon \ll 1$ means that the dynamics of the capacity is much slower than the one of the other states.

2 Proposed strategy

The proposed scheme, depicted in Fig. 2, is a dual observer scheme, which consists in two EKF interacting with each other. The EKF has been proved to be a solution showing satisfying estimation performances in many applications, maintaining a relatively low computational burden, which usually makes its use possible in real-time automotive applications. As all observers, it uses both the model of the system and the measurement to obtain an estimation of the system state.

[1]The charge and discharge current of a battery is measured in C-rate. A discharge of 1C draws a current equal to the rated capacity.

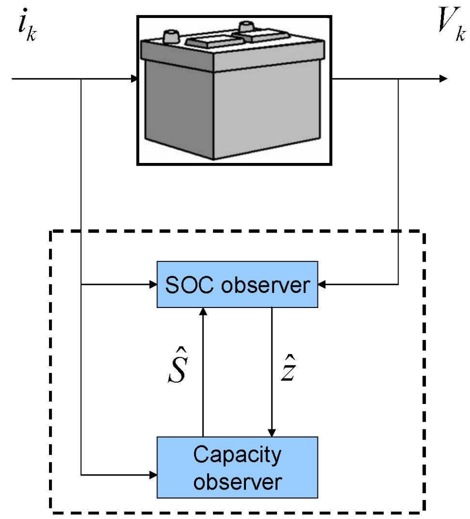

Figure 2. Scheme of the dual EFK

2.1 The SoC observer

In order to implement the proposed technique on a microprocessor, it is necessary to discretize the system, using a suitable sampling time T. Discretizing (1)-(2) using the forward Euler method one obtains

$$\begin{cases} V_{1_{k+1}} = V_{1_k} - \frac{T}{R_1C_1}V_{1_k} + \frac{T}{C_1}i_k \\ V_{2_{k+1}} = V_{2_k} - \frac{T}{R_2C_2}V_{2_k} + \frac{T}{C_2}i_k \\ h_{k+1} = F(i_k)h_k + M(1 - F(i_k)) \\ z_{k+1} = z_k - \frac{\eta T}{S_k}i_k \end{cases} \quad (4)$$

$$V_k = OCV(z_k) - V_{1_k} - V_{2_k} + h_k - R_0 i_k \quad (5)$$

with $F(i_k) \triangleq e^{-\left|\frac{\eta_k i_k \gamma T}{S_k}\right|}$.

The SoC observer uses the measurements of the input current i_k and the output voltage V_k, and the model of the battery. The estimated value of the capacity $\hat{S}_k$ is considered like a measurement coming from the capacity observer (as discussed later). This part of the estimation scheme is similar to the ones used in works on SoC estimation (see for instance [8] and the references therein). The state of system (4) can be expressed as follows

$$x_k = \left[V_{1_k}\ V_{2_k}\ h_k\ z_k\right]^T$$

while the state of the SoC observer is expressed as

$$\hat{x}_k = \left[\hat{V}_{1_k}\ \hat{V}_{2_k}\ \hat{h}_k\ \hat{z}_k\right]^T$$

where $\hat{V}_{1_k}$, $\hat{V}_{2_k}$, $\hat{h}_k$, $\hat{z}_k$ are the estimates of the corresponding states.

System (4)-(5) subjects to uncertainties can be expressed as

$$\begin{aligned} x_{k+1} &= f(x_k, i_k) + w_k \\ V_k &= g(x_k, i_k) + v_k \end{aligned}$$

where $w_k \sim \mathcal{N}(0,Q)$ and $v_k \sim \mathcal{N}(0,L)$ are independent random variables assumed to have a gaussian probability density function with zero mean and covariance matrices equal to Q and L, respectively. Define $A \triangleq \frac{\partial f}{\partial x} \in \mathcal{R}^{4\times 4}$ and $H \triangleq \frac{\partial g}{\partial x} \in \mathcal{R}^{1\times 4}$, while the variance matrix of the estimation error is $P \in \mathcal{R}^{4\times 4}$. The EKF algorithm is defined by the prediction step and the correction step as follows

Prediction step

$$\begin{aligned} \hat{x}_k^- &= f(\hat{x}_{k-1}, i_{k-1}) \\ P_k^- &= A_k P_{k-1} A_k^T + Q \end{aligned} \tag{6}$$

Correction step

$$\begin{aligned} K_k &= P_k^- H_k^T (H_k P_k^- H_k^T + L)^{-1} \\ \hat{x}_k &= \hat{x}_k^- + K_k (V_k - g(\hat{x}_k^-, i_k)) \\ P_k &= (I - K_k H_k) P_k^- \end{aligned} \tag{7}$$

where K_k is the Kalman optimal gain. The superscript minus in the equations means that the corresponding variables are still in the prediction step, and their value has not yet been corrected using the measurements. For a precise analysis of the EKF the reader can refer to the survey reported in [15].

2.2 The capacity observer

The capacity observer uses the measurement of i_k and the SoC estimate $\hat{z}_k$, combined with a model of the capacity evolution, to track the battery aging. Knowing that the value of the capacity evolves very slowly, a different sampling time, equal to NT, $N \in \mathbb{N}$ is used for this part of the observer: this EKF generates an output each N sampling times of the 'fast' SoC observer. The resulting overall scheme is then a multirate observer: while the SoC is evaluated at a sampling rate T, it is not necessary to run an EKF so often for estimating the capacity, the estimate of which is then updated each N sampling times of the SoC. The 'slow dynamics' describing the evolution of the capacity can be modelled as:

$$S_{N(k+1)} = S_{Nk} - N(k_0 S_{Nk} + k_1\sigma_1 + k_2\sigma_2 + k_3\sigma_3) + w_{S_k} \tag{8}$$

where for simplicity we assumed that the aging is depending linearly and independently on the different severity factors, while the noise term $w_{S_k} \sim \mathcal{N}(0,\tilde{Q})$ accounts for the model uncertainties. The output equation is

$$y_{Nk} \triangleq z_{Nk} - z_{N(k-1)} = \frac{\eta T}{S_{N(k-1)}} I_{N(k-1)} + v_{S_k} \tag{9}$$

where $I_{N(k-1)} = \sum_{j=N(k-1)}^{Nk-1} i_j$ and $v_{S_k} \sim \mathcal{N}(0,\tilde{L})$ is the measurement disturbance. It can be shown that this output equation is the multirate version of the one proposed in [11] for capacity estimation. The value y_{Nk} represents the variation of the SoC in a time interval equal to NT, and is obtained using the estimated SoC at those sampling times. The structure of the EKF algorithm is analogous to the one already analyzed for the SoC observer: in this case the only state variable is the capacity S_{Nk}, the input is the sum of the currents in the last N sampling times of the SoC observer, and the output is y_{Nk}, which means that the SoC estimation is considered as a measurement. The equations (8) and (9) can be synthetically expressed as

$$\begin{aligned} S_{N(k+1)} &= f_S(S_{Nk}, I_{Nk}) + w_{S_{Nk}} \\ y_{Nk} &= g_S(S_{Nk}, I_{Nk}) + v_{S_{Nk}} \end{aligned}$$

Define $A_S \triangleq \frac{\partial f_S}{\partial S} \in \mathcal{R}$ and $H_S \triangleq \frac{\partial g_S}{\partial S} \in \mathcal{R}$, while the variance of the estimation error is $\Pi \in \mathcal{R}$. Define also the state of the capacity observer as $\hat{S}_{Nk}$. Then, the EKF algorithm for the capacity estimation is

Prediction step

$$\begin{aligned} \hat{S}_{Nk}^- &= f_S(\hat{S}_{N(k-1)}, I_{N(k-1)}) \\ \Pi_k^- &= A_{S_{Nk}} \Pi_{N(k-1)} A_{S_k}^T + \tilde{Q} \end{aligned} \tag{10}$$

Correction step

$$\begin{aligned} K_{S_{Nk}} &= \Pi_{Nk}^- H_{S_{Nk}}^T (H_{S_{Nk}} \Pi_{Nk}^- H_{S_{Nk}}^T + \tilde{L})^{-1} \\ \hat{S}_{Nk} &= \hat{S}_{Nk}^- + K_{S_{Nk}} (V_{Nk} - g_S(\hat{S}_{Nk}^-, I_{Nk}) \\ \Pi_{Nk} &= (1 - K_{S_{Nk}} H_{S_{Nk}}) \Pi_{Nk}^- \end{aligned} \tag{11}$$

where $K_{S_{Nk}}$ is the Kalman gain, while, as in the case of the SoC observer, the superscript minus means that the corresponding variables are in the prediction step.

2.3 RUL estimation

With the use of these two observers in conjunction, it is possible to estimate the value of the capacity at the present moment. It is known that a battery is considered at its end-of-life when its capacity reaches the 80% of its nominal value S^*: such value is defined as $S_0 = 0.8S^*$. So, using the part of the dual EKF relative to the capacity estimation only in the prediction phase, it is possible to predict the future evolution of the probability density function of the capacity (that is, its mean and variance) obtaining an evaluation of the RUL. For example, it is known that, for a cumulative distribution function obtained integrating a gaussian probability density function reaches a value of about 0.027 for a value equal to the mean minus twice the standard deviation. Then, in order to obtain a probability of about $1-0.027=0.973$ of not overestimating the RUL, one can decide to define

$$RUL \triangleq (t_{RUL}-t_c) : \hat{S}(t_{RUL}) - 2\sqrt{\Pi(t_{RUL})} = S_0 \tag{12}$$

where t_c is the present time value, while t_{RUL} is the time instant when the system reaches its end-of-life and $\hat{S}(t_{RUL})$ is the predicted value of S when the system reaches its end-of-life.

The SoH of the battery can be expressed in terms of the so-called 'damage measure' ξ, which is obtained from the damage variable S. This variable is normalized between 0 (dead battery) and 1 (new battery), and it is defined as

$$\xi_{Nk} = \frac{\hat{S}_{Nk} - S_0}{S^* - S_0} \tag{13}$$

The damage measure represents the RUL of the battery in terms of reserve capacity. The RUL can then be defined as the estimated time that it takes to the battery to go from the actual value of the capacity S^* to S_0 . A dual way to represent the same concept is to express battery life through the residual life $\Lambda \triangleq 1-\xi$.

We want to remark that at the moment the modeling of the aging process is still very hard to do, and there is no approach on which everyone agrees. Moreover, an important challenge is how to evaluate the parameters of the probability density function (in our case, the variance), in order to obtain an estimate of the uncertainty that could take into account the information on the real battery.

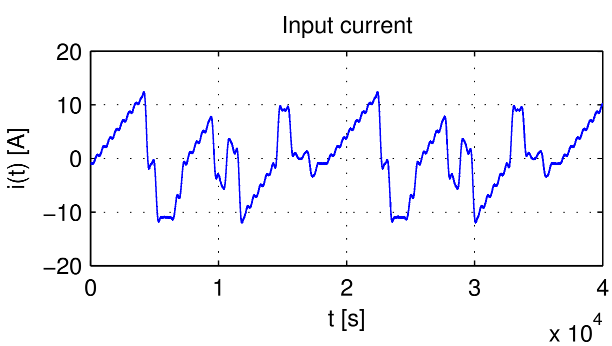

Figure 3. Time evolution of the input current for the simulation example

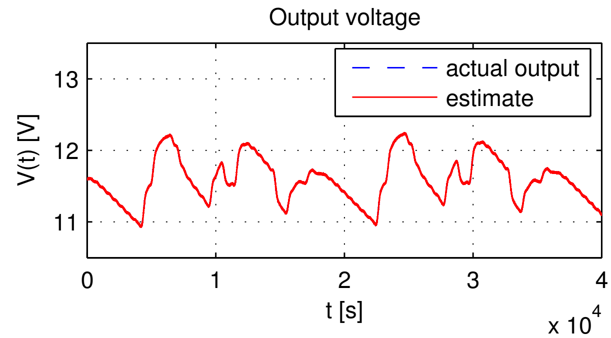

Figure 4. Time evolution of the output voltage for the simulation example

3 Simulation results

The proposed solution has been tested in simulation. The PbA battery model is defined by the following parameters: $R_0 = 2.17 \cdot 10^{-2}\ \Omega$, $R_1 = 3.54 \cdot 10^{-2}\ \Omega$, $C_1 = 1.15 \cdot 10^4\ F$, $R_2 = 3.62 \cdot 10^{-5}\ \Omega$, $C_2 = 9.52 \cdot 10^3\ F$, $M = 0.08$, $\gamma = 1$, $\eta = 1$, $\alpha = 11.564$, $\beta = 1.52 \cdot 10^{-2}$, $S^* = 60\ Ah$, identified from experiments conducted at the Battery Labs at the Ohio State University Center for Automotive Research. The used sampling times are equal to 0.5 s for the SoC observer, and to 1 hour for the capacity observer, and the simulation runs for about 11 hours. In the simulation, the parameters of the system simulating the real battery have been perturbed of small percentages (2-3%), and the initial values of some states (in particular, in this example, of V_1 and h) have also been perturbed, to show that the proposed strategy has some robustness properties. The input current profile is shown in Fig. 3, and the estimates of the states can bee seen in Fig. 5. In particular, considering that the SoC is the state variable for which the observer has been designed, the evolution of its estimation error is shown in Fig. 6.

The projection of the evolution of the capacity is based on its estimate at the present time instant, and is shown in Figs. 7 and 8. Considering that the estimation given by the EKF is a probability density function (though it is synthetically expressed using mean and variance), the evolution of such a function is shown in both a three-dimensional and bi-dimensional fashion. In particular, Fig. 7 shows the entire probability density function $p(S,t)$ for the values of interest of the capacity at every time instant in the future, while looking at Fig. 8 one can see that the remaining useful life $t_{RUL}-t_c$, according to (12), is established at the intersection of the threshold representing the lowest acceptable value for the capacity and the lower bound of the probability density function, defined here at any time instant t as $\hat{S}(t) - 2\sqrt{\Pi(t)}$. The predicted aging process for the considered battery is evaluated in a certain scenario when the battery has already been used for some time. The prediction is done starting with a value of the battery capacity of $S(t_c) = 52\ Ah$ (the nominal battery capacity is $S^* = 60\ Ah$): in this case, the threshold for which the battery is considered dead is equal to 80% of 60 Ah, i.e. $S_0 = 48\ Ah$. The variance ρ, as expected, in 400 days of prediction, passes from

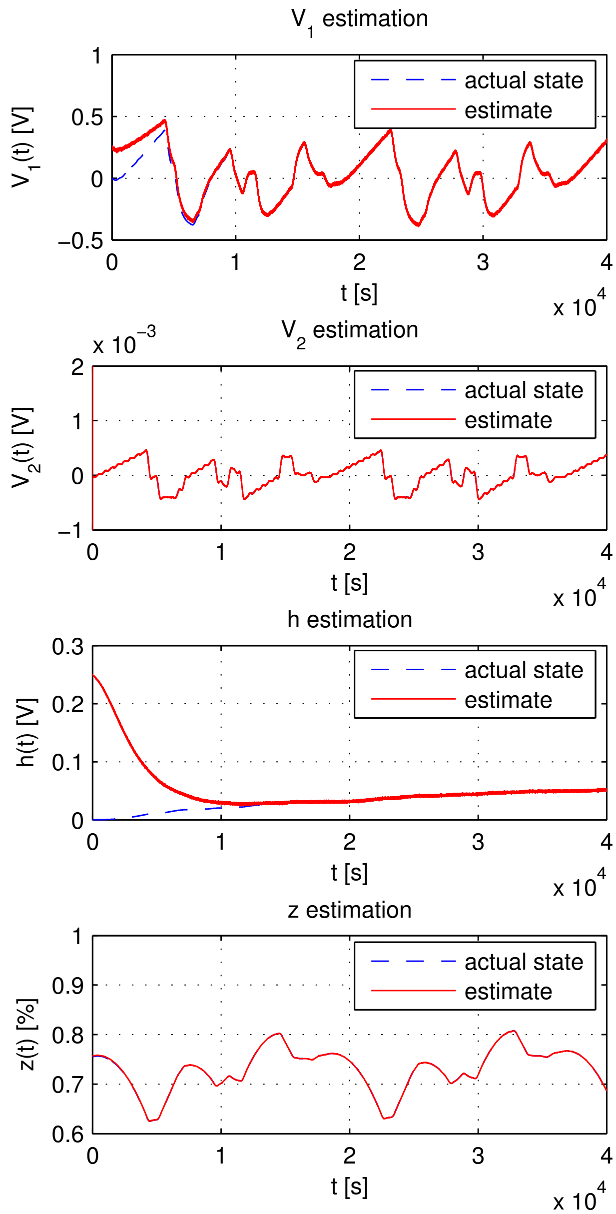

Figure 5. Time evolutions of the actual states ($V_1(t)$, $V_2(t)$, $h(t)$, $z(t)$) and of their estimates ($\hat{V}_1(t)$, $\hat{V}_2(t)$, $\hat{h}(t)$, $\hat{z}(t)$) using the proposed observer

30.6 to 1142. This means that, as time goes on, the uncertainty on the estimate increases. Using this simple model, in this case one can conclude that the estimated RUL is equal to 350 days with a confidence of 97.5%.

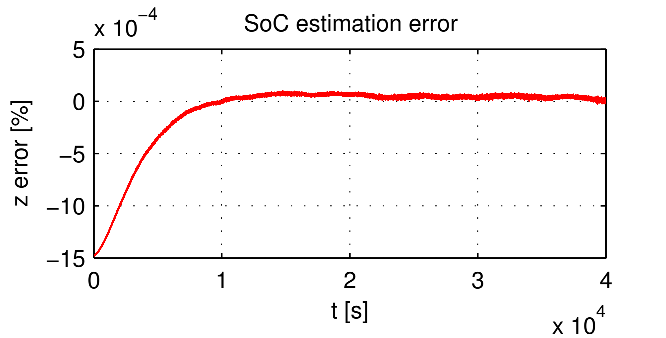

Figure 6. Time evolutions of the estimation error for the SoC: $z(t) - \hat{z}(t)$

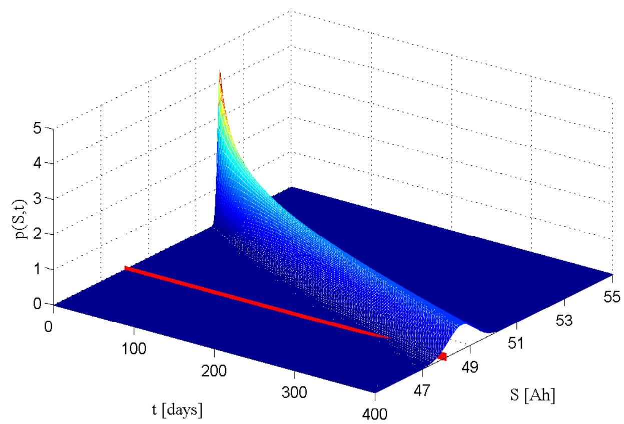

Figure 7. Three-dimensional representation of the prediction of the capacity evolution: the probability density function obtained with the EKF in prediction is represented at any time instant, while the red line is the threshold at 80% of the nominal capacity.

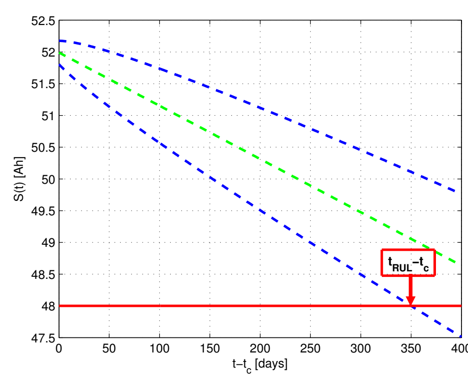

Figure 8. Bi-dimensional representation of the prediction of the capacity evolution: the dashed green line is the expected value $\hat{S}(t)$, while the dashed blue lines stand for the values of $\hat{S}(t) \pm 2\sqrt{\rho(t)}$. Finally, the solid red line is the lower threshold at 80% of the nominal capacity, and the red arrow indicates the position of $t_{RUL} - t_c$. At present time $t - t_c$ the battery has a capacity of $52\,Ah$.

4 Conclusions

In the present paper, a strategy for estimating the RUL in automotive batteries has been proposed. This strategy is based on the simultaneous estimation of the capacity and the SoC using a multirate dual EKF, and on the prediction of the future evolution of the capacity with suitable uncertainty margins. Simulations show appreciable results for both estimation and prediction. Future work will be devoted to the experimental validation of the proposed strategy, even using different observers (i.e. Moving Horizon Estimation), in order to compare different estimation methodologies on the data from real batteries.

REFERENCES

[1] Piller, S., Perrin, M., and Jossen, A., 2001. "Methods for state-of-charge determination and their applications". *Journal of Power Sources,* **96**(1), pp. 113–120.

[2] Chan, C., 2002. "The state of the art of electric and hybrid vehicles". *Proceedings of the IEEE,* **90**(2), pp. 247–275.

[3] Ruetschi, P., 2004. "Aging mechanisms and service life of lead–acid batteries". *Journal of Power Sources,* **127**(1-2), pp. 33–44.

[4] Rong, P., and Pedram, M., 2006. "An analytical model for predicting the remaining battery capacity of lithium-ion batteries". *IEEE Transactions on Very Large Scale Integration Systems,* **14**(5), p. 441.

[5] Yoshida, T., Takahashi, M., Morikawa, S., Ihara, C., Katsukawa, H., Shiratsuchi, T., and Yamaki, J., 2006. "Degradation mechanism and life prediction of lithium-ion batteries". *Journal of the Electrochemical Society,* **153**, pp. A576–A582.

[6] Uckun, S., Goebel, K., and Lucas, P., 2008. "Standardizing research methods for prognostics". In Proc. International Conference on Prognostics and Health Management.

[7] Saha, B., Goebel, K., Poll, S., and Christophersen, J., 2007. "An integrated approach to battery health monitoring using bayesian regression and state estimation". In IEEE Autotestcon.

[8] Vasebi, A., Partovibakhsh, M., and Bathaee, S., 2007. "A novel combined battery model for state-of-charge estimation in lead-acid batteries based on extended Kalman filter for hybrid electric vehicle applications". *Journal of Power Sources,* **174**(1), pp. 30–40.

[9] Plett, G., 2004. "Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs Part 1. Background". *Journal of Power Sources,* **134**(2), pp. 252–261.

[10] Plett, G., 2004. "Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs Part 2. Modeling and identification". *Journal of Power Sources,* **134**(2), pp. 262–276.

[11] Plett, G., 2004. "Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs-Part 3. State and parameter estimation". *Journal of Power Sources,* **134**(2), pp. 277–292.

[12] Srinivasan, V., Weidner, J., and Newman, J., 2001. "Hysteresis during cycling of nickel hydroxide active material". *Journal of the Electrochemical Society,* **148**, pp. A981–A983.

[13] Isidori, A., 1995. *Nonlinear control systems*. Springer.

[14] Serrao, L., Onori, S., Rizzoni, G., and Guezennec, Y., 2009. "A novel model-based algorithm for battery prognosis". In Proc. 7th IFAC SAFEPROCESS.

[15] Welch, G., and Bishop, G., 1995. "An introduction to the Kalman filter". *University of North Carolina at Chapel Hill, Chapel Hill, NC.*