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Harmonic Pulse Testing as a Monitoring Tool during Hydraulic Stimulation of an Enhanced Geothermal System

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ABSTRACT

Harmonic pulse testing is a technology that has similar capabilities as conventional well testing. It intends to determine the hydraulic parameters such as storativity and transmissivity. In comparison with well testing, pulse testing requires more time due to the number of pulses required. The advantage, however, is that it can be employed during ongoing field operations. This makes it an ideal monitoring tool.

An EGS stimulation operation was performed in Pohang, South Korea, with the purpose to monitor the seismicity and to locate the activated zone for further stimulation treatments and drilling operations. The purpose was not to increase substantially the injectivity but we employed harmonic pulse testing to demonstrate its feasibility for monitoring injectivity. Prior to the stimulation, a test was performed to obtain the baseline properties. Then, tests were performed during injection cycles with increasing rates and pressures, in order to follow the development of injectivity.

The particular test showed a slight increase of injectivity during the stimulation treatment, which was mostly lost after completion of the test. However, the test does show the great potential of the method and has taught us the critical operational parameters. These include the pulse durations, the accuracy of the switching moment and sampling rate of pressures and injection rates.

1. INTRODUCTION

Harmonic pulse testing is characterized by periodic sequences of alternating production or injection rates [Kuo, 1972; Rosa and Horne, 1997; Hollaender et al, 2002; Renner and Messar, 2006; Ahn and Horne, 2010; Fokker et al, 2013, Fokker et al, 2018]. It has similar goals as conventional well testing but there are some important differences. In particular, the interpretation is through the analysis of the rate and induced pressure signal in the frequency domain, rather than the pressure and pressure derivative interpretation. The analysis in the frequency domain allows extraction and analysis of each periodic component of the pressure response in relation to the corresponding periodic component of the rate. As a result, harmonic pulse testing can be employed as a monitoring tool during ongoing operations. It does not require the interruption of production or injection, nor the knowledge of previous rate history. Harmonic tests aim at assessing well and reservoir properties, such as skin, wellbore storage, reservoir compressibility and permeability. The main drawback of harmonic pulse testing is that it requires much longer times than conventional testing to obtain the same information (Hollaender et al., 2002). For this very reason, it is inadequate for exploration and appraisal wells, but it is suited as an alternative to conventional well testing for monitoring well performance.

The first enhanced geothermal system (EGS) in Korea was launched in Pohang at the end of 2010 [Park et al, 2017]. Two boreholes (PX-1 and PX-2) were drilled to more than 4 km depth. The first stimulation was performed in PX-2 in 2016 [Park et al, 2016] followed by four more stimulations in 2016 and 2017. The present paper is concerned with the stimulation of PX-1 conducted in August 2017. During this stimulation, a number of harmonic pulse tests were performed, both as a baseline before stimulation and during stimulation on top of a background injection rate. This makes the test an ideal candidate for demonstrating the utility of harmonic pulse testing as a monitoring technology.

We will first briefly review the principles of harmonic pulse testing. Then we will describe the test that was performed in well PX-1 in August 2017, and the results from the harmonic pulse tests. In the last section we will put our findings in a broader context, come to conclusions and sketch an outlook for further developments.

2. HARMONIC PULSE TEST PRINCIPLES

An ideal harmonic well test employs a sinusoidal signal on the flow rate and records the resulting pressure response. This response does not only show a magnitude but also a phase. The phase shift is the relative delay of the pressure cycle with respect to the imposed flow cycle. The amplitude ratio between imposed rate and pressure response and the phase shift depend on the reservoir properties in the volume of investigation, such as fluid mobility, total compressibility, skin and wellbore storage. Therefore, this response carries information about these properties.

In well operation procedures it is much easier to generate rectangular pulses than sinusoidal signals, with alternating periods of fixed rates. These can be decomposed into an array of harmonic pulses with distinct frequencies, using Fourier transformation. An experiment using square pulses then corresponds to the simultaneous execution of a number of harmonic tests. Pulses with cycle time T (duration T/2 with a high rate and T/2 with a low rate) have harmonic components with cycle times T; T/3; T/5, etc.; the number of harmonic components that can be identified depends on the sampling rate. If the governing equations are linear, the pressure response is composed of contributions with the same cycle times and it is a linear superposition of the responses of the individual components. Now, the amplitudes and the phases of the components must be quantified for both the imposing rate signal and for the resulting pressure signal. This is readily done using a Fast Fourier Transform (FFT) algorithm (also called Discrete Fourier Transform), which unravels the signal by picking out the imposed frequencies and the corresponding reservoir response. In order to maximize the information provided by harmonic pulse test interpretation, pressure data should be adequately pre-processed adopting detrending methodologies [e.g. Ahn & Horne, 2010 or Viberti 2016] with the aim of separating pure periodic components of the signal from non-periodic components.

An analytical solution of the diffusivity equation with harmonic boundary conditions, incorporating both wellbore storage effects (wellbore storage coefficient C) and skin (S) is available from other publications [Fokker et al, 2016, 2017]; the derivation is summarized in the Appendix. The pressure response function for the harmonic component with angular frequency ω is given as

$$R = \frac{p_{well}}{\tilde{q}} = \frac{K_0[\xi] + S}{k + i\omega W_S \cdot (K_0[\xi] + S)} \tag{1}$$

where K_0 and K_1 are modified Bessel functions of the second kind. For the other symbols in this equation we refer to the Appendix.

The ratio of Eq. (1) is a complex number. Its absolute value describes the amplitude of the pressure response to the rate constraint; its argument describes the phase delay of the response. A positive skin has the effect of increasing the amplitude and changing the phase of the signal over the full frequency spectrum. Wellbore storage reduces the amplitude predominantly at higher frequencies due to the term with ω in the denominator of Eq. (1).

Equation (1) can be used as a type curve to estimate reservoir properties by minimizing an objective function which penalizes the difference between amplitude ratio and phase difference as determined from the FFT analysis of measurements and as calculated with the model parameters. The objective function should include the responses of all frequencies for which they can be faithfully identified – and the standard deviation associated to these numbers.

It should be noted that the uniqueness of the matching parameter set is not guaranteed. In particular, several combinations of the (k, S) couple could give very similar results when using this analytical approach. As a consequence, in the absence of an observer well, which is not affected by skin, the use of a pulse test alone is not recommended. However, if a kh value is available from a conventional test, the match can be assessed with a good degree of confidence. This makes harmonic pulse testing an efficient well performance monitoring technique.

A longer pulse cycle time T allows for a better description of the spectrum at low frequencies, which could significantly improve the interpretation. This is related to the increasing reservoir investigation radius with longer times [Ahn and Horne, 2010]. Moreover, the optimal duration of each semi-cycle (T/2) should be significantly longer than the duration of wellbore storage effects. Finally, the shorter the duration of the semi-cycle, the more difficult it is to maintain a constant rate and the more challenging it is for the on-site operator to make frequent rate changes at the right time.

3. POHANG PX-1 TEST DESCRIPTION

The Pohang Enhanced Geothermal System (EGS) development in Korea was started in 2010 [Park et al, 2017]. Two wells were drilled: the PX-1 and the PX-2 well. The reservoir at the target depth (around 4300 m depth) is mainly composed of granodiorite and granitic gneiss. The rock mass is highly fractured. The first stimulation treatment was performed on the PX-2 well in early 2016. Seismicity was closely monitored during the operations. A stimulation of PX-1 followed in December 2016, and a second stimulation in PX-2 in March/April 2017.

In August 2017 the second stimulation of well PX-1 was conducted. In this operation, also a number of approaches were tested. Soft stimulation was applied – which was defined in the present context as injection pulses with increasing injection rates. The goal of this strategy was to minimize induced seismic events. We focus, however, on the monitoring of the effectiveness of the stimulation treatments. This monitoring was programmed in the form of pressure falloff tests after a number of stimulation pulses, but also in the form of harmonic pulse tests.

Figure 1 presents the full history of injection rates and resulting pressures. The wellhead pressures were measured with a pressure sensor (accuracy 0.1 MPa) – we used standpipe pressures in the present analysis. The rates were monitored by measuring the number of the strokes of the mud pump. The first activity was a straight injection test. This was followed by HPT-1, starting Aug 7th at 14:00. Pulses with a 60-minute cycle were employed – 30 minutes injection and 30 minutes shut-in during each cycle. This test was almost immediately followed by a harmonic pulse test with 6-minute pulses; HPT-2. The two together were meant to provide a baseline for the further treatment of the reservoir and the associated monitoring. A first fracture opening test was performed on Aug 8th. This was followed by a second test on Aug 9th, with a harmonic pulse test on top of every injection cycle. This way, the development of injectivity was scheduled to be followed during the stimulation. We coined these short-period monitoring tests HPT-3a – HPT-3g. After a long shut-in on Aug 10th, the actual soft stimulation test was performed on Aug 11th. The last four cycles of this treatment could be interpreted as a harmonic pulse test – HPT-4, as the stimulation injection pulses had the same magnitude and duration. Cycle time for this test was 120 minutes, with alternating high and lower injection rates, lasting each for 1 hour. The same interpretation could be given for the last train of pulses on Aug 13th-14th, with an even larger number of 12 useable cycles of 120 minutes (HPT-5). The operational characteristics of the harmonic pulse tests are summarized in Table 1.

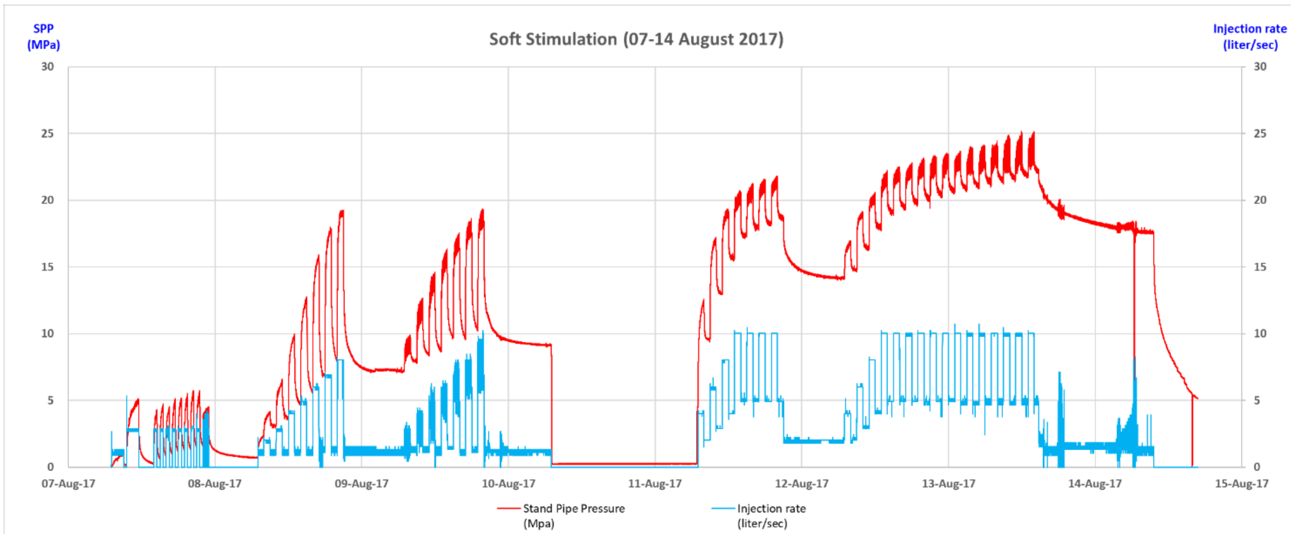

Figure 1 Pohang PX-1 soft stimulation record

Table 1 Summary of Harmonic Field Tests in Pohang PX-1 Soft Stimulation Treatment

ID	Start time	End time	Cycle time	Average injection rate	Injection rate pulse amplitude
HPT-1	2017-08-07 14:00	2017-08-07 22:00	60 min	0.00145 m^3/s	0.00145 m^3/s
HPT-2	2017-08-07 22:00	2017-08-07 23:00	6 min	0.00145 m^3/s	0.00145 m^3/s
HPT-3a	2017-08-09 07:00	2017-08-09 08:00	6 min	0.0021 m^3/s	0.0008 m^3/s
HPT-3b	2017-08-09 09:00	2017-08-09 10:00	6 min	0.0030 m^3/s	0.0009 m^3/s
HPT-3c	2017-08-09 11:00	2017-08-09 12:00	6 min	0.0039 m^3/s	0.0009 m^3/s
HPT-3d	2017-08-09 13:00	2017-08-09 14:00	6 min	0.0050 m^3/s	0.0010 m^3/s
HPT-3e	2017-08-09 15:00	2017-08-09 16:00	6 min	0.0059 m^3/s	0.0010 m^3/s
HPT-3f	2017-08-09 17:00	2017-08-09 18:00	6 min	0.0070 m^3/s	0.0010 m^3/s
HPT-3g	2017-08-09 19:00	2017-08-09 20:00	6 min	0.0079 m^3/s	0.0010 m^3/s
HPT-4	2017-08-11 13:00	2017-08-11 21:00	120 min	0.0075 m^3/s	0.0025 m^3/s
HPT-5	2017-08-12 14:00	2017-08-13 14:00	120 min	0.0075 m^3/s	0.0025 m^3/s

4. POHANG HARMONIC PULSE TEST RESULTS

The interpretation of a harmonic well test starts with the identification of the frequency information of the imposed injection rates and the resulting pressures. This is done with a Fast Fourier Transform (FFT) algorithm. To make the time traces suitable for FFT, two actions are required: the selection of the period of the test to be used, and a detrending of the signal. The selected period should contain sufficient pulses to be able to identify the frequencies, but should also exclude startup phenomena which are often seen in the first one or two cycles. Detrending is particularly important for the pressure traces, as these may be influenced by progressive pressurization of the reservoir or by other slow processes.

4.1 Baseline tests: HPT-1 and HPT-2

Examples of the power spectra of tests HPT-1 and HPT-2 are given in Figure 2. What we see is a clear "comb" of magnitude peaks, corresponding to the frequencies associated with the leading frequency and its odd multiples. For HPT-1, many more frequencies than in HPT-2 have pressure amplitudes that exceed the background level. This demonstrates some important issues: firstly, the method is constrained to a maximum frequency or minimum cycle time, due to the wellbore storage effect. Secondly, the case with the larger cycle time does not only have a smaller leading frequency but also many more higher frequencies. As a final note on the frequency spectra, a quality measure in the form of a standard deviation for these magnitudes can be derived from a comparison of the peak strength with the background level of frequencies next to it, i.e., frequencies which were not produced by the setup of the harmonic test. As an example, from Figure 2 one can already conclude that for high frequencies (for HPT-1 above 0.02 Hz; for HPT-2 above 0.04 HZ) the pressure responses do not have amplitudes that can be related to the imposed injection pulses. Therefore the standard deviation of the pressure amplitudes for these high frequencies are of the order of the value itself and they do not contribute a significant signal.

When comparing the frequency information of the pressure response to the frequency information of the imposed rates we obtain the reservoir response. The aim of the test is to quantify reservoir properties from this response by comparing to responses calculated with Eq. 1. This is visualized in Figure 3. An acceptable fit to the HPT-1 traces could be created with the parameters listed in Table 2. A consistent difference between the measured and modelled delay can be due to a synchronization error between the clocks of rate and pressure monitors. The parameters were also used to assess the response from test HPT-2. This fit is also acceptable – note that much fewer frequencies give meaningful response for this test.

Harmonic pulse testing is in particular sensitive to the reservoir permeability and to the wellbore storage coefficient. This is demonstrated in Figure 4 by presenting traces for different values of these parameters. The permeability has the largest influence towards the lower-frequency regime, where the wellbore storage does not have an effect. The wellbore storage, on the other hand, mainly influences the response of larger frequencies; in particular it determines the maximum frequency that can sensibly be used in the analysis. For a better sensitivity of the technique, therefore, a small wellbore volume is helpful. Downhole shut-in and downhole gauges would be very advantageous to promote this sensitivity.

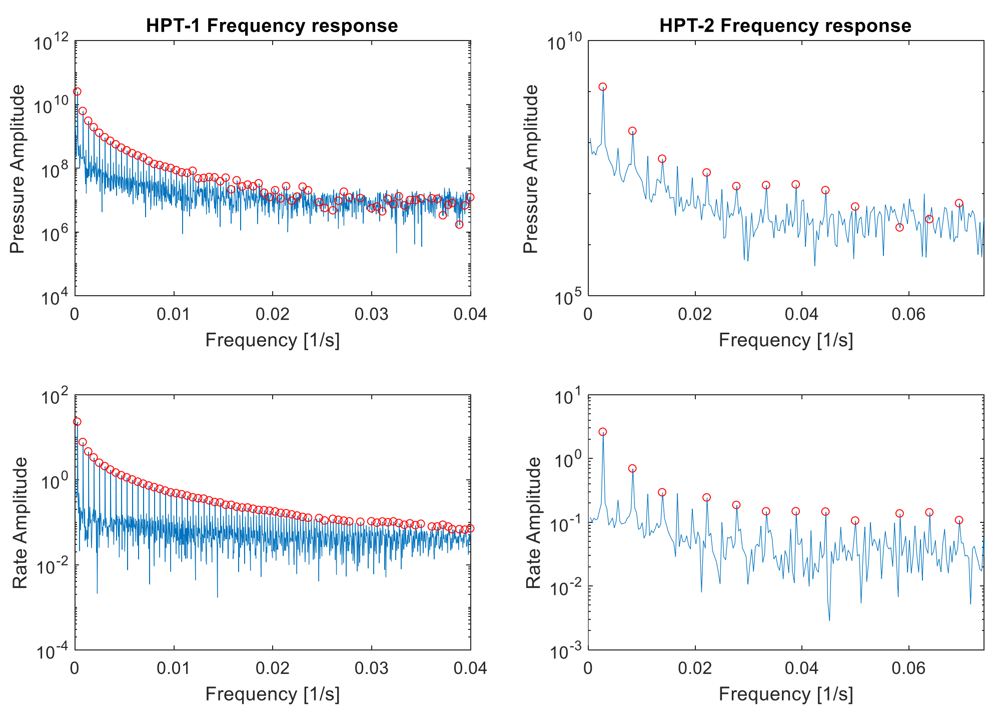

Figure 2 Amplitudes of the frequency content of the rate and pressure traces of HPT-1 and HPT-2

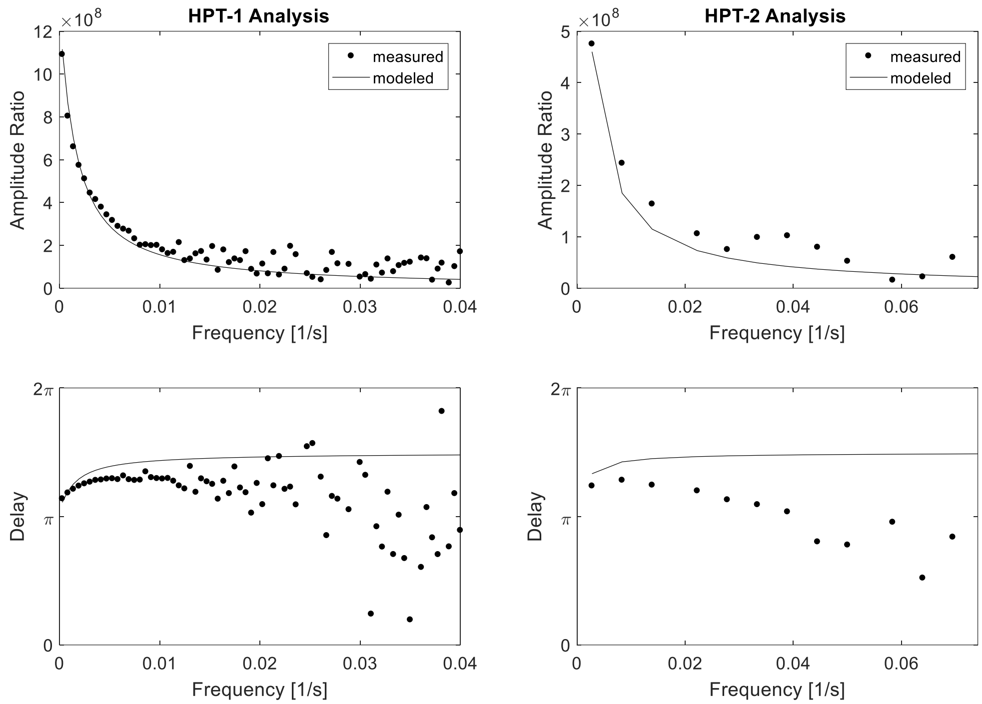

Figure 3 Measured and modelled responses of HPT-1 and HPT-2

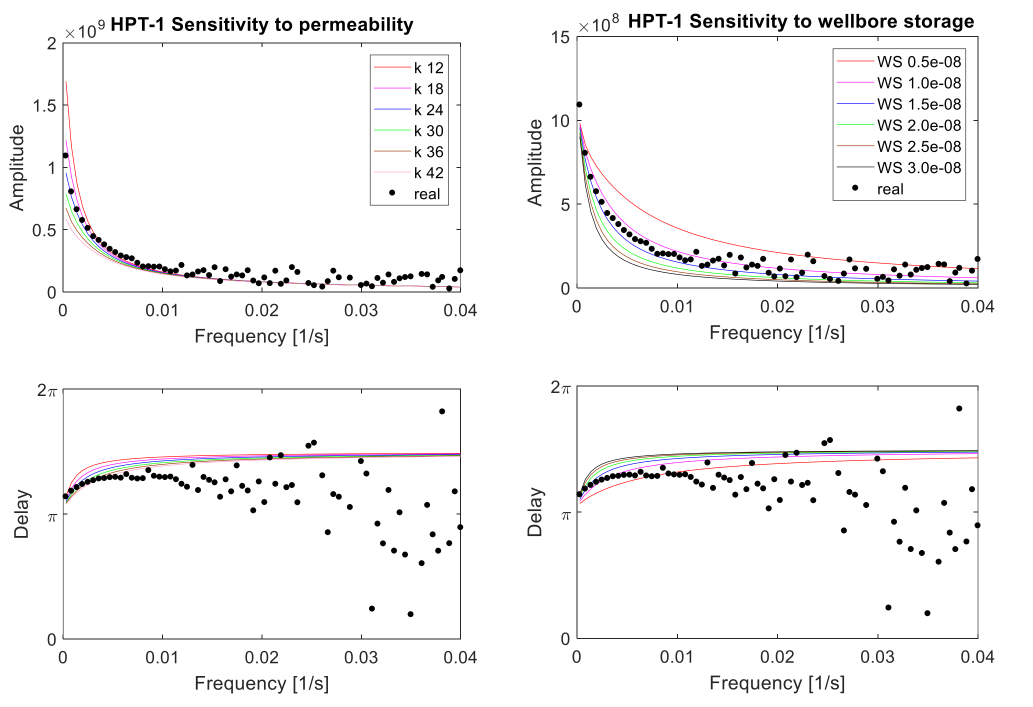

Figure 4 Results of HPT-1 with sensitivity of the model response to permeability (left) and wellbore storage (right)

Table 2 Parameters used in model responses

Test	Permeability	Skin [-]	Wellbore storage	Reservoir compressibility
HPT-1 – HPT-3	10 md	0	0.0015	4.5 10^{-5} bar^{-1}
HPT-4	30 md	0	0.0020	4.5 10^{-5} bar^{-1}
HPT-5	40 md	0	0.0020	4.5 10^{-5} bar^{-1}

4.2 Monitoring tests: HPT-3

During the sequence of tests HPT-3a – HPT-3g a fracture opening test was performed. Pulses of one hour with increasing injection rates were applied with pauses of one hour between them. On top of the background injection rate, harmonic pulse tests were applied with cycle times of 6 minutes (cf Table 1). A zoomed representation of tests HPT-3d – HPT-3f is given in Figure 5 (left). In Figure 5 (right) the frequency analysis is presented. Along with the pressure responses (the symbols in the figure) the prediction with the parameter values as determined in the baseline test, HPT-1, is provided.

The analysis provided in Figure 5 demonstrates that harmonic pulse testing can indeed be employed as a monitoring tool during well operations. Even though only few frequencies provide useful information, the effect of the increasing background rates can be distinguished. Especially the leading frequency shows that there is an effect: the later tests, with the larger rates, show smaller pressure/rate amplitudes. This points at larger permeability or larger injectivity during the later stages. The soft stimulation, however, does not seem to be extremely effective at this timescale. The effect can be well explained by opening of existing fractures at higher pressures.

4.3 Soft Stimulation Tests: HPT-4 and HPT-5

The last two stages of the operation sequence employed alternating high-rate and low-rate injection pulses of one hour each. Following a buildup in rate during 3 stages, the high and the low rates were repeated at the same level, thus constituting harmonic pulse tests with 120 minute cycle time. For both tests, a large number of frequencies with magnitude peaks could be identified. The small number of only 4 cycles for HPT-4 apparently did not seriously harm the effectiveness of the test. In contrast to the monitoring tests HPT-3, however, it was not possible to use the same parameters as in HPT-1 for the modelled response. The response observed in the last tests points to an increased permeability, presumably due to opening fractures. An assessment of the stimulation effectiveness in terms of persistently opened fractures would have required an additional test at low background rate and low injection rate amplitude after completion.

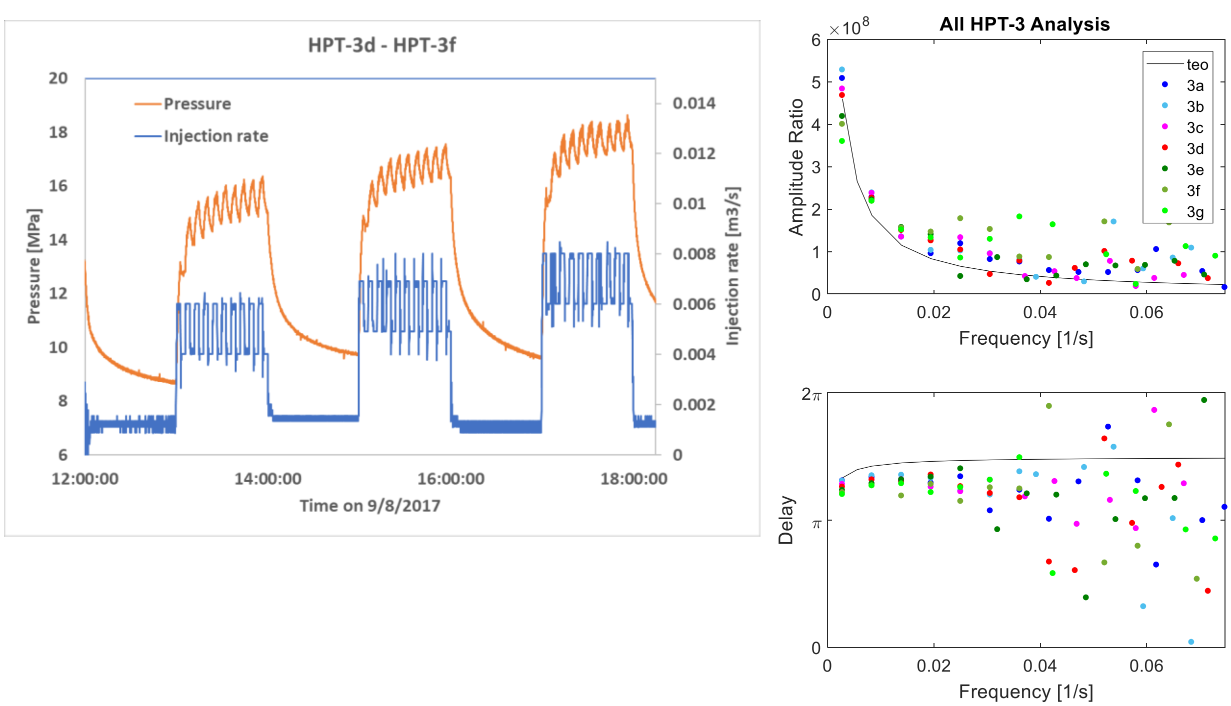

Figure 5 Left: Time traces of injection rates and pressure responses for tests HPT-3d – HPT-3f. Right: Analysis of all HPT-3 spectra

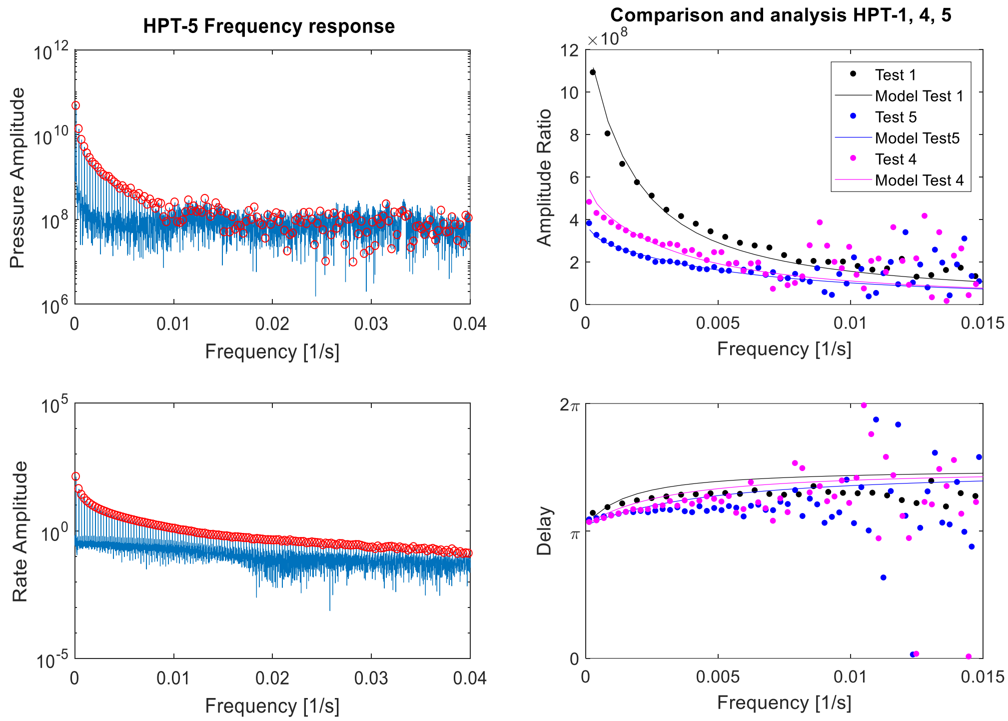

Figure 6 Left: Frequency spectrum of HPT-5. Right: Comparison of responses of HPT-1, HPT-4 and HPT-5. Different sets of parameters were used to compute the model responses (see Table 2).

5. CONCLUSIONS

We have demonstrated that harmonic pulse testing can be employed as a monitoring tool during well operations. The identification of discrete frequencies with FFT isolates the imposed harmonic signal and the associated pressure response from other effects like slow pressurization of the reservoir or influence from nearby operations.

Harmonic pulse testing is not an alternative to conventional well testing but rather a complementary technique. Well testing is a sensitive technique targeted at the identification of various reservoir characteristics like reservoir boundaries, dual porosity systems, and mechanical skin. In addition, harmonic pulse testing requires significantly more time to obtain the same information – this is due to the requirement of employing a number of cycles that have the same duration as a conventional well test when a similar reservoir size is to be probed. The additional time, however, is less of an issue when the test can be employed on top of ongoing operations if they do not have to be stopped. Also, similar technology as in conventional well testing can be employed in pulse testing, making the interpretation more intuitive for people with well testing interpretation skills [Fokker et al, 2018]. What is even more important: well testing can not easily be applied to producing wells while harmonic pulse testing can.

The present study already indicated the importance of the geomechanical effects during the monitoring phase. When more sizeable fracturing is taking place, the effect will be even larger. This calls for an extension of the present theory to also include growing fractures in it. We are currently working on such an extension. With a history matching or data assimilation of measured data, the potential of the technique is then the quantification of the mechanical parameters like the storativity ratio in dual porosity systems and the fracture network compressibility.

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APPENDIX THEORY OF HARMONIC TESTING

In a homogeneous reservoir containing slightly compressible fluid, the flow is described by the diffusivity equation:

$$\phi c\frac{\partial p}{\partial t} = \nabla\left[\frac{k}{\mu}\nabla p\right]$$

where ϕ is the rock porosity, c is the total compressibility, k is the rock permeability, μ is the fluid viscosity, p is the pressure and t is the time.

When a piecemeal homogeneous subsurface is assumed, the diffusivity equation is linear. We obtain

$$\frac{\partial p}{\partial t} = \kappa\nabla^2 p$$

$$\kappa = \frac{k}{\phi\mu c}$$

Under the assumption of linearity the pressure and flow solution of a reservoir with many wells and changing production rates can then be added to the solution of the harmonic test. The Fourier transformation will pick out the signal present in the imposed frequency. Furthermore, there will be no frequency mixing; frequencies can be treated independently. Therefore we consider each frequency component independently. The final pressure is then a superposition of the responses to all the frequency components present in the imposed flow rate, added to the background signal.

Considering each frequency independently we write the pressure solution for each frequency as the product of a space-dependent and a time-dependent function:

$$p(\mathbf{r},t) = p_\omega(\mathbf{r})e^{i\omega t}$$

The angular frequency is defined as $\omega = \frac{2\pi}{T}$, with T the cycle time of the imposed harmonic signal. This results in a time-independent differential equation for p_ω:

$$i\omega p_\omega(\mathbf{r}) = \kappa\nabla^2 p_\omega(\mathbf{r})$$

For the scaling of the volumetric rates q we take

$$\frac{\mu q}{2\pi h} = \tilde{q} = \tilde{q}_\omega e^{i\omega t}$$

The thickness of the contributing reservoir layers is indicated by h. The rate is taken positive for injection.

For an infinite reservoir with radial symmetry the diffusivity equation can be rewritten into radial coordinates. The first boundary condition for each component then is a harmonic signal on the flow rate corrected for the wellbore storage effect. The second boundary condition is a zero pressure at large distances from the well, as the net flow of the harmonic signal is zero. We obtain

$$i\omega p_\omega(r) = \kappa\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial p_\omega}{\partial r}\right)$$

$$k\left[r\frac{\partial p}{\partial r}\right]_{r=r_w} = -\tilde{q} + W_S\frac{\partial}{\partial t}\left([p(r,t)]_{r=r_w} + \Delta p_S\right)$$

$$[p(r,t)]_{r\to\infty} = 0$$

$$W_S = \frac{\mu C}{2\pi h}$$

where r is the radial distance from the tested well, h is the producing-layer thickness, C is the wellbore storage coefficient. The term Δp_S has been added to p since the wellbore storage must be calculated using the wellbore pressure, i.e. the pressure at $r = r_w$ corrected for the additional pressure drop due to the skin.

The additional pressure drop due to skin S is (Dake, 1978 / 1994; Eq. 4.26):

$$\Delta p_S = \frac{q\mu}{2\pi kh}S$$

Recalling Darcy's law, $q = -\frac{2\pi rkh}{\mu}\frac{\partial p}{\partial r}$, the additional pressure drop due to skin can be expressed as:

$$\Delta p_S = -S\left[r\frac{\partial p}{\partial r}\right]_{r=r_w} = -S\left[r\frac{\partial p_\omega}{\partial r}\right]_{r=r_w} e^{i\omega t}$$

and the boundary condition for the differential equation becomes independent of time as well:

$$i\omega p_\omega(r) = \kappa\frac{1}{r}\frac{\partial}{\partial r}\left\{r\frac{\partial p_\omega}{\partial r}\right\}$$

$$(k + i\omega W_S S)\left[r\frac{\partial p_\omega}{\partial r}\right]_{r=r_w} = -\tilde{q}_\omega + i\omega W_S p_\omega(r_w)$$

$$[p(r,t)]_{r\to\infty} = 0$$

The solution to these equations is the zeroth-order modified Bessel function of the second kind, K_0: (using the fact that for small ξ, i.e. at the wellbore wall, $\xi K_1(\xi) = 1$)

$$p_\omega(r) = \frac{\tilde{q}_\omega K_0\left[\xi\cdot\frac{r}{r_w}\right]}{k + i\omega W_S\cdot(K_0[\xi] + S)}$$

$$\xi = r_w\sqrt{\frac{i\omega}{\kappa}}$$

The scaling of the function follows from the well boundary condition.

The skin pressure drop is calculated as

$$\Delta p_S = -S\left[r\frac{\partial p_\omega}{\partial r}\right]_{r=r_w} e^{i\omega t} = \frac{S}{K_0(\xi)} p_\omega(r) e^{i\omega t}$$

The pressure components at the wellhead can now be calculated, and we obtain the following response function:

$$R = \frac{p_{well}(t)}{\tilde{q}} = \frac{1}{\tilde{q}_\omega e^{i\omega t}}[p(r_w,t) + \Delta p_S] = \frac{K_0[\xi] + S}{k + i\omega W_S\cdot(K_0[\xi] + S)}$$

The multiplier of the oscillatory function is a complex number. Its absolute value describes the amplitude of the pressure response to the rate constraint; its argument describes the phase delay of the response. With respect to a well with no skin and no wellbore storage, a positive skin has the effect of increasing the amplitude and changing the phase of the signal over the full frequency spectrum. Wellbore storage reduces the amplitude predominantly at higher frequencies due to the term with ω in the denominator – rather than the term ξ which scales with $\sqrt{\omega}$.