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# Characterization of Mass/Heat Transfer in Fractured Geothermal Reservoirs Using Mathematical Model of Complex Dynamical System

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## ABSTRACT

A fractional advection–dispersion equation (fADE) offers a simple but effective tool for describing anomalous mass transport in a fractured reservoir. A diffusive analogy between mass and heat transfer is used to derive a new fractional heat transfer equation (fHTE) to analyze the thermal response within the reservoir. The fHTE derived here is used to model a reservoir around a fault zone with improved results over conventional models. The fracture density in the fault zone decays from the high permeability zone by a power-law. The fADE and the fHTE constitutional parameters are shown to correlate with the decay of fracture density. The fADE constitutional parameters, obtained by curve-fitting the tracer response, estimate the fHTE constitutional parameters to predict the thermal response to reinjection.

## 1. INTRODUCTION

Reinjection prevents a decline in pressure and an exhaustion of water in a geothermal reservoir. One of the major problems with this process, however, is the possibility of an early thermal breakthrough in the production well. Tracer testing is a standard method for tracing mass transport, and is a valuable tool in the design and management of production and injection operations. The advection–dispersion equation (ADE) based on Fick's law has been widely used to solve a range of problems in the analysis of tracer transport. The Fickian equation, however, fails to model highly fractured reservoirs for which heavy tailing of tracer breakthrough is often observed.

To model the non-Fickian behaviors of fractured reservoirs, several tracer transport models have been proposed. For example, to extend the ADE, the dual-porosity model has been developed to simulate solute transport through both high-permeability and low-permeability domains (Coats and Smith 1964). In a fractured geothermal reservoir, the multiple flow-path model has been used frequently (Horne and Rodriguez1981; Axelsson et al. 2001); this model is includes several flow paths that may be isolated from one another by fracture distributions. Fomin et al. (2011) derived the fractional advection- dispersion equation (fADE) that accounts for the diffusion of a solute into the surrounding rock of fractal geometry, which enables a reproduction of the tracer responses observed in geothermal fields. Since the tracer response can be an indicator of thermal breakthrough, it is reasonable to suppose that the effect of heat transfer into the surrounding rocks could be described by extension of the fADE.

With transport properties similar to those of a thermal front, a chemical front can be used as an indicator of thermal breakthrough. Migration of the cold-water front for a single-phase liquid flow that conducts heat from a semi-infinite matrix is fairly well understood through the work of Lauwerier (1955) and Bodvarsson (1972). Consequently, the temperature evolutions at the production wells have been estimated (Gringatten and Sauty 1975; Bodvarsson and Tsang 1982; Kocabas 2004). This model has been widely used to estimate the cooling effects of the re-injected fluid in several geothermal fields but leads to an overestimation of temperature decline (Aksoy et al. 2008). Shook (2001) proposed tracer analysis methods that predicted thermal breakthrough in geothermal reservoirs. This study indicates that transforming the variables of the tracer data provides a quick and simple means of predicting the beginning of cooling of the produced fluids under the assumption that heat conduction is negligible. Because transverse heat conduction may occur and play an important role in temperature change in fractured reservoirs, this study also pointed out that modifications to the method are required to extend it to fractured media.

Suzuki et al. (2014) has proposed a fractional heat transfer equation (fHTE) based on the fADE used to estimate thermal response. The fHTE including fractional derivatives is derived by drawing a diffusive analogy between mass and heat transfer. In this study, we demonstrate flow behavior of fault zone architectures in a geothermal reservoir with fluid injection. Synthetic field performance data is generated to validate the applicability of the method for estimating thermal response. The parameter of the fault structure is formulated on the basis of the fADE and the fHTE constitutional parameters.

## 2. MATHEMATICAL MODEL

Bulk permeability in a geothermal reservoir is dominated by a large-scale fault that forms the main conduit of fluid flow (Massart et al., 2010). The hydrological systems of fault zones strictly depend on the fault architecture resulting from a complex set of components, such as the fault core and the damage zone (Caine et al., 1996). It is known that the fault zone is mechanically related to the growth of the fault zone, while the damage zone of subsidiary structures surrounds the fault zone (Evans et al. 1997; Le Garzic et al. 2011). Several studies have shown that damage decays sharply with distance from the fault core (Chester et al., 2005; Mitchell and Faulkner, 2009). Savage and Brodsky (2011) suggests the fracture density, *FD*, at the damage zone decays with distance from faults according to a power law as follows

$$FD = FD_0 r^{-n} \tag{1}$$

where $FD_0$ is the fault constant (the fracture density at 1 meter from the fault), $r$ is the distance from the fault, and $n$ is the exponent describing the decay.

Brogi (2008) indicates the fault zone architectures in the Rapolano geothermal area (hinterland of the Northern Apennines, Italy), in which the fracture density reaches its maximum value close to the fault core and decreases with the distance from the slip zone. The frequency distribution histogram of fractures in the Rapolano geothermal area can be fit to a power law distribution as shown in Fig. 1. Although highly constant fracture density around the vicinity of the fault zone can be also observed, the fracture density surrounding the area decreases with the distance. On the basis of the tendency for the fracture density, let the diffusion coefficient obey power law scaling (fractal). A fractal diffusion equation using the fractal diffusion coefficient can be converted into a term of a temporal fractional derivative (Samko et al. 1993).

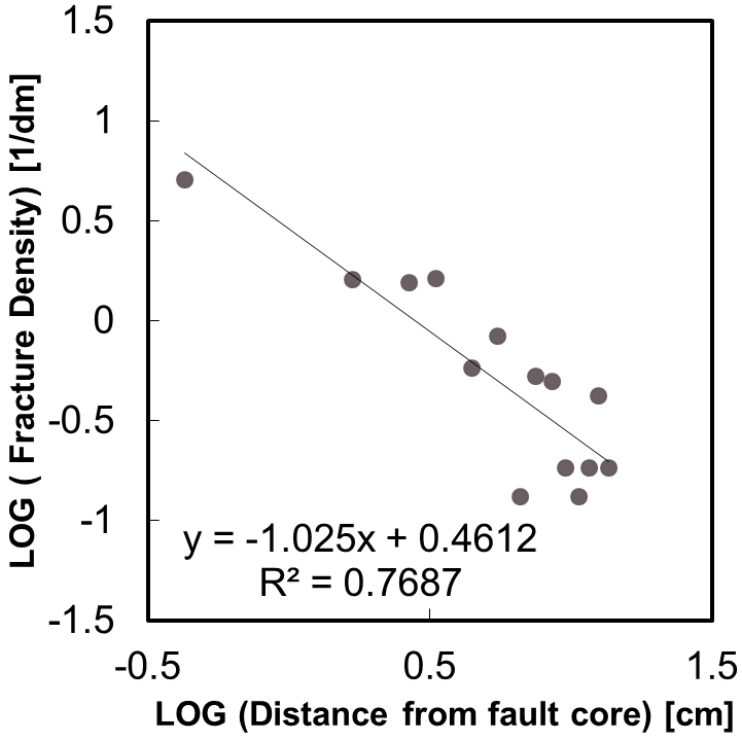



**Figure 1: Fracture density in the Rapolano geothermal area (after Borgi, 2008).**

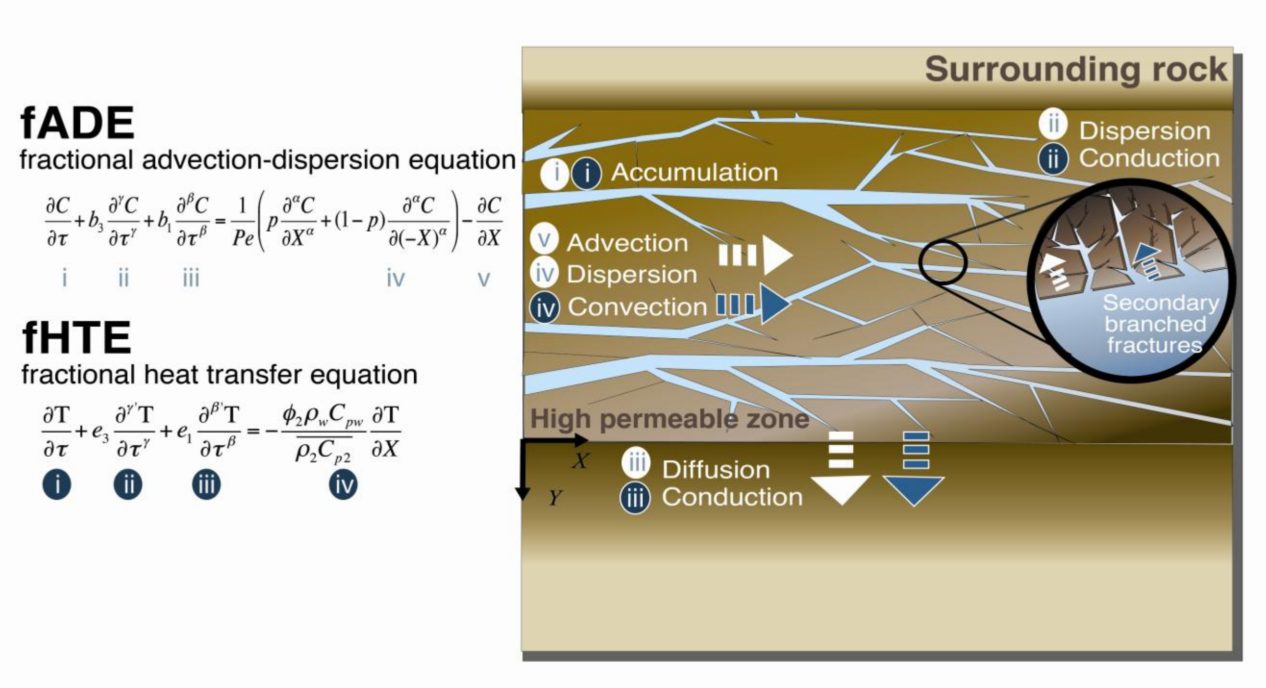



**Figure 2: Schematic of a fractured reservoir.**

Fomin et al. (2011) has proposed the fractional advection-dispersion equation (fADE) to model mass transport in a fractured reservoir (Fig. 2) using fractional derivatives as follows:

$$\frac{\partial C}{\partial \tau}+b_3\frac{\partial^\gamma C}{\partial \tau^\gamma}+b_1\frac{\partial^\beta C}{\partial \tau^\beta}=\frac{1}{Pe}\left(p\frac{\partial^\alpha C}{\partial X^\alpha}+(1-p)\frac{\partial^\alpha C}{\partial(-X)^\alpha}\right)-\frac{\partial C}{\partial X} \tag{2}$$

where $\tau$ is time, $X$ is distance from the injection zone, and $C$ is the tracer concentration in the reservoir. The coefficients $b_3$ and $b_1$ represent retardation processes caused by the diffusivities into the secondary branched fractures and into the surrounding rocks, respectively. $Pe$ is the Peclet number, the constants $\beta$ $(0 < \beta \leq 1)$, $\gamma$ $(0.5 \leq \gamma \leq 1)$, and $\alpha$ $(0 < \alpha \leq 1)$ are the orders of the fractional temporal derivatives, and $p$ $(0 \leq p \leq 1)$ is the skewed parameter. Detailed derivations of the equation can be found in the literature (Fomin et al. 2005, 2011).

Analogous behaviors of heat and mass transfer have been long recognized (Welty et al., 2007). A heat transfer equation is presented as follows (Suzuki et al., 2013):

$$\frac{\partial T}{\partial \tau}+e_3\frac{\partial^{\gamma'} T}{\partial \tau^\gamma}+e_1\frac{\partial^{\beta'} T}{\partial \tau^\beta}=-\frac{\phi_2\rho_w C_{pw}}{\overline{\rho_2 C_{p2}}}\frac{\partial T}{\partial X} \tag{3}$$

where $\overline{\rho C_{p2}}=m_2\rho_w C_{pw}+(1-m_2)\rho_r C_{p2}$; $\rho_w$ and $\rho_r$ are the density of water and rock, respectively; $C_{pw2}$ and $C_{pr2}$ are the heat capacity of water and rock in the reservoir. $T$ is temperature. The coefficients $e_3$ and $e_1$ represent retardation processes caused by the heat diffusivities into the secondary branched fractures and into the surrounding rocks, respectively. The orders $\beta$' $(0 < \beta \leq 1)$ and $\gamma$' $(0.5 \leq \gamma \leq 1)$ are the orders of the fractional derivatives.

This study discusses the effect of diffusion and heat transfer into the surrounding rocks, which are represented by the third terms on the left-hand side in Eqs (1) and (2). Thus, the heterogeneity inside the reservoir is neglected ($b_3 = e_3 = 0$, $\alpha = 1$).

## 3. SIMULATION SETUP

The influence of injected water on tracer and thermal responses for different permeability distributions of the surrounding rocks is investigated using the general-purpose reservoir simulator TOUGH2 (Pruess et al., 1999). The reservoir consists of a fault zone with high permeability, which is surrounded by impermeable rocks. The reservoir is assumed to be homogeneous with a permeability of $1 \times 10^{-13}$ $m^2$. One-dimensional advective bulk flow travels from the inlet towards the outlet. The tracer is injected at 0.2 kg/s for one day in the injection zone, after which the injection is switched to fresh water. The tracer response and temperature profile are observed in the production zone.

In this study, the fracture density surrounding the high permeability zone is expressed in terms of the permeability, which is defined by a power law in the following form

$$K_s(y) = K_r \left( y - \frac{T_r}{2} \right)^{\theta} \tag{4}$$

where $K_r$ and $T_r$ are the constant permeability and the thickness of the reservoir, $y$ is the distance form the reservoir, and $\theta$ is the index of anomalous dispersion.

## 4. RESULTS AND DISCUSSION

The tracer responses and thermal responses calculated using TOUGH2 for different $\theta$ are fitted by the fADE and the fHTE, respectively. For comparison, the best-fit curves of the classical ADE are also applied. The constitutional parameters of the models are determined by minimization of the root mean squared error.

The influence of the permeability distribution in the surrounding rocks is shown in Fig. 3. Figure 3(a) shows the tracer responses when the surrounding rocks are impermeable. The tracer response is a symmetric Gaussian distribution. This is because the tracer transport process is governed solely by advection. Both the fADE and the ADE with $b_1$ = 0 can characterize the tracer response computed by TOUGH2.

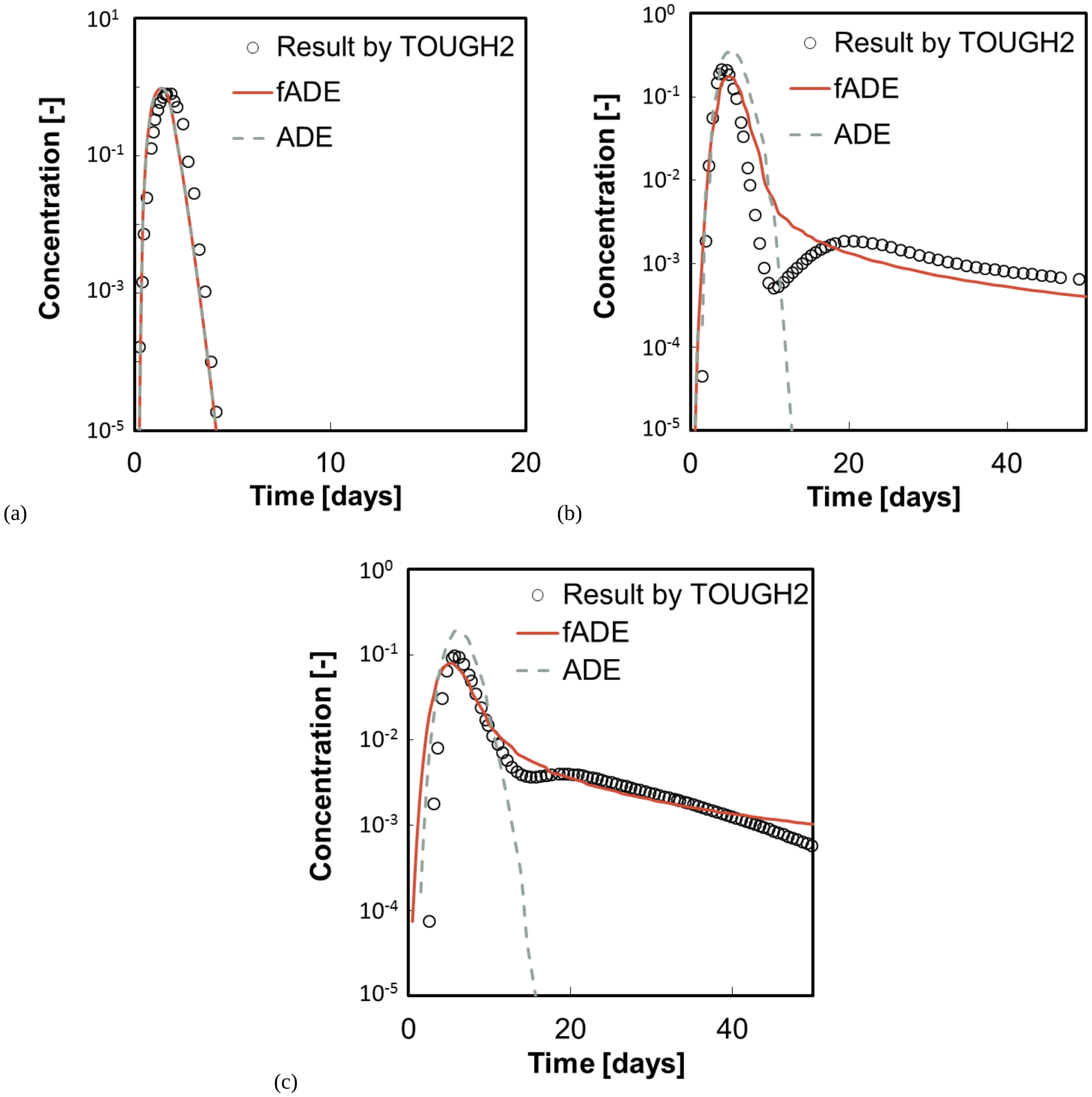



**Figure 3: Simulated tracer responses by TOUGH2 (symbols) (Fig. 3.5) and best fits with the fADE (solid lines) and the ADE (dashed lines) for (a) $K_s$ = 0, (b) constant permeability ($K_s$ = 1×10$^{-15}$), and (c) spatially varying permeability ($\theta$ = 1.5).**

Figure 3(b) shows the tracer response for a constant permeability of $K_s = 3\times10^{-15}$ m$^2$ along with the best-fit curves of the fADE and the ADE. For the case when $K_s$ is constant, the tracer responses exhibit apparently long tails and secondary peaks. The double-peak responses indicate that there are two distinct flow pathways between wells. If the highly permeability zone is surrounded by less permeable rocks, then mass exchange between the two domains occurs. In this case, the tracers will travel not only inside the highly permeable zone but also through the surrounding rocks. The first peak in the tracer response is produced by advective flow in the reservoir; the secondary peak is then produced as a result of diffusion into the surrounding rocks. It appears that the ADE solution provides no long tail and deviates significantly from the long-term tracer response. The fADE fitting result captures the overall trend of the tracer response and is well matched with the long-term behavior; however, this result shows disagreement with the double-peak behavior in the tracer response. This discrepancy is due to the derivation of the fADE based on the fractal geometry (Fomin et al., 2011); thus, the fADE does not necessarily describe the detail of the double-peak behavior.

For a spatially varying permeability distribution in the surrounding rocks, the permeability is defined according to a power law as in Eq. (4). Figure 3(c) shows the result of a spatially varying permeability for $\theta = 1.5$ along with the best-fit curves of the fADE and the ADE. Compared to the constant permeability distribution of the surrounding rocks, the spatially varying distribution produces retardation in the arrival time of the tracer peak and a decrease in the peak concentration. In addition, the previously apparent secondary peak is not observed. In this case, the flow domains are not distinguished as merely two domains. Several flow patterns are considered to occur due to the wide range of permeabilities of the surrounding rocks, and the superposition of the tracer responses for each pattern may produce a gradual decrease in the concentration profile. Therefore, the fADE, which consists the fractal coefficient, is in reasonable agreement with the calculated tracer response.

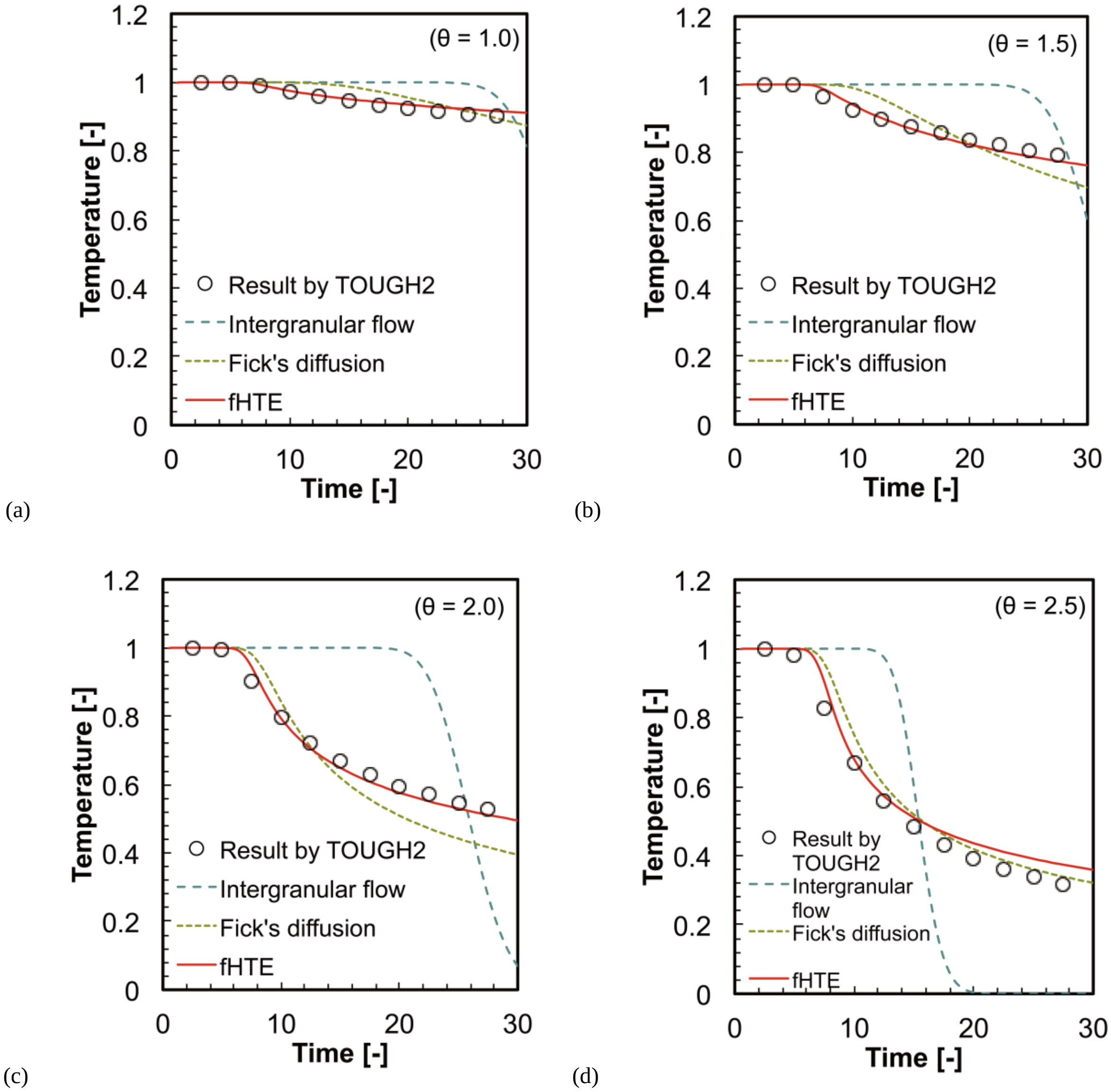



**Figure 4: Simulated temperature profiles by TOUGH2 using the fault model and the best fit with the fHTE.**

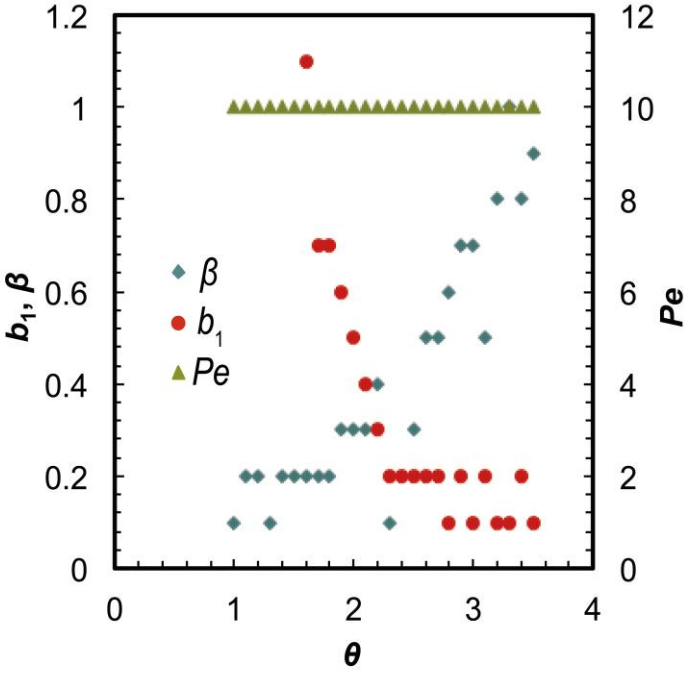



**Figure 5: Relationship between the fADE constitutional parameters and *θ*.**

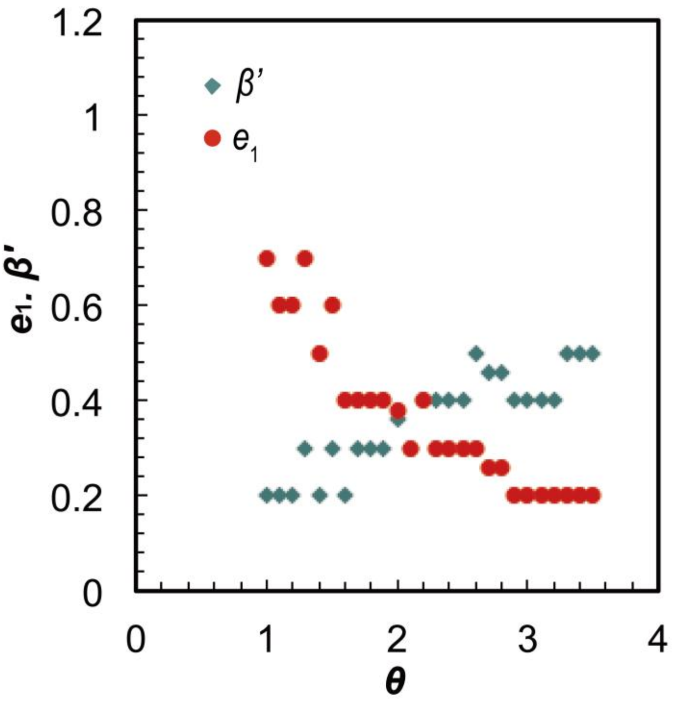



**Figure 6: Relationship between the fHTE constitutional parameters and *θ*.**

To verify the applicability of the fHTE, the equation is applied to the calculated temperature profiles. Temperature profiles for different permeability distributions in the surrounding rocks with the best-fit curves of the fADE are shown in Fig. 4. The temperature has already been normalized; the initial temperature of the reservoir is set to 1, and the temperature of the injected water is set to 0. Continuous cold-water injection is simulated until the thermodynamic state of the reservoir is in equilibrium. The fitted lines by conventional models are also plotted. Bodvarsson (1972) has derived the basic equation for temperature field for rocks with intergranular flow. Gringarten and Sauty (1975) derived procedures based on Fick's Law to determine the thermal breakthrough characteristics with vertical thermal conductivity in the cap rock and the bedrock. It appears that the solutions of the fHTE are in good agreement with the calculated temperature profile for the case of spatially varying permeability distributions in the surrounding rocks.

The best-fit fADE and fHTE constitutional parameters are plotted in Figs. 5 and 6. The constitutional parameters of the models, i.e., $\beta$, $b_1$, and *Pe* in the fADE and $e_1$, and $\beta$' in the fHTE, are determined by minimization of the root mean squared error. Smaller $\theta$ causes higher penetration from the reservoir into the surrounding rocks. *Pe* in the fADE is determined to be constant by the optimization. It may result from the constant permeability inside the reservoir in this study. The smaller $\theta$ leads to increases in the retardation parameters $b_1$ and $e_1$ and decreases in the orders of the fractional derivative $\beta$ and $\beta$', respectively. It is demonstrated that the fHTE constitutional parameters show the same dependence as the fADE constitutional parameters.

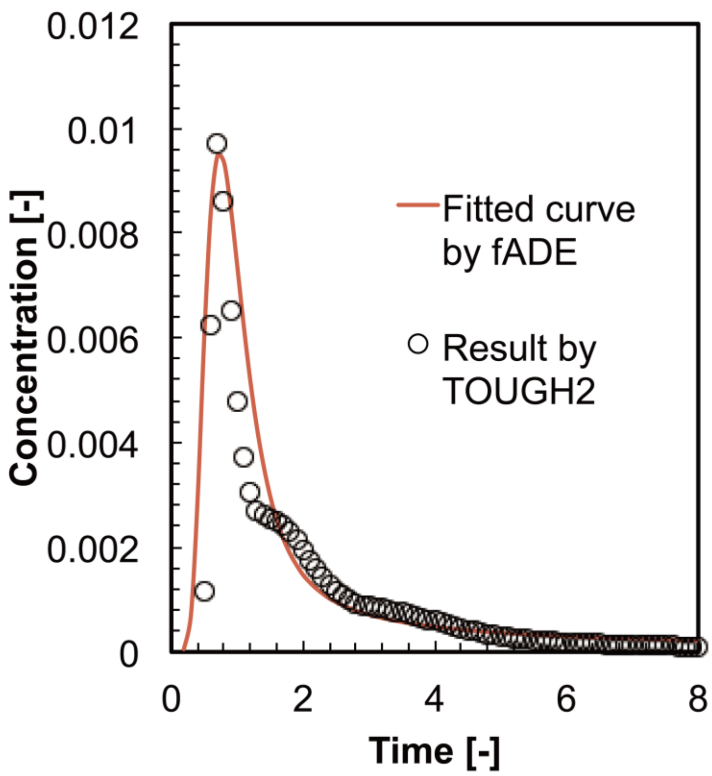
Regression equations to estimate fHTE constitutional parameters based on the fADE constitutional parameters can be obtained as follows:

$$e_1 = -0.12\beta + 0.17 b_1 + 0.25\ (\pm 0.05) \tag{5}$$

$$\beta' = 0.15\beta + 0.10 b_1 + 0.34\ (\pm 0.05) \tag{6}$$

Eqs. (5) and (6) are used to estimate $e_1$ and $\beta$' from a tracer response simulated by TOUGH2. The tracer response along with the best-fit curves of the fADE for $\theta = 1.0$ is shown as Fig. 7. The best-fit fADE constitutional parameters are $\alpha = 1.0$ , $\beta = 0.1$, $b_1 = 2.0$, and $Pe = 10.0$. Substituting the parameters into Eqs. (5) and (6), the $e_1$ and $\beta$' are estimated: $e_1 = 0.66$ and $\beta' = 0.15$. The temperature profile simulated by TOUGH2 and the predicted result are plotted in Fig. 8. The predicted result is in good agreement with the temperature profile. The correlation coefficient between both temperature profiles is $R^2 = 0.992$. We also find that the correlation coefficients between both temperature profiles are greater than 0.99 regardless of $\theta$. Hence, the fADE can offer an estimation of temperature profiles via the fHTE.

These comparisons imply that the fHTE can offer an accurate evaluation of temperature profiles and an elaborate characterization of thermal response in a fault-related subsidiary structure. As mentioned previously, the fADE can summarize complexity based on fractal geometry and can quickly and efficiently analyze mass transport in a fractured reservoir. In addition, our previous study has demonstrated that the fADE offers a method for predicting tracer responses irrespective of well intervals in a fractured reservoir (Suzuki et al., 2012). Future developments from this study will include prediction of thermal breakthrough irrespective of well intervals and optimization of the well location and/or the condition of injection. The fHTE is expected to be a powerful tool for predicting premature thermal breakthrough in a fractured reservoir.




**Figre 7: Simulated tracer responses by TOUGH2 and the best fits with the fADE for $\theta = 1.0$.**

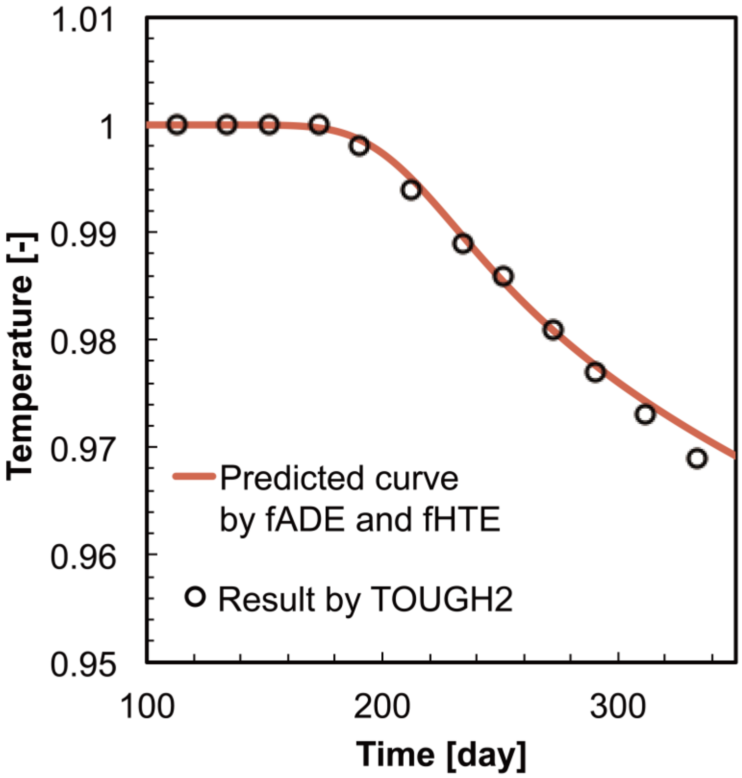



**Figure 8: Simulated temperature profiles by TOUGH2 and the estimated curves of the fHTE by using Eqs. (5) and (6) for $\theta$ = 1.0.**

## 5. CONCLUSIONS

On the basis of the derivation of the fADE, a heat transfer model using fHTE is developed for the characterization of geothermal reservoirs. We find that the fracture density in a geothermal area can be described by a power-law approximation with distance from the fault core. The fADE introduces a fractal diffusion coefficient, which decreases with distance from a high permeability zone in a reservoir to result in the diffusion into the surrounding rocks.

A general reservoir simulator, TOUGH2, was used to generate reservoir performance data. It revealed the influence of different permeability patterns in the surrounding rocks. Conventional advection–dispersion models can describe tracer and thermal responses only for impermeable surrounding rocks. An increase in the permeability of the surrounding rocks leads to a long tail in the tracer response and a gradual temperature decline. The fADE solution was found to be in reasonable agreement with the tracer response for spatially varying permeability distributions, but the agreement with constant permeability was less satisfactory. Curve fittings on the temperature profile suggest that the fHTE can be used to interpret temperature profiles in fractured media. Higher permeability of the surrounding rocks resulted in an increase in the retardation parameters and a decrease in the fractional orders in the fADE and the fHTE. A linear regression analysis is conducted to provide predictive equations of the fHTE constitutional parameters. Those equations are expected to be a powerful tool for predicting premature thermal breakthrough in a fractured reservoir by using tracer responses.

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