

A Set of New Correlations for the Compressibility of Two-Phase Water and for the Steam Z Factor in a Wide Thermodynamic Range

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ABSTRACT

The methods used in the interpretation of transient pressure tests (TPT) range from very simple analytical models up to highly complex numerical techniques. The models of the traditional TPT analysis use several simplifications, for example, that the fluid is lightly compressible and the flow has a simple geometry. Other assumptions include homogeneous rock properties, uniform pressure in the reservoir and constant extraction rates. Computer-aided current methods are capable of handling more complex scenarios with concrete geological characteristics and fluid realistic behavior. In the most advanced models, it is not necessary to assume a small compressibility and the fluid may be non-isothermal and two-phase. The TPT analyses in both cases allow studying the local behavior of the reservoir under dynamic conditions of phase change. In these models, the accurate calculation of fluid compressibility in the total rock-liquid-steam system is needed. In case that the reservoir produces only steam or the system conditions goes farther the critical point presenting a supercritical phase, the calculus of the steam compressibility is the key point in the correct interpretation of the pressure test. When the drain volume of a well contains pure steam, the traditional pressure equations based on the Theis model, cannot be used directly. This is due to the high compressibility of the steam and to the fact that all the thermodynamic properties of steam depend on pressure and temperature (p, T). This means that the differential equations of the simplest model become nonlinear. The Z compressibility factor is defined as the ratio of the volume occupied by a real gas to the volume occupied by an ideal gas at the same (p, T) conditions. The Z factor is not constant and depends on (p, T) for different gases. Under geothermal conditions the radial flow differential equation can be formulated by defining a normalized pseudopressure of the real gas, which depends on both, its dynamic viscosity and Z. In this paper, we present new correlations for the computation of the geothermal fluid compressibility for each one of its two phases. A new correlation to compute approximately the Z factor is also included. In the case of the steam, its compressibility behaves in a strange way when its temperature is between the critical point and 450°C. These correlations cover a wide thermodynamic range, [1, 400] bar and [10, 450] °C, which includes some observed singularities, the traditional geothermal enthalpy as well as reservoirs with very high enthalpy at supercritical temperature conditions.

1. INTRODUCTION

In advanced models for pressure test analysis, it is not necessary to assume a small compressibility and the fluid may be non-isothermal and two-phase. In wells that produce pure steam, the small compressibility hypothesis is totally erroneous. The TPT analysis in both cases allows studying the local behavior of the reservoir under dynamic conditions of phase change. For two-phase flow systems, the Garg's model (1980) is completely analogous to the Theis model (1935) from a mathematical point of view, but it includes the accurate calculation of steam compressibility in the total rock-liquid-steam system. In case that the reservoir produces only steam or the system conditions goes farther the critical point presenting a supercritical phase, the calculus of the steam compressibility is the key point in the correct interpretation of the pressure test. In this paper, we present new correlations for the computation of the geothermal water compressibility of each one of its two phases. In the case of steam, its compressibility behaves in a strange way when its temperature is between the critical point and 450°C. The set of practical correlations that are able to reproduce this behavior is published for the first time, covering a wide thermodynamic range which is detailed in the next sections.

The rock compressibility coefficient measures the change in the bulk volume V_B when the hydrostatic pressure p changes with respect to an initial volume; it is the reciprocal of the bulk modulus K_B . When the test is done at constant temperature and drained conditions, the rock isothermal bulk compressibility C_B is defined as:

$$C_B = - \frac{1}{V_B} \frac{\Delta V_B}{\Delta p} = \frac{1}{K_B} \quad [Pa^{-1}] \quad (1)$$

Similarly, if V_f , p_f and ρ_f are the volume, pressure and density of the fluid respectively, the water isothermal compressibility is defined as:

$$C_f = - \frac{1}{V_f} \left(\frac{\partial V_f}{\partial p_f} \right)_T = \frac{1}{\rho_f} \left(\frac{\partial \rho_f}{\partial p_f} \right)_T \quad (2)$$

The compressibility of any geothermal phase is always a function of both, pressure and temperature.

2. COMPRESSIBILITIES OF WATER IN PRESSURE TESTS ANALYSIS (TPT)

The fluid compressibility plays an important role in TPT of geothermal wells. Buildup, Drawdown, Falloff, Interference, are the most common pressure tests (Horne, 1995). Besides the diffusion equation, fluid compressibility affects directly different physical properties of the fluid, such as wellbore storage, hydraulic diffusivity, fluid pressure, skin factor, radius of investigation, dimensionless time, storativity ratio in double porosity reservoirs, etc. As an example, the radial equation to compute the flowing bottom well pressure p_{wf} in the international system of units is:

$$p_{wf}(t) = p_i - 0.183234 \frac{Q_v \mu_f}{k b_a} \left(\log [t] + 0.868589 s + \log \left[\frac{k}{\phi \mu_f C_t r_w^2} \right] + 0.351378 \right) \quad (3)$$

Where Q_v , μ_f , s , k , ϕ , p_i , C_t , t , r_w , and b_a are volumetric fluid flow rate, viscosity, skin factor, permeability, porosity, initial pressure, fluid/rock compressibility, time, wellbore radius, and reservoir thickness, respectively. This equation shows that the pressure difference during the TPT is inversely proportional to the total compressibility $C_t = C_B + C_f$ of the reservoir.

2.1 Correlations for the Compressibility of Liquid Water

We describe here a set of practical correlations developed to compute water compressibility in a wide thermodynamic range. The raw data basis comes from the IAPWS-95 (*International Association for the Properties of Water and Steam*) equation of state (Wagner and Pruss, 2002). This formulation contains the thermodynamic properties of water for general and scientific use.

2.1.1 First Correlation for the Liquid Phase

The first model built for the liquid phase is as follows:

$$C_{liq1}(p, T) = 0.51606886334 \times 10^{-4} - 2.51668013407 \times 10^{-8} p - 2.09034801372 \times 10^{-7} T + 1.59289240484 \times 10^{-10} p T + 1.84524385704 \times 10^{-9} T^2 - 5.99776925278 \times 10^{-13} p T^2 \quad (4)$$

The two-dimensional correlation (4) for the liquid phase compressibility is approximately valid for pressures between 1 bar and 220 bar and for temperatures from 10 to 210°C (compressed liquid). It is not valid for values larger than 210°C, because the error increases with temperature. Figure 1 illustrates the behavior of this compressibility range.

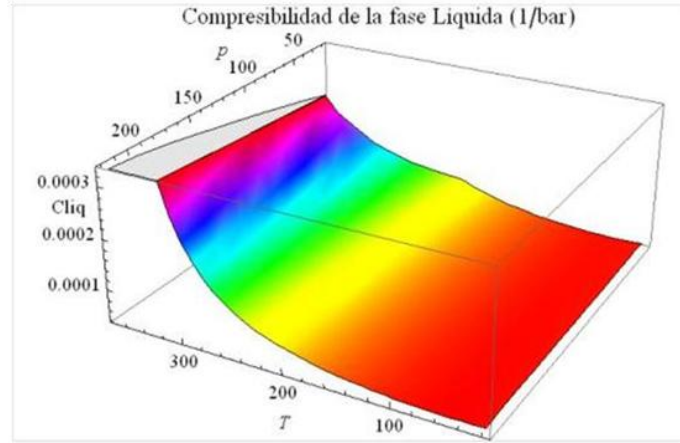


Figure 1: Compressibility of liquid water $C_{liq}(p, T)$ between [1, 300] bar and [10, 370] °C.

2.1.2 Second Correlation for the Liquid Phase

With a different mathematical model, we constructed the following correlation that is valid for pressures from 20 bar up to 300 bar and temperatures from 210°C up to 370°C (compressed liquid). The correlation is not valid for temperatures larger than 370°C.

$$C_{liq2}(p, T) = \frac{a_0 + a_1 T + a_2 T^2 + a_3 p + a_4 p T + a_5 p^2}{b_0 + b_1 T + b_2 p} + c_0 \quad (5)$$

Where the constant coefficients are:

$$a_0 = 93.1663311586, a_1 = -0.1968009864, a_2 = -0.1189515785 \times 10^{-3}, a_3 = -0.4047179477,$$

$$a_4 = 0.00214683589, a_5 = -0.1049247203 \times 10^{-3}, b_0 = 1.2926119842 \times 10^6,$$

$$b_1 = -4139.4544963629, b_2 = 1157.3957523284, c_0 = \begin{cases} 0, & \text{if } T \leq 260^\circ\text{C} \\ 6.1870252 \times 10^{-8} T, & \text{if } T > 260^\circ\text{C} \end{cases}$$

2.2 Correlations for the Compressibility of Steam

We describe another group of correlations to calculate the steam compressibility. Figure 2 illustrates the behavior of the vapor phase and figure 3 shows its supercritical phase. The computation was done in the range of 20 bar up to 180 bar for pressures, and for temperatures between 200°C and 350°C. The correlations have the general form:

$$C_{vap1}(p, T) = \alpha_0(p) + \alpha_1(p)T + \alpha_2(p)T^2 + \alpha_3(p)T^3 + \alpha_4(p)T^4 + \alpha_5(p)T^5 \quad (6)$$

These coefficients are functions of pressure $\alpha_n(p)$ and are written in Table 1:

Table 1.- Functional Coefficients at different pressures for the correlation (6)

p (bar)	$\alpha_0(p)$	$\alpha_1(p)$	$\alpha_2(p)$	$\alpha_3(p) \square 10^{-8}$	$\alpha_4(p) \square 10^{-11}$	$\alpha_5(p) \square 10^{-10}$
20	0.2873563	-0.002926293	0.00001397944	-3.011388	2.44876	0
30	0.3612855	-0.003862092	0.00001746083	-3.546081	2.713991	0
40	0.5518567	-0.006161136	0.00002746525	-5.484529	4.123256	0
50	0.8256363	-0.009329163	0.00004100774	-8.061324	5.962759	0
60	1.332468	-0.01517458	0.00006618992	-12.89028	9.438075	0
80	3.682279	-0.04195056	0.0001805821	-34.63108	24.94097	0
100	10.37273	-0.1169838	0.0004962977	-93.70659	66.40901	0
120	42.19327	-0.4740797	0.001999556	-375.0362	263.8818	0
130	21.88433	-0.232275	0.0009254172	-163.9096	108.8718	0
150	72.54727	-0.7647016	0.00302328	-531.2027	349.9518	0
170	3838.536	-50.37704	0.2644192	-69382.27	91010.81	-4.774319
180	675.4677	-8.255423	0.04031741	-9834.567	11981.66	-0.5832577

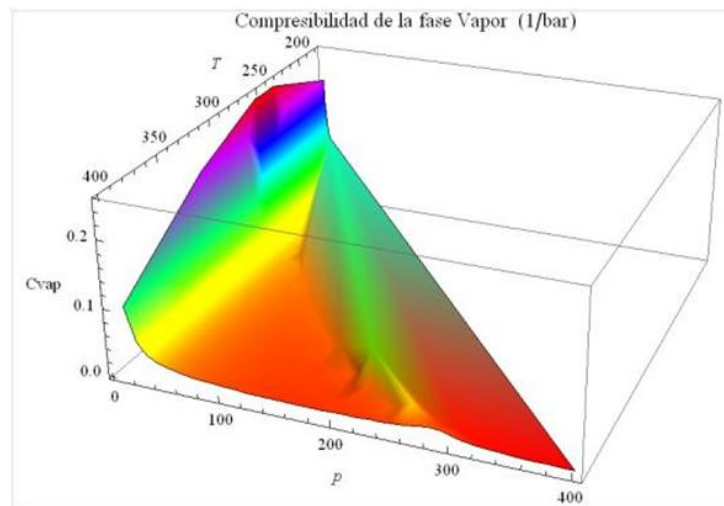


Figure 2: Compressibility of steam $C_{vap}(p, T)$ between [1, 400] bar and [200, 400] °C.

Starting from 180 bar steam compressibility behaves erratically when T increases. Figure 2 shows that at 300°C $C_{vap}(p, T)$ will form a fold that continues until 400°C, which is very difficult to model. This strange behavior of steam compressibility occurs in a neighboring region to the critical point. For example, at $T = 320^\circ\text{C}$, the correlation for compressibility is more complex than the previous one:

$$C_{vap2}(p, T) = \sum_{k=0}^{11} \beta_k(T) p^k \quad (7)$$

Expanding this expression (7):

$$C_{\text{vap}2}(p, T_0) = -5.592918 \cdot 10^{-20} p^{11} + 3.79771 \cdot 10^{-17} p^{10} - 1.137487 \cdot 10^{-14} p^9 + 1.978352 \cdot 10^{-12} p^8 - 2.213567 \cdot 10^{-10} p^7 + 1.66816 \cdot 10^{-8} p^6 - 8.617871 \cdot 10^{-7} p^5 + 3.047846230929007 \cdot 10^{-5} p^4 - 7.2457952665566 \cdot 10^{-4} p^3 + 0.01114754890018806 p^2 - 0.10369336479407769 p + 0.5159308339752458 \quad (8)$$

Similar correlations can be obtained using the same technique (7) as a template. Figures 4, 5, 6 and 7 show this strange behavior as the temperature increases. The data points are of the IAPWS-95, the curves correspond to the model we created.

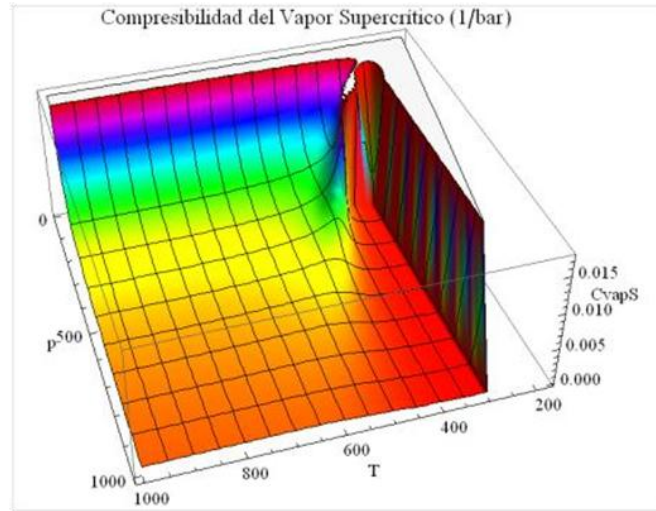


Figure 3: Compressibility of steam $C_{\text{vap}}(p, T)$ covering the supercritical region. between [1, 400] bar and [200, 400] °C.

The following figures illustrate the steam compressibility behavior between 320°C and 400°C. The dots correspond to the general model (7) and the continuous curve to the IAPWS-95 formulation.

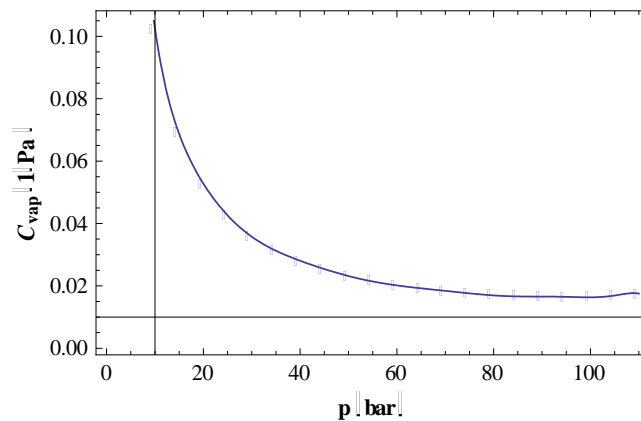


Figure 4: Compressibility of steam for $T = 320^\circ\text{C}$.

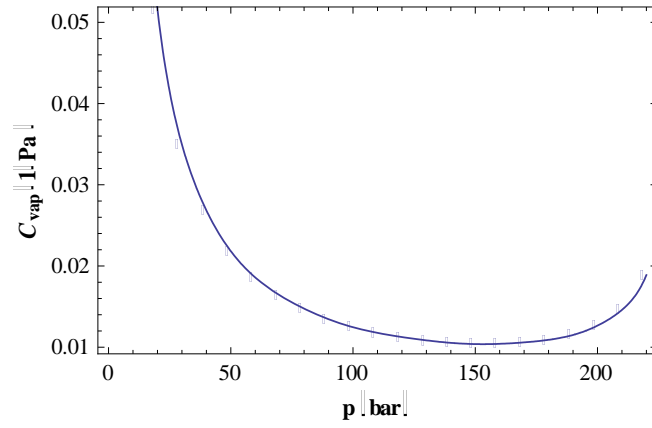


Figure 5: Compressibility of steam for $T = 380^{\circ}\text{C}$.

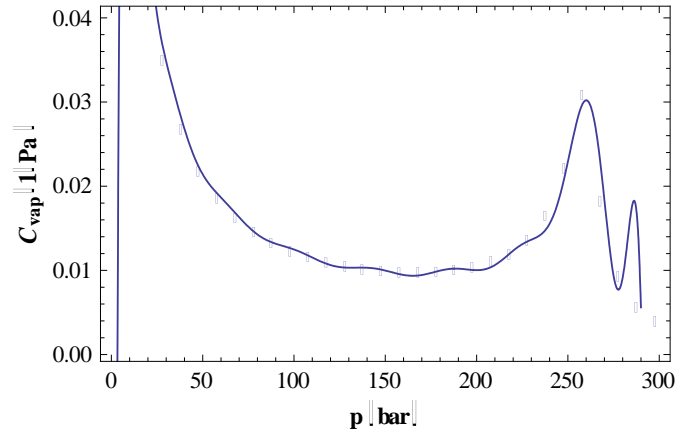


Figure 6: Compressibility of steam for $T = 390^{\circ}\text{C}$.

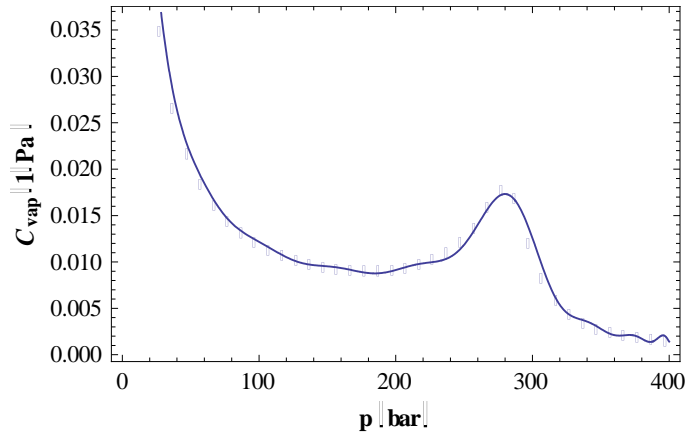


Figure 7: Compressibility of steam for $T = 400^{\circ}\text{C}$.

For the supercritical water phase, we built the next two-dimensional correlation (Figure 3):

$$C_{\text{vap}2}(p, T) = -0.9308259 + 0.9396242 e^{-p} - 0.00001556435 p + 2.926128 \cdot 10^{-8} p^2 - 4.855833 \cdot 10^{-11} p^3 + 2.233625 \cdot 10^{-14} p^4 - 0.00001933079 T + 1.841308 \cdot 10^{-8} p T + 8.662136 \cdot 10^{-12} p^2 T + 1.458951 \cdot 10^{-8} T^2 - 1.609355 \cdot 10^{-11} p T^2 - 2.16006 \cdot 10^{-12} T^3 \quad (9)$$

3. CORRELATIONS FOR THE Z COMPRESSIBILITY FACTOR FOR TPT IN STEAM WELLS

When a geothermal reservoir region has only a gas phase or when the drain volume of a well contains pure steam, the traditional pressure equations which are based on the Theis model, cannot be used directly. This is due to the high compressibility of the steam and to the fact that all the thermodynamic properties of steam depend on pressure and temperature (p, T). This means that the differential equations of the simplest model become nonlinear. There are three ways to address this problem. The first is to use an extended two-phase model by considering a maximum saturation of the gas phase, very close to one ($S_g = 0.999$); in this way, the two-phase mixture practically contains only steam. The second one is to solve directly the differential equations using nonlinear numerical methods. The third one is to consider the vapor as a real gas and use a model which includes the vapor with this characteristic. In this paper we include a new correlation to compute approximately the steam Z factor.

3.1 The Steam Z Compressibility Factor

The Z compressibility factor is defined as the ratio of the volume occupied by a real gas to the volume occupied by an ideal gas at the same (p, T) conditions. However, Z is not constant and depends on those conditions for different gases. Using this definition and the van der Waals equation of state we deduce that:

$$Z = \frac{p v}{R T} = \frac{v}{v - b} - \frac{a}{R T v} \quad (10)$$

Where v is the molar volume, T is temperature in K, $R = 8.314472$ J/mol/K is the ideal gas constant, a and b are the van der Waals constants, which can be computed in terms of the critical pressure and the critical temperature of steam:

$$a = \frac{27}{64} \frac{R^2 T_c^2}{p_c}; \quad b = \frac{R T_c}{8 p_c} \quad (11)$$

The thermodynamic dependency of Z can also be established in terms of its reduced pressure and reduced temperature, which are defined as $p_r = p/p_c$ and $T_r = T/T_c$, where the critical pressure and critical temperature of steam are $p_c = 221.15$ bar y $T_c = 647.27$ K, its critical density is $\rho_c = 315.46$ kg/m³. Figure 8 illustrates the behavior of $Z(p_r, T_r)$ for steam and other gases.

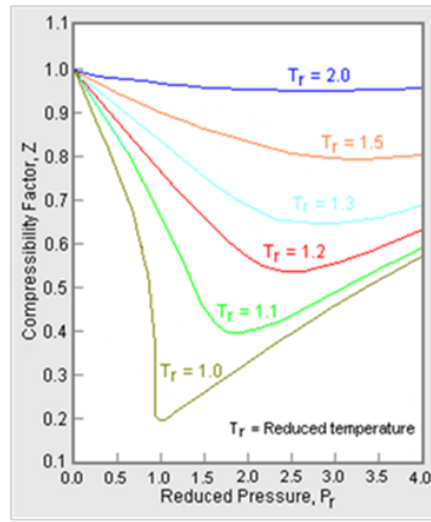


Figure 8: The Z factor as function of p_r, T_r for different gases: steam, methane, CO₂, N₂, etc.

Under reservoir conditions at 300 °C and 50 bar, the steam Z factor is 0.902; at 100 bar, $Z = 0.776$, while at 150 bar, $Z = 0.56$; on average it seems that the Z factor decreases with pressure. Under given thermodynamic conditions of $Z(p, T)$ the radial flow differential equation can be formulated by defining a steam normalized pseudopressure $m(p)$ (Horne, 1995), which also depends on its dynamic viscosity:

$$m(p) = 2 \int_{p_0}^p \frac{p}{Z(p, T) \mu(p, T)} dp \quad (12)$$

Where p is the measured pressure of the test and p_0 is a reference pressure for example, the minimum p obtained from the test. Equation (12) clearly shows the key importance of computing the Z factor in steam wells.

3.2 Correlations for the Steam Z Compressibility Factor

Using a table of numerical values obtained from equation (10) and verified with the numerical formula that can be consulted in the WolframAlpha web page (www.wolframalpha.com), we built the following general correlation for $Z(p, T)$ in the two-dimensional range $212.4 \leq T \leq 450$ °C and $20 \leq p \leq 220$ bar. Figure 9 shows in a 3D graphic the behavior of Z in the mentioned region:

$$Z(p, T) = 0.95115782784 - 0.00664086125 p - 3.70368391114 \times 10^{-8} p^3 + 0.0001150956332 T + 0.0000132895865 p T + 1.52046481158 \times 10^{-8} p^2 T + 0.0000022127265 T^2 - 5.90954955214 \times 10^{-9} T^3 \quad (13)$$

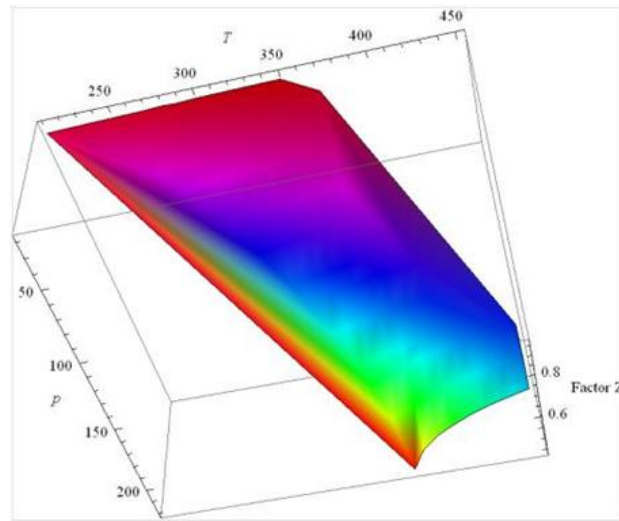


Figure 9: Three-dimensional graphic of $Z(p, T)$ between [20, 220] bar and [212.4, 450] °C.

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