Stress-Permeability Relationships in Low Permeability Systems: Application to Shear Fractures

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Keywords: shear fractures, boundary element method, stress-permeability models

ABSTRACT

For tight or low matrix permeability reservoirs, such as geothermal fields, faults and fractures are the main conduits for flow. Parallel plate models are typically used to model heat and mass transport in fractures. Using such a model, the cubic law specifies that the permeability ($k$) is equivalent to $b^2/12$, where $b$ is the average fracture aperture. However, the limitation of this approach is that it fails to take into account the roughness within the fracture plane that results to permeability variation. An important component of understanding fracture surface roughness is identifying its source. There are several mechanisms by which rough fractures can be created, through chemical effects, mechanical effects, inelastic effects, and stress.

In this study, the fracture generation mechanism considered was the application of stress. The spatial variation of crack opening depended on the following: applied stress values and direction, stress interactions, and initial shape of the fracture or fault. Analytical models for stress-dependent permeability were evaluated. The displacement discontinuity boundary element method (DDM) integrated with the complementarity algorithm was used as a consistent model that can simulate crack opening and slip from imposed stress conditions under elastic behavior. For future work, it is recommended that results from the DDM method be compared to the analytical models of stress-permeability relationships.

1. INTRODUCTION

Reservoirs found in tight rocks have gained more interest in the last few years. This interest is driven by the increasing importance of shale gas in the U.S. energy mix and the potential of using enhanced geothermal systems (EGS) to increase geothermal energy usage. Both shale gas and EGS are made possible by hydraulic fracturing, where permeability is increased by using high pressure fluid to create new fractures or induce slip on preexisting fractures (McClure and Horne, 2011). An MIT report on the potential of EGS estimated that EGS could provide 100 GWe of generating capacity in the next 50 years if appropriate investments in research and development were made (Tester et al., 2006).

Given the great potential of EGS applications, understanding the fundamental physics of flow within fractures found in tight reservoirs is important in optimizing production from these types of energy resources. Often, it is assumed that the aperture and permeability within the fault or fracture are constant. However, this assumption is an oversimplification because real faults and fractures have rough surfaces which alter the flow paths (Ritz et al., 2012). Flow experiments conducted on rough fracture surfaces have demonstrated channeling flow where only 30 percent of the fracture area is conductive to flow (Ishibashi et al., 2012). Thus, 70 percent of the fracture area is not conductive. Figure 1 includes a picture from Ishibashi et al. (2012) and shows the simulated fluid flow distribution on two natural fractures. The simulated fluid flow distribution in Figure 1 illustrates significant channeling with most of the fluid exiting at port 4 of fracture B. This observation was consistent with the results of an actual flow experiment.

![Figure 1: Experiments on flow channeling (from Ishibashi et al., 2012). Left: photograph of a granite rock sample with two intersecting fractures. Right: fluid flow simulation results showing flow channeling within rough fractures.](image-url)
Aside from affecting flow distributions, flow channeling can potentially have a significant impact on heat transfer processes. In geothermal reservoirs, where most of the heat comes from the adjacent rock matrix, flow channeling reduces the cross-sectional area available for heat conduction. Thus, the total contact area between the moving fluid and the hot rock is reduced. The heat transfer effects of flow channeling can potentially alter the amount of energy produced in EGS reservoirs.

An important component of understanding fracture surface roughness is identifying its source. There are several mechanisms by which rough fractures can be created. The creation can be through chemical effects, mechanical effects, inelastic effects, and stress application. Figure 2 and Figure 3 show photographs of two sample fracture surfaces in shale that were taken before and after the application of stress. Applied stresses include normal and shear stresses. Comparing the samples before and after shearing, it can be observed that the application of stress caused significant changes on the texture of the surfaces (Gutierrez et al., 2000). In addition, shearing under lower normal stress results to a greater increase in surface roughness. This experiment also illustrates the complexity and heterogeneity of real fracture surfaces.

Another generation mechanism is through chemical means. This includes the dissolution, precipitation, and alteration of minerals in rocks. It has been observed that increasing the degree of alteration reduces the roughness of the fracture surface (Sauzse, 2002). The extent of these chemical effects will depend on the thermodynamic state of the reservoir which includes pressure and temperature. Rock and fluid types can likewise influence the chemical interactions in the subsurface.

Aside from chemical and mechanical means, surface roughness can also be generated by breakage and plasticity caused by rubbing the two fracture surfaces. These frictional effects can also be considered as inelastic behavior. Depending on the rock strength and rock type, these inelastic effects could affect flow significantly. Small particles formed during the breaking of rocks can block pores and reduce permeability. Gutierrez et al. (2000) have observed that high normal stresses can lead to gouge formation and permeability reduction in shale. This occurs when the applied effective normal stress exceeds the unconfined compressive strength of the intact rock. Lee and Cho (2002) also observed fine-grained gouge material formation in shear experiments performed on marble and granite.

Figure 2: Effect of shearing on the fracture surface of Kimmeridge shale samples (from Gutierrez et al., 2000). Left: photograph of the top of sample 1 before shearing. Right: photograph of the same sample after shearing with an applied effective normal stress ($\sigma_{n,e}$) of 1 MPa.

Figure 3: Effect of shearing on the fracture surface of Kimmeridge shale samples (from Gutierrez et al., 2000). Left: photograph of the bottom of sample 3 before shearing. Right: photograph of the same sample after shearing with an applied effective normal stress ($\sigma_{n,e}$) of 6 MPa.
A key component in understanding rough fractures is the accurate modeling of the opening and slip within fractures at different applied stresses. In this study, different analytical models relating shear displacement to permeability and hydraulic aperture were evaluated. Furthermore, the displacement discontinuity boundary element model (DDM) with integrated complementarity (Ritz et al., 2012) was proposed as an alternative model to correlate stresses to displacements for rough fracture surfaces.

2. ANALYTICAL MODELS

The main generation mechanism considered in this study is the application of stress. The resulting spatial distribution of the stress-dependent aperture depends on the following: applied stress values and direction, stress interactions, and the initial shape of the fracture. There are several analytical models that describe the correlation between the applied stress and displacements. The analytical models are: Barton et al. (1985), Willis-Richards et al. (1996), and Dempsey et al. (2013). Using the cubic law, as shown in Equation 1, the equivalent permeability (k) can be derived from the hydraulic aperture (b_{hydraulic}) (Witherspoon et al., 1980).

\[ k = \frac{b_{hydraulic}^2}{12} \]  

(1)

2.1 Barton et al. (1985)

This system of stress-aperture relationships uses the Barton JRC-JCS concept based on data from friction experiments (Barton, 1973; Barton and Choubey, 1977; Bakhtar and Barton, 1984; Barton et al., 1985). JRC is the joint roughness coefficient and JCS is the joint wall compression strength. The JRC is a measure of the surface roughness, with a value that ranges from 0 to 20. Smooth planar surfaces have a JRC value of 0 while extremely rough surfaces have a JRC value of 20. On the other hand, JCS takes the weathering of the joint wall into account (Barton and Choubey, 1977). If there is no weathering of the joint wall material, JCS is equal to the unconfined compression strength of the intact rock (σ_c). Equation 2 shows the relationship between the hydraulic aperture (b_{mech}) and the mechanical aperture (b_{mech}) (Esaki et al., 1999).

\[ b_{hydraulic}[\mu m] = \frac{(b_{mech}[\mu m])^2}{JRC^2} \]  

(2)

The mechanical aperture is related to the initial aperture (D_{n,0}) and the aperture change due to the normal stress (D_{n,n}) and shear stress (D_{n,s}) as shown in Equation 3. To calculate D_{n,0}, Equation 4 is used (Barton et al., 1985). Moreover, the hyperbolic variation of D_{n,n} versus the effective normal stress (σ_{eff}) is shown in Equation 5 (Bandis et al., 1983). In Equation 5, D_{max} is the maximum closure and K_{nl} is the normal stiffness. The change in the shear dilation angle (d_{n,s}) value is determined using Equation 6 (Barton et al., 1985). d_{n,s} is related to the mobile JRC coefficient (JRC_{mob}), which is a function of the shear displacement (D_{s}) as presented in Equation 7. The relationship of the ratio of JRC_{mob} to the peak JRC coefficient (JRC_{peak}) to the ratio of D_{s} to the peak shear displacement (D_{s,peak}) can be derived from standard tables as presented in Barton et al. (1985). Based on Equation 8, D_{n,s} is equal to the tangent of d_{n,s} (Esaki et al., 1999).

\[ b_{mech} = D_{n,n} + D_{n,s} + D_{n,0} \]  

(3)

\[ D_{n,0} = \frac{JRC}{5} [0.2 \frac{σ_c}{JCS} - 0.1] \]  

(4)

\[ D_{n,n} = \frac{σ_{eff} D_{max}}{K_{nl} D_{max} + σ_{eff}} \]  

(5)

\[ d_{n,s} = \frac{1}{2} JRC_{mob} \log \left( \frac{JCS}{σ_{eff}} \right) \]  

(6)

\[ \frac{JRC_{mob}}{JRC_{peak}} = f \left( \frac{D_s}{D_{s,peak}} \right) \]  

(7)

\[ D_{n,s} = \tan(d_{n,s}) \]  

(8)

2.2 Willis-Richards et al. (1996)

The Willis-Richards et al. (1996) model is another analytical model that relates the equivalent fracture aperture to the normal and shear displacements (Willis-Richards et al., 1996; Rahman et al., 2002; Kohl and Megel, 2007; McClure, 2012). Unlike the Barton et al. (1985) model, the Willis-Richards et al. (1996) model does not have separate expressions for the mechanical and hydraulic apertures. Instead, the same expressions are used with different values for the constants (McClure, 2012). Equation 9 shows the expression for the hydraulic aperture in terms of D_{n,n}, D_{n,s}, and the residual aperture (D_{n,res}). The residual aperture is the aperture at high effective normal stresses. A hyperbolic joint closure law, shown in Equation 10, is used to model the normal displacement due to surfaces in contact. Here, D_{n,n} is initial unstimulated aperture and σ_{eff} is the reference effective normal stress at which the aperture is reduced by 90 percent (Willis-Richards et al., 1996). Furthermore, Equation 11 shows the expression for D_{n,s}, where D_s is the shear displacement and φ_{dli} is the effective shear dilation angle. φ_{dli} is the shear dilation angle at low effective normal stress (Rahman et al., 2002). The expression for D_{n,s} assumes a penny-shaped circular crack and elastic behavior.

\[ b_{hydraulic} = D_{n,n} + D_{n,s} + D_{n,res} \]  

(9)
Dempsey et al. (2013) proposed an empirical model based on the results of shearing experiments performed by Lee and Cho (2002). Figure 4 shows the measured fracture permeability versus shear displacement from the hydromechanical experiments. It can be observed that, initially, the fracture permeability increases as the shear displacement increases. Furthermore, there is a threshold shear displacement value at which the increase in permeability commences. However, the permeability eventually ceases to increase due to asperity degradation and gouge production (Dempsey et al., 2013). In addition, the threshold shear displacement value increases as the applied normal stress increases. This observation is consistent with the Coulomb criterion which states that it is harder for a fracture to slip at higher effective normal stresses. Equation 12 shows the expression for the sigmoidal logarithmic change in permeability ($\Delta k_x$) due to the shear displacement ($D_s$). Here, $\Delta k_{x,max}$ is the maximum change in the logarithm of the permeability; $d_5$ and $d_{95}$ are the shear displacements at 5% and 95% total increase in permeability, respectively (Dempsey et al., 2013).

\[
D_{h,n} = \frac{D_{h,0}}{1 + \hat{\theta}_{n,ref}} \\
D_{h,s} = D_s \tan \left( \frac{\phi_{dil}}{1 + \hat{\theta}_{n,ref}} \right)
\]

(10)

(11)

2.3 Dempsey et al. (2013)

Dempsey et al. (2013) proposed an empirical model based on the results of shearing experiments performed by Lee and Cho (2002). Figure 4 shows the measured fracture permeability versus shear displacement from the hydromechanical experiments. It can be observed that, initially, the fracture permeability increases as the shear displacement increases. Furthermore, there is a threshold shear displacement value at which the increase in permeability commences. However, the permeability eventually ceases to increase due to asperity degradation and gouge production (Dempsey et al., 2013). In addition, the threshold shear displacement value increases as the applied normal stress increases. This observation is consistent with the Coulomb criterion which states that it is harder for a fracture to slip at higher effective normal stresses. Equation 12 shows the expression for the sigmoidal logarithmic change in permeability ($\Delta k_x$) due to the shear displacement ($D_s$). Here, $\Delta k_{x,max}$ is the maximum change in the logarithm of the permeability; $d_5$ and $d_{95}$ are the shear displacements at 5% and 95% total increase in permeability, respectively (Dempsey et al., 2013).

\[
\Delta k_x[\log \text{scale}] = \frac{\Delta k_{x,max}[\log \text{scale}]}{1 + \exp[\ln(10)][1 - 2^{(\frac{d_s - d_5}{d_{95} - d_5})}]}
\]

(12)

2.4 Evaluation of analytical models

The main goal of this evaluation was to highlight the general behaviors of the stress-permeability models found in literature. To evaluate the three analytical models presented, results from these models were compared to measured data shown in Figure 4. The Lee and Cho (2002) data was chosen for the evaluation because they included the effects of the normal and shear stresses on permeability. Furthermore, the experiment was conducted on granite which is applicable to geothermal systems. Though the Lee and Cho (2002) data was performed at low effective normal stress conditions, they still provided a method for demonstrating the effectiveness of the models in capturing the mechanisms of surface roughness generation due to applied stresses. Nonlinear least squares optimization was used to determine the best fit parameters for the different analytical models. The trust-region-reflective algorithm was applied to ensure that the parameter values obtained were within a specified realistic range.

Results for the Barton et al. (1985) model are shown in Figure 5. The parameters used for the generated results are: $JRC = 5$, $JCS = 150$ MPa, $\sigma_c = 160$ MPa, $D_{max} = 0.01$ mm, $K_n = 10$ MPa/m, $JRC_{peak} = 11.5$, and $D_{s,peak} = 9.0$ mm. Overall, the Barton et al. (1985) model captures the general shape of the permeability versus stress curve. The Barton model includes the threshold shear displacement behavior before the permeability increases and the stagnant permeability behavior at high shear displacement values. Because the Barton model is derived from friction experiments, its conceptual basis is consistent with experimental observations.
However, this model does not capture the variation in low shear displacement behavior for different applied normal stresses. The calculated permeability decrease due to the increase in applied normal stress is not large enough. In addition, the three results for the different applied normal stresses do not converge at high shear displacement values.

Figure 6 shows the comparison of the results from the Willis-Richards et al. (1996) model with the measured data. Values of the best fit parameters are the following: $\sigma_{\text{ref},n} = 10 \text{ MPa}$, $D_{\text{n,ref}} = 0.053 \text{ mm}$, $D_{\text{n,ref}} = 0.0 \text{ mm}$, and $\phi_{\text{dil}} = 3.2^\circ$. Because the aperture change is a simple linear function of the shear displacement, the sigmoidal permeability behavior cannot be observed in the generated results. Moreover, the threshold at small shear displacements is not represented.

A comparison of the results using the Dempsey et al. (2013) model versus the measured data is presented in Figure 7. Parameter values that provide the best fit to the data are: $\Delta k_{\text{max}} = 2.08 \log(\text{cm}^2)$, $d_5 = 1.33 \text{ mm}$, and $d_{\text{avg}} = 7.74 \text{ mm}$. The Dempsey model is designed to fit the general behavior of this dataset, so it conforms to the overall shape of the dataset. However, the effect of normal stress application is not modeled sufficiently.

These results show that analytical models can be inadequate in describing the permeability change due to applied stresses. This limitation is inherent in the empirical and simplified nature of these models. Thus, care must be taken in assuming that a particular analytical model is valid for geomechanical simulations. Furthermore, there is also a difficulty in deriving or estimating parameter values for each model. In addition, these models do not take stress interactions with neighboring fractures into account. Finally, it is also difficult to determine the applicability of these models at different scales. Because most of these models have been derived from laboratory experiments, the applicability at the reservoir scale has not been demonstrated sufficiently.

**Figure 5:** Fracture permeability versus shear displacement at different applied normal stress values. Results from the Barton et al. (1985) model (solid lines) are compared with measured data (markers with dashed lines) from Lee and Cho (2002).
Figure 6: Fracture permeability versus shear displacement at different applied normal stress values. Results from the Willis-Richards et al. (1996) model (solid lines) are compared with measured data (markers with dashed lines) from Lee and Cho (2002).

Figure 7: Fracture permeability versus shear displacement at different applied normal stress values. Results from the Dempsey et al. (2013) model (solid lines) are compared with measured data (markers with dashed lines) from Lee and Cho (2002).

3. DDM WITH INTEGRATED COMPLEMENTARITY

As shown in the previous section, analytical models can provide inaccurate results for stress-permeability relationships. An attractive alternative would be to use a consistent physical model that describes the aperture change due to applied stresses. Assuming elastic behavior, the fracture opening and slip from imposed stress conditions can be simulated using the displacement discontinuity boundary element method (DDM) with integrated complementarity (Ritz et al., 2012). The complementarity algorithm is integrated with the original DDM model to ensure that the fracture surfaces do not overlap. The DDM method can handle nonplanar fractures and stress
interactions between fractures. This model can handle problems at different scales. In addition, the model utilizes elastic properties that are typically available from standard core characterization tests. Furthermore, because this is a physical model, it is self-consistent and systematic. Additionally, the spatial variation of the fracture aperture can be defined because the fracture is divided into elements that can have distinct aperture values. Once the aperture is determined, the cubic law can be used to calculate the equivalent fracture permeability value at each element.

3.1 Displacement Discontinuity Boundary Element Method (DDM)

Cauchy’s Formula (Pollard and Fletcher, 2005) can be used to determine the equivalent traction vector \( \vec{t} \) from the stress tensor \( \sigma \) acting on a plane described by a normal vector \( \vec{n} \) as presented in Equation 13. Aside from Cartesian coordinates, \( \vec{t} \) can also be resolved in terms of the normal \( \vec{t}_n \) and shear \( \vec{t}_s \) traction vectors (Equation 14). Equations 15 and 16 show the magnitude of the normal \( \sigma_{nn} \) and shear \( \sigma_{ns} \) traction vectors, respectively. In addition, \( \sigma_{nn} \) and \( \sigma_{ns} \) are related by the Coulomb criterion for frictional sliding of crack surfaces (Equation 17); where \( \mu \) is the coefficient of friction and \( S_f \) is the frictional strength. Sliding initiates when \( \sigma_{ns} \) is equal to \( -\mu\sigma_{nn} + S_f \). The condition \( \sigma_{nn} \leq 0 \) is present to ensure that the surfaces are in contact, which implies that the normal stress is compressive. As the compressive applied normal stress increases, the resistance to sliding increases. A distinction must be made between the shear failure of preexisting cracks and intact rocks. In this study, slip on preexisting fractures was considered. Typical values for \( \mu \) range from 0.5 to 0.8, with 0.6 as a good initial estimate (Pollard and Fletcher, 2005). On the other hand, the \( S_f \) value for a prefractured rock is negligible up to depths of approximately 8 km (Ritz et al., 2012). At higher depths, a standard \( S_f \) value of 50 MPa can be used (Byerlee, 1978). In addition, Pollard and Fletcher (2005) reported a \( S_f \) range of 0.28 to 1.1 MPa based on laboratory data on friction experiments.

\[
\vec{t} = \sigma \vec{n}
\]  
\[\vec{t} = \vec{t}_n + \vec{t}_s\]  
\[\sigma_{nn}^2 = |\vec{t}_n|^2 = |\vec{t} \cdot \vec{n}|^2\]  
\[\sigma_{ns}^2 = |\vec{t}_s|^2 = |\vec{n} \times (\vec{t} \times \vec{n})|^2\]  
\[\sigma_{ns} \leq -\mu\sigma_{nn} + S_f, \quad \sigma_{nn} \leq 0\]  

Aside from Cauchy’s Formula and the friction equation, \( \sigma_{nn} \) and \( \sigma_{ns} \) are related to the displacement discontinuities that result from the stress interactions within the curvilinear crack. Displacement discontinuities include opening (\( D_n \)) and slip (\( D_s \)). The displacement discontinuity boundary element method (DDM) provides a way to model quasistatic slip on cracks. This method is applicable to cracks situated in an infinite homogeneous, isotropic, and elastic matrix material. Figure 8a shows a crack of length \( 2w \) embedded in an infinite homogeneous material with applied remote stresses. The main advantage of using the boundary element method (BEM) is that only the crack surface needs to be discretized. In contrast, both the crack and the matrix have to be discretized in the finite element (FE) method (Ritz et al., 2012). In the DDM, curvilinear cracks are discretized into linear elements with constant displacement discontinuity as shown in Figure 8b (Crouch and Starfield, 1983; Ritz et al., 2012; Ritz and Pollard, 2012). Each element has a length of \( 2h \). Stress components are applied at the midpoint of the element while displacement discontinuities are uniform throughout the element. The positive sign convention for the displacement discontinuities and stress components are shown in Figure 8c.

Equation 18 shows the relationship between the stress components \( \sigma_{nn}, \sigma_{ns} \) and the displacements \( D_n, D_s \), where \( A_{nn}, A_{ns}, A_{nn}, \) and \( A_{ss} \) are the influence coefficients of the \( j \) neighboring elements on element \( i \). The effective stress on element \( i \) is the sum of the effects of all the other elements. The expressions for the influence coefficients can be found in Crouch and Starfield (1983). These influence coefficients represent the interactions among the elements. In order to obtain the displacement discontinuities from the traction boundary conditions, the inverse of Equation 18 must be taken. The resulting expression is Equation 19, where \( C_{nn}, C_{ns}, C_{nn}, \) and \( C_{ss} \) are determined from the inverse of the original influence coefficients.

\[
[s_{nn}]_i = [A_{nn}]_i[D_n]_j + [A_{ns}]_i[D_s]_j
\]  
\[
[s_{ns}]_i = [A_{ns}]_i[D_n]_j + [A_{ss}]_i[D_s]_j
\]  
\[
[D_n]_i = [C_{nn}]_i[s_{nn}]_j + [C_{ns}]_i[s_{ns}]_j
\]  
\[
[D_s]_i = [C_{sn}]_i[s_{nn}]_j + [C_{ss}]_i[s_{ns}]_j
\]  

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Figure 8: The displacement discontinuity boundary element method (DDM).  (a) Infinite two-dimensional plane with an embedded crack of length $2w$.  $\sigma_{xx}^\infty$, $\sigma_{yy}^\infty$, and $\sigma_{xy}^\infty$ are the applied remote stresses.  The positive stress sign convention is shown here.  (b) Discretization of the crack surface into $N$ elements, each with length equal to $2h$.  The normal ($n$) and shear ($s$) directions are shown here.  (c) The positive sign convention for the displacement discontinuities and stress components.  The displacement discontinuities are opening ($D_n$) and slip ($D_s$) while the stress components are normal ($\sigma_{nn}$) and shear ($\sigma_{ns}$) (from Ritz et al., 2012)

3.2 Complementarity Algorithm

The main disadvantage of using the original DDM formulation is the nonphysical handling of frictional contact surfaces under compression. In the original DDM, the interpenetration of fracture surfaces is allowed (positive $D_n$ values). To address the interpenetration issue, Ritz et al. (2012) have used the complementarity optimization algorithm (Ferris and Munson, 2000) to reevaluate the results from the DDM. The detailed conditions for sticking, slipping, and opening can be found in Ritz et al. (2012). In the complementary problem, pairs of inequalities are evaluated with one inequality forced to satisfy an equality (Ritz et al., 2012). An example of an inequality pair is $D_n \leq 0 \perp \sigma_{nn} \leq 0$, where either $D_n$ or $\sigma_{nn}$ is zero depending on whether the crack surfaces are in contact or not. Because one of the relationships satisfies an equality to zero, the product of each relationship pair is zero. To find the other pairs of inequalities, $D_s$ and $\sigma_{ns}$ must be decomposed into their left and right components. Equation 20 shows the left ($D_s^L$) and right ($D_s^R$) components for $D_s$. Furthermore, the Coulomb friction equation can be expanded to separate the positive and negative $\sigma_{ns}$ values as presented in Equation 21. Rearranging Equation 21, the left ($\sigma_{ns}^L$) and right ($\sigma_{ns}^R$) shear stress components can be obtained as shown in Equation 22.

$$D_s = D_s^L - D_s^R$$ (20)

$$\sigma_{ns} \leq -\mu \sigma_{nn} + S_f, \quad \sigma_{ns} \geq 0 \quad (\text{right - lateral})$$ (21)

$$\sigma_{ns} \geq -\mu \sigma_{nn} + S_f, \quad \sigma_{ns} \leq 0 \quad (\text{left - lateral})$$ (22)

Thus, three complementarity relationship pairs presented in Equation 23 can be formulated. The form of the complementarity problem is shown in Equation 24, where $f_i \geq 0 \perp Z_i \geq 0$ for all elements $i = 1 \text{ to } N$. Applying this form to the relationship pairs, the expressions for $f(x)$ (Equation 25) and $Z$ (Equation 26) are straightforward. Because three relationship pairs need to be solved, three equations are needed to relate $f(x)$ and $Z$. The first two equations are from the DDM equations. The third equation is found by adding $\sigma_{ns}^R$ and $\sigma_{ns}^L$ from Equation 22 to obtain Equation 27.
\[-D_n \geq 0 \perp -\sigma_{nn} \geq 0 \]
\[D^R_n \geq 0 \perp \sigma_{nn}^R \geq 0 \]
\[\sigma_{nS}^L \geq 0 \perp D^L_S \geq 0 \]

\[f(z) = MZ + Q \]  \hspace{1cm} (24)

\[f(z) = \begin{bmatrix} -D_n \\ D^R_n \\ \sigma_{nn}^L \\ \sigma_{nS}^L \\ D^L_S \end{bmatrix} \]

\[Z = \begin{bmatrix} -\sigma_{nn} \\ \sigma_{nn}^R \\ \sigma_{nn}^L \\ \sigma_{nn}^S \end{bmatrix} \]

\[\sigma_{nS}^L + \sigma_{nS}^R = -2\mu\sigma_{nn} + 2S_f \]  \hspace{1cm} (27)

**4. CONCLUSIONS AND RECOMMENDATIONS**

Flow experiments have demonstrated that flow channeling can occur within rough fractures. Flow channeling can reduce the surface area available for conductive heat transfer in low permeability systems. Thus, it is important to describe the aperture distribution accurately within each fracture. One of the main mechanisms for changing the aperture and permeability distributions is the application of stress. There are various stress-permeability analytical models available in literature. These analytical models provide a simple method of determining the permeability change due to stress application on fractures. However, these stress-permeability models can be inaccurate because these are either empirical or simplified expressions. Inaccurate fracture permeability distributions can lead to inaccurate temperature predictions.

To evaluate the analytical models, the calculated permeability evolution with respect to shear displacement was compared to data for slip on preexisting fractures at different effective normal stresses. The general behavior of increasing permeability with increasing slip was represented in the model results. Though the overall trend was captured, the effect of the effective normal stress and the behavior at low shear displacement values were not described adequately. A consistent physical model, the displacement discontinuity boundary element method (DDM) with integrated complementarity (Ritz et al., 2012), was proposed as an alternative to analytical stress-permeability models. The DDM model is capable of modeling the spatial variation of opening and slip within a rough fracture at specified applied remote stress conditions. Moreover, the DDM model can also account for stress interactions for multiple cracks. Furthermore, the DDM model can be applied at different problem scales.

The next part of this research will be focused on implementing and evaluating the DDM model for slip on preexisting cracks. Results from the DDM model will be compared to existing analytical models. Further improvements will be applied on the DDM model of Ritz et al. (2012). This includes the utilization of higher order DDM methods to handle rough surface geometry profiles. Additionally, the DDM model will be edited to include mixed initial boundary conditions. Mixed initial boundary conditions are needed for nonzero initial aperture distributions. Finally, the DDM model will be extended to three dimensions in a method similar to Kaven et al. (2012).

**REFERENCES**


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