A NEW DOUBLE POROSITY FRACTAL MODEL FOR WELL TEST ANALYSIS WITH TRANSIENT INTERPOROSITY TRANSFERENCE FOR PETROLEUM AND GEOTHERMAL SYSTEMS

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ABSTRACT

A new double porosity model for Naturally Fractured Reservoirs (NFRs) assuming fractal fracture network behavior and its solution is presented. Primary porosity is idealized as Euclidian matrix blocks (slabs or spheres) and Secondary porosity is defined by any post-depositional geological phenomenon such as fractures and vugs.

In order to provide a framework, the generalized radial flow model solution for well test analysis for petroleum and geothermal systems in Laplace and Real space was developed. Development of an appropriate wellbore storage model for fractal reservoirs is also shown.

For this model, the dimensionless fractal fracture area parameter was developed. In addition, interporosity skin factor between matrix blocks and fractal fracture network is introduced. Relationship of convergence between interporosity skin under transient transference regime and pseudosteady state transference regime is discussed. An analytical general solution was obtained in Laplace space; besides, analytical solutions in real space that describe the behavior of NFRs at different stages and different cases of flow are also presented. Early, intermediate and late-time approximations are used to obtain reservoir and fractal fracture network parameters. A synthetic example is presented to illustrate the application of this model.

INTRODUCTION

Behavior of NFRs

NFRs are multi-porous systems, caused by chemical or tectonic events or both. Porous media identified in NFRs are matrix, fractures, faults and vugs, from micro to mega scales.

Transient pressure behavior in non-fractal double porosity systems has three flowing periods: fracture network expansion (early times), interaction between porous media (intermediate times) and single system behavior (late times). A characteristic of the transient pressure behavior of these systems is that in a semilog plot two parallel straight lines will be shown. The first straight line correspond to the first radial flow due to fractures expansion and the second straight line is attributed to a radial flow when all porous systems are acting as a single one. Besides, a third straight line non-parallel to the other two is observed, which describes the interaction between porous media (intermediate times). Shape of such straight line depends on the flow regime conditions, i.e., transient or pseudosteady state interporous transference. This topic has been under debate by some authors and it is part of the discussion on the present work.

Nature of flow in multi-porous systems obeys to the fact that flow in each porous medium behaves differently in terms of gradient pressure from the other media. Such behavior is known as transient interporous flow; this flow regime was studied previously by de Swaan (1976), Najurieta (1980), Cinco-Ley et al. (1982), Serra et al. (1982) and Streltsova (1982).

On the other hand, practice has shown that pressure gradients in all porous media apparently behave in the same way. Such flow transference is known as pseudosteady-state flow. Barenblatt et al. (1960) and Warren et al. (1963) developed this theory.

However, Cinco-Ley et al. (1985) showed that the apparent pseudosteady-state transference behavior can be attributed to a presence of interporous skin between matrix and fracture network. Such interporous skin is produced by a film created by mineralization or interaction between fluids in the face of the matrix blocks. Mineralization has been observed in outcrops, where precipitation and other chemical phenomena create a skin between different porous media. Interporous skin is defined as:

\[ S_{\text{intD}} = \frac{k_r x_d}{k_d h_l} \]  

Where:
\[ k_1 = \text{permeability of medium 1}, \]
\[ h_1 = \text{characteristic length of medium 1}, \]
\[ k_d = \text{permeability of damaged zone}, \]
\[ x_d = \text{thickness of damaged zone}. \]

\[ V_b = \alpha_d b^{3-d_e} r^{d_e-1} \Delta r. \quad (2) \]
Where:
\[ \alpha_d = \text{Area of a unit sphere of in } d_e \text{ dimensions}; \]
\[ \text{it is defined as:} \]
\[ \alpha_d = \frac{2 \pi^{d_e/2}}{\Gamma\left(\frac{d_e}{2}\right)}. \quad (3) \]
\[ d_e = \text{Euclidean dimension}, \]
\[ b = \text{extent of the flow region}, \]
\[ r = \text{radial distance from the centre of the source} \]
\[ \text{measured along the flow paths}. \]
\[ \Gamma(x) = \text{gamma function of } x. \]
\[ \Delta r = \text{width between the surfaces}. \]

Since the term \( \Delta r \) in eq. 2 represents the width between the surfaces, it can be deduced that the exposed to flow area is given by:
\[ A_{\text{exp,flow}} = \alpha_d b^{3-d_e} r^{d_e-1}. \quad (4) \]

### Porosity of a Unitary Fracture
Conceptually it represents the volumetric fraction occupied by a single fracture regarding the total rock volume. It is given by:
\[ \phi_{uf} = \frac{\text{fracture volume}}{\text{total bulk volume}} = \frac{V_{uf}}{V_b}. \quad (5) \]

### Fractal Fracture Network Porosity
It represents the volumetric fraction of all fractures in the rock. It is defined as:
\[ \phi_{fb} = \frac{\text{fracture network volume}}{\text{total bulk volume}} = \frac{V_{fb}}{V_b}. \quad (6) \]
Assuming fractures with the same characteristics all over the bulk, fractal fracture network volume, can be expressed as:
\[ V_{fb} = n_f(r)V_{uf}\Delta r, \quad (7) \]
where:
\[ n_f(r) = \text{number of fractures into fractured bulk}, \]
\[ V_{uf} = \text{unitary fracture volume}. \]
Moreover, the number of fractures into fractured bulk can be expressed using a power-law model:
\[ n_f(r) = ar^{D_{fb}-1}. \quad (8) \]
Where:
\[ a = \text{site density parameter}, \]
\[ D_{fb} = \text{fractal dimension of the fracture network}. \]
Therefore, fracture network volume is expressed as:
\[ V_{fb} = ar^{D_{fb}-1}V_{uf}\Delta r. \quad (9) \]
Combining eq. 2 and 7, porosity of the fracture network is given by:
\[ \phi_{fb} = \frac{ar^{D_{fb}-d_e}V_{uf}}{\alpha_d b^{3-d_e}}. \quad (10) \]
**Darcy’s Law in Fractal form**

Chang et al. (1990) proposed the following equation to express Darcy’s law in a fractal form:

\[ q = -k(r) \frac{\partial p}{\partial r} \mu \]  \hspace{1cm} (11)

Prior expression uses permeability as a function of radius; it can be expressed as:

\[ q = \frac{aVm k_{fb} \beta}{\phi_{fb} \mu} \frac{\partial p_{fb}}{\partial r} \]  \hspace{1cm} (12)

Where:
- \( p_{fb} \) = pressure in fractal fracture network.
- \( k_{fb} \) = permeability of the fractal fracture network.
- \( \mu \) = viscosity of flowing fluid.
- \( \beta \) = grouping parameter, defined as:
  \[ \beta = D_{fb} - \theta - 1 \]  \hspace{1cm} (13)

Where:
- \( \theta \) = conductivity index; related to the spectral exponent of the fractal fracture network.

**PROPOSED MODEL**

**Model development**

Based on eqs. 2, 10 and 12 the diffusivity equation for a double porosity fractal reservoir with transient interporosity transfer is derived:

\[ \frac{1}{r^{D_{fb}-1}} \frac{\partial}{\partial r} \left( r^\beta \frac{\partial p_{fb}(r,t)}{\partial r} \right) = -\frac{k_{ma} A_{fb}}{k_{fb}} \int_0^t \frac{\partial p_{ma}(\tau) (\nabla \Delta p_{ma})_{sat}}{d\tau} d\tau = \frac{1}{\eta_{fb}} \frac{\partial p_{fb}(r,t)}{\partial t} \]  \hspace{1cm} (14)

Where:
- \( p_{ma} \) = pressure in matrix blocks.
- \( k_{ma} \) = permeability of the Euclidean matrix blocks.
- \( A_{fb} \) = fractal fracture network area per unit of bulk volume, it is defined for slab matrix blocks as:
  \[ A_{fb} = \frac{2}{h_{ma} + h_f} \]  \hspace{1cm} (15)

Where:
- \( h_{ma} \) = strata height,
- \( h_f \) = fracture width.

For cube matrix blocks \( A_{fb} \) is defined as:

\[ A_{fb} = \frac{6h_{ma}^2}{(h_{ma} + h_f)^2} \]  \hspace{1cm} (16)

\( \eta_{fb} \) = fractal fracture network hydraulic diffusivity coefficient, it is defined as:

\[ \eta_{fb} = \frac{k_{fb}}{\phi_{fb} \mu k_{fb}} \]  \hspace{1cm} (17)

Where:

\( c_{fb} \) = fractal fracture network total compressibility.

**Dimensionless variables**

The following dimensionless variables in field units are used in the present study.

**Dimensionless radius**

\[ r_D = \frac{r}{r_w} \]  \hspace{1cm} (18)

Where:
- \( r_w \) = wellbore radius.

**Dimensionless time**

\[ t_D = \frac{r_D t_p}{\phi_{fb} r_w^{\beta -1}} \]  \hspace{1cm} (19)

Where:
- \( \phi_{fb} \) = fractal fracture network porosity.
- \( \phi_{ma} \) = porosity of matrix blocks.
- \( c_{ma} \) = matrix blocks total compressibility.

For an oil-filled system, dimensionless pressure in the fracture network:

\[ p_{fbD}(r_D.t_D) = \frac{\alpha_{D_{fb}} (0.03281)^\theta a_{V_{Vl}} k_{fb} \Delta p_{fb}(r,t)}{887.22 q_{B_o} r_w^{1-\beta} \phi_{fb}} \]  \hspace{1cm} (21)

Where:
- \( q \) = flow rate,
- \( B_o \) = formation volume factor of oil,
- \( p_i \) = initial pressure.

**Dimensionless pressure in the matrix**

\[ p_{maD}(r_D.t_D) = \frac{\alpha_{D_{fb}} (0.03281)^\theta a_{V_{Vl}} k_{fb} \Delta p_{ma}(r,t)}{887.22 q_{B_o} r_w^{1-\beta} \phi_{fb}} \]  \hspace{1cm} (22)

For gas reservoirs, dimensionless pressure in the fracture network:

\[ p_{fbD}(r_D.t_D) = \frac{\alpha_{D_{fb}} (0.03281)^\theta a_{V_{Vl}} k_{fb} \Delta p_{fb}(r,t)}{9387.5 q_{g} \mu_{g} Z T r_w^{1-\beta} \phi_{fb}} \]  \hspace{1cm} (23)

Where, \( \Delta p_{fb} \) is given by:

\[ \Delta p_{fb} = p_i^2 - p_{fb}^2(r,t) \]  \hspace{1cm} (24)

\( q_g \) = gas flow rate,

\( Z \) = real gas deviation factor,

\( T \) = temperature.

**Dimensionless pressure in the matrix**

\[ p_{maD}(r_D.t_D) = \frac{\alpha_{D_{fb}} (0.03281)^\theta a_{V_{Vl}} k_{fb} \Delta p_{ma}(r,t)}{9387.5 q_{g} \mu_{g} Z T r_w^{1-\beta} \phi_{fb}} \]  \hspace{1cm} (25)

where:

\[ \Delta p_{ma} = p_i^2 - p_{ma}^2(r,t) \]  \hspace{1cm} (26)

For geothermal reservoirs (steam), dimensionless pressure in the fracture network:
\[ p_{fbD}(r_D,t_D) = \frac{\alpha_{Dfb}(0.03281)^{\theta} \alpha V_{mf} M_{fb} \Delta m}{81361 W \mu Z T R_{w}^{1+\beta}} (p_{fb}) \]  

(27)

Where:
\[ \Delta m(p_{fb}) = p_{f}^2 - p_{fb}^2(r,t) \]  

(28)

\[ W = \text{mass flow rate}, \quad M = \text{Molecular weight.} \]

Dimensionless pressure in the matrix:
\[ p_{maD}(r_D,t_D) = \frac{\alpha_{Dma}(0.03281)^{\theta} \alpha V_{mf} M_{fb} \Delta m}{81361 W \mu Z T R_{w}^{1+\beta}} (p_{ma}) \]  

(29)

And:
\[ \Delta m(p_{ma}) = p_{f}^2 - p_{ma}^2(r,t) \]  

(30)

For this work the oil-filled case was considered.

Similar expressions should be obtained for the other cases. The substitution of variables from eq. 18 to eq. 22 into Eq. 14 yields:
\[ \frac{\partial^2 p_{fbD}(r_D,t_D)}{\partial t_D^2} + \beta \frac{\partial p_{fbD}(r_D,t_D)}{\partial t_D} - r_D^{\theta} A_{fbD}[1 - \omega_0] \left[ \frac{\partial p_{maD}(r_D,t)}{\partial t} \right] F(\eta_{maD} H_D, t_D - \tau) d\tau = \]  

(31)

\[ = r_D^{\theta} \frac{\partial p_{fbD}(r_D,t_D)}{\partial t_D} \]

(32)

Where, dimensionless storativity ratio, \( \omega \) is defined:
\[ \omega = \frac{\phi_{fb} \phi_{ma}}{\phi_{fb}} ; \]

(33)

Dimensionless matrix hydraulic diffusivity:
\[ \eta_{maD} = \frac{k_{maD}(\phi_{fb})}{\phi_{ma} R_{fb}} ; \]

(34)

Dimensionless block size, for slabs:
\[ H_D = \frac{r_w^2}{h_{ma}} ; \]

(35)

and for spheres:
\[ H_D = \frac{r_w^2}{d_{ma}} ; \]

(36)

Where:
\[ d_{ma} = \text{sphere diameter}. \]

Dimensionless fractal fracture area, \( A_{fbD} \):
\[ A_{fbD} = \frac{A_{fb} h_{fb} V_{fb}^{\theta}}{V_{ma}} . \]

(37)

Fluid transfer function, assuming slabs:
\[ F(\eta_{maD} H_D, t_D - \tau) = \frac{4\eta_{maD}}{H_D} \sum_{n=1} e^{- \frac{\eta_{maD}(2\alpha + 1) \tau}{2 \eta_D}} , \]

(38)

Or, if spheres as matrix blocks, fluid transfer function is:
\[ F(\eta_{maD} H_D, t_D - \tau) = \frac{4\eta_{maD}}{H_D} \sum_{n=1} e^{- \frac{\eta_{maD}(2\alpha + 1) \tau}{2 \eta_D}} . \]

(39)

\[ f(s) = \frac{A_{fbD}[1 - \omega] F(\eta_{maD} H_D, s)}{1 + \frac{H_D S_{ma-fbD}}{\eta_{maD}} F(\eta_{maD} H_D, s)} + \omega, \]

(40)

Where:
\[ S_{ma-fbD} = \text{Interporous skin between matrix and fractures.} \]

Moreover, for slab matrix blocks:
\[ F(\eta_{maD} H_D, s) = \frac{\eta_{maD}}{H_D} \tan \left( \frac{1}{2} \frac{H_D}{\eta_{maD}} \right) . \]

(41)

And, for spheres as matrix blocks:
\[ F(\eta_{maD} H_D, s) = \frac{\eta_{maD}}{H_D} \coth \left( \frac{1}{2} \frac{H_D}{\eta_{maD}} \right) - 2 \eta_{maD} H_D . \]

(42)

It can be verified that, for radial flow and non-fractal object, i.e., \( D_{fb} = 2 \), and \( \theta = 0 \), eqs. 42 and 43 converge to the model proposed by Cinco-Ley et al. (1985); in addition to the previous conditions, if no interporous skin exists, the model converges to the one proposed by Cinco-Ley et al. (1982). Figure 3 shows the convergence of this model to the one developed by Cinco-Ley et al. (1982) assuming slabs.
Figure 3: Pressure and pressure derivative function behavior for some values of $D_{fb}$, and $\theta$ without interporous skin and its convergence to the model proposed by Cinco-Ley et al., (1982).

It is important to point out that in general, the relationship $D_{fb} = \theta + 2$ in double porosity fractal models will provide a response like the typical shown by double porosity reservoirs assuming radial flow, i.e., a semilog straight line during fractal fracture network expansion (short times) parallel to the semilog straight line during single system behavior (late times). Therefore, even when the typical double porosity pressure behavior -which assumes radial flow- be observed, a fractal behavior might be taking place. Figure 4 shows the pressure behavior on a semilog plot, for $D_{fb}$, and $\theta$ values that satisfies this condition, assuming slabs as matrix blocks.

Figure 4: Semilog plot of the pressure behavior for some values of $\theta$ that satisfy the condition of $D_{fb} = \theta + 2$.

If there is no interporous skin, i.e., $S_{ma-frD} = 0$, this model acquires a general shape of transient interporous transfer. A particular case of this model in terms of the matrix-fracture interaction parameter, $\lambda$, was presented by Oelfareju (1996). Figure 5 and Figure 6 show the impact of interporous skin in the response of pressure and pressure derivative function, when $D_{fb} \neq \theta + 2$ and $D_{fb} = \theta + 2$, respectively; assuming slabs as matrix blocks. It can be observed that, even for low interporous skin values ($S_{ma-frD} = 0.1$), the pressure derivative function shows a valley shape during interaction between media. Such behavior has been attributed to pseudosteady state transference conditions.

Figure 5: Impact of interporous skin in pressure and pressure derivative function, when $D_{fb} \neq \theta + 2$.

For practical application, phenomena around wellbore such as skin at wellbore and wellbore storage are incorporated in Laplace space as follows:

$$p_{wD}(s, C_D, S_{well}) = \frac{p_{wD}(s) + \frac{S_{well}}{S}}{1 + S_{well}S C_D + C_D S^2 p_{wD}(s)}.$$  \hspace{1cm} (47)

Figure 7 and Figure 8 show the effect of wellbore storage and skin around wellbore in the examples shown in Figure 5 and Figure 6, respectively.

Figure 6: Behavior of pressure and pressure derivative function, when $D_{fb} = \theta + 2$.

Figure 7: Impact of interporous skin in pressure and pressure derivative function, for $D_{fb} = \theta + 2$,

Figure 8: Behavior of pressure and pressure derivative function, when $D_{fb} \neq \theta + 2$, considering phenomena around wellbore.
Solutions at early, intermediate and late times

In order to provide models to compare transient pressure data during fracture expansion, interaction between matrix and fractal fracture network and single system behavior, particular solutions for each one of these periods were developed. In addition, such solutions are useful to calibrate the numerical inversion of eq. 43.

Solution at early times

At early times, only fracture network expansion is acting; during this period interporous transference function is approximated as:

\[ f(s) \approx \omega, \]

Hence, solution at wellbore is given by:

\[ p_{wD}(t_D) = \frac{\Gamma\left(\frac{D_{fb}}{\theta + 2} - 1, \frac{\omega}{(\theta + 2)^2 t_D}\right)}{(\theta + 2)\Gamma\left(\frac{D_{fb}}{\theta + 2}\right)}, \]  
(48)

where:

\[ \Gamma(a, x) = \text{Incomplete Gamma Function}. \]

If \( D_{fb} = \theta + 2 \) pressure derivative function will yield a slope equals to zero. Then eq. 48 is approximated:

\[ p_{wD}(t_D) = \frac{[\ln(t_D) + 2\ln(\theta + 2) - \ln(\omega) - 0.57721]}{(\theta + 2)}. \]  
(49)

Otherwise:

\[ p_{wD}(t_D) = \frac{1}{(\theta + 2)[v - 1]} \frac{(\theta + 2)^{2v - 2} \omega v^{-1}}{v - 1}\Gamma(v)\left[1 - v \right], \]  
(50)

Where:

\[ v = \frac{D_{fb}}{\theta + 2}. \]  
(51)

Solution at intermediate times

At intermediate times, beginning of matrix flow contribution takes place. This phenomenon occurs under transient-state conditions.

\[ f(s) \approx A_{D\beta D}[1 - \omega] \left[ \frac{\eta_{m\alpha D}}{H_D S} \right]^{1 - v}. \]  
(52)

Then, approximated solution for \( D_{fb} = \theta + 2 \) is:

\[ p_{wD}(t_D) = \frac{2}{\theta + 2} \left[ \frac{1}{4} \ln(t_D) + \ln(\theta + 2) - \frac{1}{2} \ln\left( A_{D\beta D}\right) - \frac{\eta_{m\alpha D}}{H_D} \right] + \frac{1}{2\sqrt{\pi}} S_{m\alpha - \beta D}^{\frac{1}{3}} H_D^{\frac{1}{3}} t_D^{\frac{1}{2}} - 0.4329 \]  
(53)

For \( D_{fb} \neq \theta + 2 \):

\[ p_{wD}(t_D) = \frac{\Gamma\left(\frac{1 - \beta}{\theta + 2}\right)}{(\theta + 2)\Gamma\left(\frac{3 - v}{2}\right)} \left( A_{D\beta D}[1 - \omega] \left[ \frac{\eta_{m\alpha D}}{H_D} \right]^{v - 1} \right). \]  
(54)

\[ \left[ \frac{1}{t_D^2} + \frac{S_{m\alpha - \beta D} H_D^{\frac{1}{3}}}{\eta_{m\alpha D}} \left[ \frac{3 - v}{2} \right] \right] \frac{\Gamma\left(\frac{2 - v}{2}\right)}{\Gamma\left(\frac{2 - 1}{2}\right)} \]  

In order to have a straight line shape, previous expression can be written as:

\[ t_D^{\frac{v}{2}} p_{wD}(t_D) = \frac{\Gamma\left(\frac{1 - \beta}{\theta + 2}\right)}{(\theta + 2)\Gamma\left(\frac{3 - v}{2}\right)} \left( A_{D\beta D}[1 - \omega] \left[ \frac{\eta_{m\alpha D}}{H_D} \right]^{v - 1} \right). \]  
(55)

\[ \left[ \frac{S_{m\alpha - \beta D} H_D^{\frac{1}{3}}}{\eta_{m\alpha D}} \left[ \frac{3 - v}{2} \right] \right] \frac{\Gamma\left(\frac{2 - v}{2}\right)}{\Gamma\left(\frac{2 - 1}{2}\right)} \]  

b) On the other hand, for severe damage between matrix blocks and fractal fracture network, i.e., pseudosteady-state transference equivalence, interporous transference function is given by:

\[ f(s) \approx \frac{A_{D\beta D} \eta_{m\alpha D}}{S_{m\alpha - \beta D} H_D S}. \]  
(56)

Then, if \( D_{fb} = \theta + 2 \):

\[ p_{wD}(t_D) = \frac{2}{(\theta + 2)} \left[ \ln\left[\frac{\eta_{m\alpha D}}{H_D}\right] - \frac{\eta_{m\alpha D}}{H_D} \right] - 0.57721 \]  
(57)

Otherwise:

\[ p_{wD}(t_D) = \frac{(\theta + 2)^{2v - 2} \Gamma\left(\frac{1 - \beta}{\theta + 2}\right)}{\Gamma\left(\frac{3 - v}{2}\right)} \left( A_{D\beta D}[1 - \omega] \left[ \frac{\eta_{m\alpha D}}{H_D} \right]^{v - 1} \right). \]  
(58)
Solution at late times

At late times, the flow is dominated by matrix under pseudosteady-state flow, it yields a single system behavior; for this period of time, interporous transference function is given by:

\[ f(x) \approx 1. \]

Hence solution of eq. 31 is given by:

\[ p_{wD}(t_D) = \frac{1}{\Gamma(\theta+2)} \frac{1}{(\theta+2)^2 t_D} \left[ \frac{D_f}{\theta+2} \right] \]

\[ (\theta+2) \Gamma(\theta+2) \tag{60} \]

If \( D_f = \theta+2 \), eq. 60 can be approximated:

\[ p_{wD}(t_D) = \frac{[\ln(t_D) + 2\ln(\theta+2) - 0.57721]}{(\theta+2)}. \tag{61} \]

Otherwise:

\[ p_{wD}(t_D) = \frac{1}{(\theta+2)^{1-v}} \frac{1}{[v-1][v]} T_D^{1-v}. \tag{62} \]

Adjust of analytical solutions in real space shown in previous section, to the general solution numerically inverted for different scenarios are shown from Figure 8 to Figure 13.

Transient interporous transference without interporous skin

Figure 8 and Figure 9 show the pressure behavior of a double porosity fractal reservoir, in semilog scale for \( D_f = \theta+2 \) and log-log scale for \( D_f \neq \theta+2 \), respectively. For this case, the interporous transference is assumed to be free, i.e., no interporous skin. Both figures show two parallel straight lines at early and late times. Interaction between porous media is represented by a non-flat straight line between the two parallel straight lines.

Figure 8: Convergence from the short, intermediate and long times solutions to the general solution, when \( D_f = \theta+2 \), with no interporous skin.

Figure 9: Convergence from the short, intermediate and long times solutions to the general solution, when \( D_f \neq \theta+2 \), with no interporous skin.

Transient interporous transference with low interporous skin

Figure 10 and Figure 11 show the pressure behavior of a double porosity fractal reservoir, in semilog scale for \( D_f = \theta+2 \) and log-log scale for \( D_f \neq \theta+2 \), respectively. For this case, interporous skin is low. As it was shown in previous section, the presence of a valley shape in the pressure derivative function may propitiate an interpretation that flow transference occurs under pseudosteady state conditions. In both cases, it can be observed that straight line, which corresponds to the intermediate times, has a non-flat slope, which indicates that flow transference is occurring under transient interporous transference, even when interporous skin is present.

Figure 10: Convergence from the short, intermediate and long times solutions to the general solution, when \( D_f = \theta+2 \), with low interporous skin.

Transient interporous transference with severe interporous skin

Figure 12 and Figure 13 show the convergence of this model to the pseudosteady-state interporous transference (severe interporous skin) for \( D_f = \theta+2 \) and \( D_f \neq \theta+2 \), respectively. A flat
straight line when matrix blocks and fractal fracture network are interacting is the characteristic behavior of the media interaction under apparent pseudosteady-state conditions.

**EXAMPLE OF APPLICATION**

A drawdown test was carried out in Well A. The behavior of pressure and pressure derivative function are shown in Figure 14. Reservoir and well data are given in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>$q$</td>
<td>2,000 bpd</td>
</tr>
<tr>
<td>$B_o$</td>
<td>1.6 bbl/c.y/ bbl/c.s</td>
</tr>
<tr>
<td>$\mu$</td>
<td>6 cp</td>
</tr>
<tr>
<td>$r_w$</td>
<td>0.5 ft</td>
</tr>
<tr>
<td>$\phi_{fb}$</td>
<td>0.01 fraction</td>
</tr>
</tbody>
</table>

**Table 1: Reservoir and well data for the example.**

Pressure derivative function in Figure 14 does not show the fractal fracture expansion, i.e., behavior at early times. Then, analyzing late time response (see Figure 15) and comparing such behavior with eq. 62, it can be conclude that, $v \approx 0.8362$, hence, following relation between fractal parameters was deduced: $D_{fb} = 0.8362\theta + 1.6724$. Then, the methodology described by Flamenco et al. (2003) was applied and resulting parameters are: $\theta_{\text{min}} = 0$, $\theta_{\text{max}} \approx 1.588$, 

$\left(\frac{\mu k_{fb}}{u}ight)_{\text{min}}^{-1} / \alpha V_{uf} k_{fb}^{-\nu} \approx 0.000127$ and $\left(\frac{\mu k_{fb}}{u}ight)_{\text{max}}^{-1} / \alpha V_{uf} k_{fb}^{-\nu} \approx 0.451$.

**Figure 14: Pressure and pressure derivative function behavior for synthetic example.**

Pressure derivative function in Figure 14 does not show the fractal fracture expansion, i.e., behavior at early times. Then, analyzing late time response (see Figure 15) and comparing such behavior with eq. 62, it can be conclude that, $v \approx 0.8362$, hence, following relation between fractal parameters was deduced: $D_{fb} = 0.8362\theta + 1.6724$. Then, the methodology described by Flamenco et al. (2003) was applied and resulting parameters are: $\theta_{\text{min}} = 0$, $\theta_{\text{max}} \approx 1.588$, 

$\left(\frac{\mu k_{fb}}{u}ight)_{\text{min}}^{-1} / \alpha V_{uf} k_{fb}^{-\nu} \approx 0.000127$ and $\left(\frac{\mu k_{fb}}{u}ight)_{\text{max}}^{-1} / \alpha V_{uf} k_{fb}^{-\nu} \approx 0.451$.

**Figure 15: Log-log plot of the late time pressure behavior for synthetic example.**
Similarly, intermediate times can be analyzed plotting $t^{1/2} \Delta p(t)$ vs $\sqrt{t}$, and comparing it with the straight line given by eq. 55, after doing the correspondent transformations to field units. Figure 16 shows the $t^{1/2} \Delta p(t)$ vs $\sqrt{t}$ plot for this example.

![Graph](image)

Figure 16: Specialized plot for intermediate times of the pressure behavior for synthetic example.

CONCLUSIONS

1. A fractal flow model that describes transient interporous behavior in double porosity systems was developed. With this model it is possible to consider spheres or slabs as matrix blocks.

2. Main advantage of using fractal models is that these are the best way to represent randomness in the fracture network distribution within the reservoir.

3. Analytical solutions in real space to describe pressure behavior during early, intermediate and late times were developed. Solutions during intermediate times are used to characterize parameters useful for reservoir engineering studies; such as matrix block, fractal fracture network area per unit of bulk volume and interporous skin.

4. Advantages of using transient interporous transference models with interporous skin were discussed.

NOMENCLATURE

$A_{\text{exp.flow}}$ Exposed to flow area.

$A_{f_{\text{fb}}}$ Fractal fracture network area per unit of bulk volume.

$A_{f_{\text{D}}}$ Dimensionless fractal fracture area.

$a$ Site density parameter.

$B_o$ Formation volume factor of oil.

$b$ Extent of the flow region.

$C$ Wellbore storage.

$C_D$ Dimensionless wellbore storage.

$C_{\text{Swell}}$ Storage around damaged zone at wellbore.

$c_{\text{SwellD}}$ Dimensionless storage around damaged zone at wellbore.

$c_{f_{\text{fb}}}$ Fractals fracture network total compressibility.

$c_{\text{ma}}$ Matrix blocks total compressibility.

$D_{f_{\text{fb}}}$ Fractal dimension of the fracture network.

$d_e$ Euclidean dimension.

$d_{\text{ma}}$ Sphere diameter.

$f(H_{\text{maD}}, H_D, t_D)$ Fluid transference function.

$f(s)$ Interporous transference function.

$H_D$ Dimensionless block size.

$h_1$ Characteristic length of medium 1.

$h_f$ Fracture width.

$h_{\text{ma}}$ Strata height.

$k_1$ Permeability of medium 1.

$k_{\text{d}}$ Permeability of interporous damaged zone.

$k_{f_{\text{fb}}}$ Permeability of the fractal fracture network.

$k_{\text{ma}}$ Permeability of the Euclidean matrix blocks.

$M$ Molecular weight.

$n_f(r)$ Number of fractures into fractured bulk.

$p_{f_{\text{fb}}}$ Pressure in fractal fracture network.

$p_i$ Initial pressure.

$p_{\text{ma}}$ Pressure in matrix blocks.

$q$ Flow rate.

$q_g$ Gas flow rate.

$r$ Radial distance from the centre of the source (measured along the flow paths).

$r_D$ Dimensionless radius.

$r_w$ Wellbore radius.

$r_{\text{we}}$ Effective wellbore radius.

$S_{\text{intD}}$ Interporous skin.

$S_{\text{ma-fbD}}$ Interporous skin between matrix and fractures.

$S_{\text{well}}$ Skin around wellbore.

$s$ Laplace space parameter.

$T$ Temperature.

$t$ Time.

$t_D$ Dimensionless time.

$x_d$ Thickness of damaged zone.

$V_b$ Bulk volume.

$V_{f_{\text{fb}}}$ Fractal fracture network volume.

$V_{uf}$ Unitary fracture volume.

$v$ Grouping parameter.

$W$ Mass flow rate.
REFERENCES


APPENDIX A

Generalized Radial Flow Model for Well Test analysis

Barker (1988) proposed a general radial flow model with the shape:
\[
\frac{\partial^2 p_D(r_D,t_D)}{\partial r_D^2} + \frac{d_e}{r_D} \frac{\partial p_D(r_D,t_D)}{\partial r_D} - \frac{\partial p_D(r_D,t_D)}{\partial t_D} = 0;
\]
(A.1)
following boundary conditions are established: Initial condition:
\[
p_D(r_D,0) = 0,
\]
(A.2)
Inner boundary:
\[
\frac{\partial p_D(1,t_D)}{\partial r_D} = -1,
\]
(A.3)
Outer boundary:
\[
\lim_{r_D \to \infty} p_D(r_D,t_D) = 0.
\]
(A.4)
Solution in Laplace Space of eq. A.1 is given by:
\[
p_D(r_D,s) = \frac{s^{2-d_e}}{K_{2-d_e} \left( \frac{r_D}{s} \right)}.
\]
(A.5)
In Real Space:
\[
p_D(r_D,t_D) = \frac{r_D^{2-d_e}}{2\Gamma \left( \frac{d_e}{2} \right)} \left[ \frac{4-d_e}{2} \right].
\]
(A.6)
Dimensionless variables are the same established in the main text, with \( \theta = 0 \) and taking the convergence of fractal fracture network dimension to the Euclidean dimension, i.e. \( D_{fb} = d_e \).

APPENDIX B

Wellbore Storage for Fractal Models

Total flow rate is given by:
\[
q_{B_o} = -C \frac{dp_{wf}(t)}{dt} + \frac{a_d b^{3-d_k} k_{fb}}{\mu} \left( r_{D,D} - 0 \right) \frac{\partial p_{fb}}{\partial r} \bigg|_{r=r_w}
\]
(B.1)
Using dimensionless variables and fractal fracture network definitions:

\[
\left( \beta_D \frac{\partial p_{fb}}{\partial r_D} \right)_{D_D} = C_D \frac{\partial p_{wD}}{\partial t_D} - 1, \quad \text{(B.2)}
\]

Where dimensionless Wellbore Storage for fractal reservoirs is defined as:

\[
C_D = \frac{\phi_{fb} C}{a V_{nf}(\phi_I) r_w D_\phi}.
\]  \quad \text{(B.3)}

**Wellbore Storage in Damaged Zone**

Wellbore Storage in the entire damaged zone in a fractal reservoir is given by:

\[
C_{\text{Swell}} = C + \frac{a V_{nf}(\phi_I) r_{we} D_\phi}{\phi_{fb}} - \frac{a V_{nf}(\phi_I) r_w D_\phi}{\phi_{fb}}, \quad \text{(B.4)}
\]

Where effective wellbore radius is defined as:

\[
r_{we} = r_w e^{-Swell}.
\]  \quad \text{(B.5)}

Substituting effective wellbore radius definition in (B.4), the following expression results:

\[
C_{\text{SwellD}} = C_D - 1 + D_\phi Swell + 1, \quad \text{(B.7)}
\]

Where:

\[
C_{\text{SwellD}} = \frac{\phi_{fb} C_{\text{Swell}}}{a V_{nf}(\phi_I) r_w D_\phi}.
\]  \quad \text{(B.8)}