POSSIBILITY TO USE CO₂ AS EGS FLUID IN HUNGARY

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ABSTRACT
It is well known that in the Pannonian Basin the Earth’s crust is shallower and thinner than the continental average in Europe. After sub-crustal erosion sank and thinned the Pannonian basin, it was filled with tertiary sediment. The basin is now only about 23km thick. Its sedimentary layers have weak thermal conductivity, averaging about 2W/m, with temperatures greater than 200°C down to a depth of 4000m. This last fact favors the creation of Enhanced Geothermal Systems.

Hungary has another advantage: natural gas reservoirs with a high CO₂ content, often more than 90%. CO₂’s low critical pressure makes it ideal as a functional heat-bearing fluid in EGS systems. It can be used to implement a super critical Rankine cycle for electric power production. By positing a Hele-Shaw flow in a single equivalent fracture, with one-dimensional heat conduction in the surrounding rock, we can estimate the potential flow, heat transfer and thermal power of the proposed EGS system.

INTRODUCTION
Because the earth’s crust is thin and sunken within the Carpathian arc, with a higher than average geothermal gradient and greater terrestrial heat flow, Hungary has favorable natural conditions for geothermal energy production and use.

Unfortunately, it’s hard to circulate water efficiently through a fractured reservoir with very narrow cracks -- the flow rate would be too low. Given the need to find another inexpensive, abundant and heat-bearing fluid of lower viscosity and acceptable specific heat, CO₂ appears to be a good alternative. Western Hungary has large, easily accessible CO₂ reservoirs, previously used as artificial gas caps so as to enhance depleted petroleum reservoirs. Using carbon dioxide as a heat-transfer fluid creates certain problems, however, which can only be fully answered through further research, experiment and practice.

We can model the flow in a fractured reservoir, between an injection and a production well, by replacing the fracture system with an equivalent plane fracture bounded by two parallel planes. The flow must be laminar in the narrow gap, as described by the Navier-Stokes equation. A two-dimensional laminar flow pattern develops in the fracture -- the so-called Hele-Shaw flow. In the case of incompressible fluid the flow can be treated by complex variable function.

The most important questions of this study are
- What kind of flow system will be developed between and around the wells?
- How will injection affect to the pressure distribution in the reservoir?

This problem was first investigated by Bodvarsson (1975), later by Bodvarsson, Preuss and O’Sullivan (1985), then by Ghassemi and Tarason (2004), and more recently by Bobok and Toth (2009).

THE CONCEPTUAL MODEL
To get a preliminary result without suitable or reliable input data, it’s best to replace the existing large, horizontal, fractured reservoir model with one that has a single equivalent fracture, bounded by parallel plane walls.

It is assumed that the reservoir is not over pressured, and that the pressure distribution is hydrostatic along its depth. Horizontal extension of the equivalent fracture is much greater than the distance between the two boreholes. The injection well occurs in the plane fracture as a source, and the production well is a sink. The flow in the fracture is steady, laminar and two dimensional. This is the so-called Hele-Shaw flow.
THE MATHEMATICAL MODEL

An orthogonal coordinate system was chosen. The xy plane is parallel to the fracture walls at a point halfway between them. Z is the transverse direction, 2b is the gap between the plates, the x-axes are directed to the source and the sink. Because water cannot be compressed, the flow and the heat transfer can be determined separately.

In a porous reservoir, Darcy’s law can describe the flow of a fluid, where \( \text{rot } \mathbf{v} = 0 \) means the flow is non-rotational and \( \text{div } \mathbf{v} = 0 \) means the flow is incompressible.

For a two-dimensional flow we can write

\[
\frac{\partial v_x}{\partial y} - \frac{\partial v_y}{\partial x} = 0
\]

(1)

\[
\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = 0
\]

(2)

These two equations become identities if they are fulfilled at any point of the xy plane.

From these are obtained the Cauchy-Riemann equations

\[
\frac{\partial \Phi}{\partial x} = \frac{\partial \Psi}{\partial y} ; \quad \frac{\partial \Phi}{\partial y} = -\frac{\partial \Psi}{\partial x}
\]

(5)

Fulfillment of Eq. (5) is equivalent to the existence of an analytic complex variable function \( W(\xi) \), the so-called complex potential

\[
W(\xi) = \Phi(x,y) + i \cdot \Psi(x,y)
\]

(6)

of which the real part is the velocity potential \( \Phi \) and the imaginary part is the stream function \( \Psi \).

The \( \Psi = \text{const} \) curves are the streamlines.

The complex potential of the Hele-Show flow between the source and the sink can be written applying the method of hydrodynamic singularities:

\[
W = \frac{Q}{2\pi} \ln \frac{\xi + a}{\xi - a}
\]

(7)

Fig.1. shows the two singularities at the point \( x = -a \), the source of Q, and at the point \( x = a \), the sink of capacity of -Q. Using the exponential form of \( \xi \) the real and imaginary parts of \( W \) can be separated easily.

![Figure 1. The two singularities.](image)

Thus the equation of the streamlines is

\[
\phi_1 - \phi_2 = \frac{2\pi k}{Q} = \text{const}
\]

(9)

Looking at Fig.1, we can see that

\[
tg(\phi_1 - \phi_2) = \frac{y}{x + a} - \frac{x - a}{y} = \frac{1}{x - a} - \frac{1}{x + a}
\]

(10)

After some manipulation we obtain

\[
x^2 + \left( y - \frac{a}{C} \right)^2 = a^2 \left( 1 + \frac{1}{C^2} \right)
\]

(11)

The streamlines’ shapes are a family of circles between the source and the sink, with centers at \( x = 0 \) and \( y = \frac{a}{C} \). It’s shown in Fig.2.

![Figure 2. Sub-Dividing intoPart-Channels.](image)
If CO$_2$ flows in the fractured reservoir because of the compressibility

\[ v_x = \frac{\partial \phi}{\partial x}, \quad v_y = \frac{\partial \phi}{\partial y} \quad \text{are valid,} \quad (12) \]

\[ v_x = \frac{\partial \rho}{\partial y}, \quad v_y = -\frac{\partial \rho}{\partial x} \quad \text{are not valid,} \quad (13) \]

So there is no complex-function potential!

Temporally the only possibility is to neglect compressibility in the determination of streamlines. With W thus defined the calculation is the same as above. The streamlines are circles.

The equivalent fracture is divided by the circles into part-channels. In every part-channel the flow rate is the same. Compressibility is taken into account in determining the pressure loss and the heat transfer coefficient. The mechanical energy equation can be written for an element arc in the part-channel. The curvilinear coordinate is

\[ \frac{v^2}{2} + \int \frac{dp}{\rho} + \Phi = \text{const} \quad (14) \]

where
- \(p\) is the pressure
- \(\rho\) is the density
- \(v\) is the flow speed
- \(\Phi\) is the potential

The continuity equation

\[ \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho v) = 0 \quad (15) \]

where
- \(\rho\) is fluid density,
- \(t\) is time,
- \(v\) is the flow velocity vector field.

The Boyle-Mariotte equation

\[ \frac{P_1}{P} = \frac{\rho_1}{\rho} \quad (16) \]

where
- \(P\) is the pressure
- \(\rho\) is the density

**Figure. 3. Infinitesimal volume element schema.**

Integration between the injection well (1) and the production well (2) can be derived using

\[ p_1^2 - p_2^2 = \lambda \left( \frac{s_2 - s_1}{4R_H} \right) p_1 \cdot p_1 \cdot v_1^2 \quad (17) \]

\[ R_H = \frac{a \cdot b}{2(a + b)} \quad (a >> b) \quad (18) \]

\[ R_H = \frac{b}{2 \left( \frac{1}{a} + \frac{b}{a} \right)} \quad (19) \]

where
- \(p\) is the pressure
- \(\rho\) is the density
- \(v\) is the flow speed
- \(\lambda\) is the coefficient of friction
- \(a\) is the distance between the source and the sink (Fig.1)
- \(b\) is the distance between the gaps (Fig.2)
- \(s\) is the width of the gap (Fig.2)
- \(R_H\) is the hydraulic radius for the Reynolds number

The other half of the aperture of the fracture is derived using

\[ p_1^2 - p_2^2 = \lambda \left( \frac{s_2 - s_1}{b} \right) 2 \cdot p_1 \cdot p_1 \cdot v_1^2 \quad (20) \]

The difficulty with this is that it is only a rough approximation, which considers CO$_2$ as an incompressible fluid. The Hele-Shaw flow’s velocity can be determined from a velocity potential, but the stream function does not satisfy the Laplace equation. Thus, the flow has no regular complex potential function. The Poisson equation is valid for the stream function, because of the compressibility of the fluid.

One possible simplification would be to neglect compressibility as the first step in determining the constants’ streamlines between the two wells. The plane of the fracture can be divided into part-channels by the streamlines. In the part-channels the
flow can be considered as one-dimensional, using a curvilinear coordinate system. The compressibility of this one-dimensional flow is accounted for in the calculation of the pressure losses and the heat transfer coefficients between the reservoir rock and the fluid.

Because the flow is not isothermal, the carbon dioxide is warmed up continuously as it flows toward the production well, making the relevant calculations more complex. The temperature increases during expansion, so this change of the thermal state must be polytropic. The determination of the pressure loss of a polytrophic gas flow was elaborated by Zsuga (2012).

These are the main difficulties in calculating a compressible Hele-Shaw flow. Before any preliminary experiments, further research would need for a more sophisticated simulation model.

SUMMARY

This paper is a pre-feasibility study, made to determine the possibility of using CO₂ as EGS fluid in Hungary. A fractured limestone reservoir was chosen, of which there are many in southeastern Hungary. In 2009, the formal pre-feasibility study “Cold Front Propagation in a Fractured Geothermal Reservoir” investigated the idea of using a doublet well to supply geothermal energy for EGS power plants. In that paper, the fracture system was replaced by an equivalent fracture adapted to a Hele-Shaw flow. Water was used as the working fluid.

This paper investigates the use of CO₂ as a functional and heat-transfer fluid in an EGS system. Although the CO₂ heat-transfer coefficient is lower than that of water, CO₂ as a working fluid has some advantages: CO₂ is less viscous, loses less pressure throughout the process and – a pleasant surprise – can use take advantage of wetness in dolomite or limestone to dissolve them, thereby widening the gaps. Using our analytical model, one could describe the flow and heat transfer in a single equivalent fracture and describe one-dimensional heat conduction in the surrounding rock.

REFERENCES


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