ABSTRACT

Using the injection volume and earthquake magnitude exceedance relations outlined in Shapiro et al. (2010) and an extensive Paradox Valley (Northern Colorado) injection and seismic dataset first documented in Ake et al. (2005) we computed an $M_{\text{max}}$ of 1.2 at the 95% exceedance probability. This result is remarkably close to the observed $M_{\text{max}}$ of 0.9 (at the 92% exceedance probability) for the Paradox Valley 1991 14-day 11,000 m$^3$ initial injection sequence. In an effort to understand the difference between predicted $M_{\text{max}}$ (5.7 at the 95% exceedance probability) and observed $M_{\text{max}}$ (4.3 at the 55% exceedance probability) for the Paradox Valley 1996-2000 >2,000,000 m$^3$ injection sequence, we investigate the impact of parametric variability on hazard estimates. Key parameters in these injection-related seismicity hazard estimates are the maximum pore pressure necessary to create displacement along randomly-oriented cracks, $C_{\text{max}}$, poroelastic uniaxial storage coefficient, and cumulative injection volume at the time of the end of injection. Predicted induced $M_{\text{max}}$ is much less sensitive to crack density, background earthquake activity rate, and earthquake recurrence $b$ values derived from later injection. Real-time seismicity monitoring during injection is essential to confirm or update these key parameters to obtain robust estimates of $M_{\text{max}}$.

INTRODUCTION

We begin by estimating the pre-injection “tectonic potential” (Shapiro et al., 2007) using Paradox Valley (see Figure 1, from Ake, et al., 2005) injection and seismicity data summarized in Ake, et al. (2005). More recently, Denlinger et al. (2010) produced an effective data summary validating the relationship between Paradox Valley injection, changes in local stress, and seismicity. A key result of this work is that it establishes a systematic link between injection and seismicity (given criteria proposed by Suckale, 2010). We use two injection histories for hazard modeling: first an initial injection volume of 11,000 m$^3$ over 14 days in July 1991 with a hydrostatic pressure of 42 MPa at 4.3 km injection depth which produced an initial $M_{\text{max}}$ of 0.9 with all seismicity occurring within several hundred meters of the injection well and second, a sustained injection (typically 1290 L/min and 81 MPa at 4.3 to 4.8 km) starting in May 1996 which culminated with the observed $M_{\text{max}}$ to date at Paradox Valley of 4.3 in 2000. Here we use these and other data to develop a probabilistic model of the relationship between injection and seismicity, including a parametric sensitivity analysis.
Shapiro et al. (2010) estimate injection related seismicity levels using tectonic potential, cumulative injection volume at the time of the end of injection, background earthquake activity rate normalized for reservoir volume, a poroelastic uniaxial storage coefficient, and the volume and duration of crustal perturbation. Tectonic potential is computed as a function of critical maximum pressure and concentration of pre-existing cracks (Shapiro et al., 2010).

Other time and injection dependent methods for estimating the injection induced seismic hazard include Convertito et al. (2012) and Bachman et al. (2012). Convertito et al. (2012) consider the injection related hazard at the Geysers geothermal field over a certain time window conditioned on a total monthly injection rate (typically 12,000 m$^3$ per month) and seismicity recorded over three years (with a lower bound of magnitude of 1.2) in a volume of approximately 400 km$^3$. The utility of this approach in more modest injection efforts with less seismic data may be problematic.

Bachman et al. (2012) use three models to forecast seismicity related to the Basel injection experience; the statistics based Epidemic Type Aftershock (ETAS) (Ogata 1988) and Reasenberg and Jones (Reasenberg & Jones (1989) models, and the physics based model of Shapiro (Shapiro and Denske, 2009 and Shapiro et al., 2010).

The Shapiro and ETAS models incorporate injection data. Bachman et al. (2012) formally evaluated these different approaches and found that the Shapiro model best forecast injection induced seismicity but a better model might be obtained by incorporating all three approaches.

Here we apply the Shapiro approach to the Paradox injection and seismicity data set and suggest strategies to improve the hazard model via parametric uncertainty analysis. We suggest that improvements in the Shapiro model parameterization, especially estimates for the critical maximum pressure parameter ($C_{max}$) and use of the Guttenberg Richter $b$ value from later injection may result in the same uncertainty reduction that would be obtained by including other models as suggested by Bachman et al. (2012) in the hazard estimate. The use of a single approach would simplify the hazard estimate and ensure internal consistency in parameterization by relying on one model.

**METHODOLOGY**

Our statistical model for estimating injection induced earthquake exceedance probability starts with the
Shapiro et al. (2010) models and assumption that “in the case of an arbitrary non decreasing injection rate, the induced seismicity is an inhomogeneous Poisson process with a temporal event rate that is not constant”. These models use Guttenberg-Richter statistics which are based on earthquakes per area which we convert to an earthquake exceedance probability per volume. Shapiro et al. (2010) compute occurrence probability of n events with magnitude larger than M in a specified time interval for the important case of the probability \( P(0, M, t) \) of the absence of an event with a magnitude larger than a given \( M \) in the time interval from the start to the end of injection.

\[
P(0, M, t) = \exp(-Qc(t)10^{-\Sigma-bM})
\]  

(1)

\( Qc(t) \) is the cumulative injected volume at the time at the end of injection, \( b \) is the Gutenberg-Richter slope of the cumulative magnitude-recurrence for the site, \( \Sigma \) is their seismogenic index,

\[
\Sigma = a - \log \left( F, S \right)
\]  

(2)

\( F \) is the “tectonic potential” from Shapiro et al. (2007) and \( S \) is the poroelastic uniaxial storage coefficient, which is relatively well constrained to the range of \( S=10^{-6} \) to \( 0.5 \times 10^{-7} \) m\(^3\)/Pa for limestone (Domenico and Schwartz, 1997). The validation of our initial probabilistic hazard estimates support the use of these values for Paradox Valley.

The actual value of \( a \) (a Gutenberg-Richter type statistic) used in Shapiro et al. (2010) to predict the maximum magnitude of induced seismicity, \( M_{\text{max}} \), depends on the potential volume of perturbed crust and the duration that the pore-pressure perturbation exists in that volume. It is assumed that \( a \) is consistent with a power law type size distribution of existing cracks. Since \( a \) is usually derived from earthquake recurrence calculations in units of 1/ (km\(^2\)*year), it is necessary to determine the seismogenic thickness appropriate for the region to convert \( a \) to units of 1/ (km\(^2\)*year). Then the duration that pore-pressure perturbations will remain elevated during and after injection must be determined along with the depth range and lateral extent of the pore-pressure perturbation. With this information (duration and perturbed pore-pressure volume), \( a \) can be normalized (multiplied by volume and time) to yield the nondimensional \( a \) required for application in Shapiro et al. (2010). A conservative approach to estimate the volume is to use the maximum seismogenic diffusivity associated with induced seismicity of 10 m\(^3\)/s from Talwani et al. (2007) to calculate the fracture extent over the duration of elevated pore pressure, and thus maximum lateral extent of the pore-pressure perturbation; this maximizes the nondimensional estimate of \( a \).

The tectonic potential has the advantage of only depending on the tectonic activity of the injection region. It is a function of the critical maximum pressure parameter, \( C_{\text{max}} \), and concentration of pre-existing cracks \( N \),

\[
F_{c} = \frac{C_{\text{max}}}{N}
\]

(3)

Rothert and Shapiro (2007) developed the concept of critical pressure, \( C \), which is the pore pressure necessary to create displacement along a crack. For a network of random pre-existing cracks, Rothert and Shapiro (2007) indicate that in the first approximation \( C \) is uniformly distributed between its minimal and maximal values, \( C_{\text{min}} \) and \( C_{\text{max}} \), with values on the order of \( 10^{5} \) to \( 10^{6} \) Pa, respectively. \( C_{\text{max}} \) is usually larger than the injection caused pressure perturbation (excluding maybe a small volume around the injection source). Rothert and Shapiro (2007) estimate \( C_{\text{max}} \) from induced seismicity at their sites (their Table 1).

Table 1: \( C_{\text{max}} \) Values from Rothert and Shapiro (2007).

<table>
<thead>
<tr>
<th>Site</th>
<th>Overpressure (MPa)</th>
<th>Cmax (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fenton Hill</td>
<td>14</td>
<td>8</td>
</tr>
<tr>
<td>Soult-Sous-Forêts</td>
<td>12</td>
<td>4</td>
</tr>
<tr>
<td>Cotton Valley</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>Paradox Valley</td>
<td>34</td>
<td>16</td>
</tr>
</tbody>
</table>

Nominally, the ratio of \( C_{\text{max}} \) to the applied overpressure is the ratio of minimum effective stress to maximum effective stress from Pine and Batchelor (1984). SR, given by

\[
SR = \frac{1+\sin \phi}{1-\sin \phi}
\]

(4)

where \( \phi \) is the friction angle.

The use of \( C_{\text{max}} \) is the result of simplifying assumptions: fractures created during injection are not important, stress corrosion, tectonic loading and stresses leading to the recharging of critical cracks
are much slower than the process of pore pressure relaxation, and that the number of events from the start of injection to current time is given by a spatial integral of pressure perturbations (including those occurring from strongly nonlinear fluid rock interactions) which can be estimated from mass conservation.

Although Rothert and Shapiro (2007), Shapiro et al. (2007), Shapiro and Dinske (2009), and Shapiro et al. (2010) never document the units they use for \( a \) in Equation (2), Cladouhous (pers. comm., 2011) inquired and found that \( a \) was considered nondimensional in these publications. Thus, appropriate use and specification of \( a \) is based on the unit volume of potentially induced seismicity and the duration that pore pressures remain elevated in response to injection, to yield a simple probability distribution for \( M_{max} \).

We validate and expand this approach by using the extensive Paradox Valley injection and seismicity data set and assess the impact of parametric variability.

We begin with a derivation of \( a \) using injection and induced seismicity from Paradox Valley where an injection well was operated with an initial injection volume of 11,000 m³ over 14 days injection with a hydrostatic pressure of about 42 MPa at a depth of 4.3 km (Ake et al., 2005). We then use the modified Shapiro et al. (2010) approach to probabilistically evaluate the hazard potential for the four year-period at Paradox Valley leading up to the observed \( M_{max} \) at Paradox Valley of 4.3. Injection rates and pressures were typically 855 to 1290 L/min and 77 to 81 MPa at 4.3 to 4.8 km over that four year period.

Shapiro et al. (2010) estimate \( \Sigma = 2.5 \) from the first 9 days of an unspecified injection period at Paradox Valley. We calculate \( C_{max} \) using the eight years of injection spanned by the analyses of Shapiro et al. (2007). The effective radius of injection pressure induced seismicity for this period is about 2 km (Denlinger personal communication, 2010), into a formation thickness of 200 m (Bremkamp and Harr, 1988), which is a small fraction of the depth range of the seismogenic crust for the region of about 15 km (Ake et al., 2005). This formation thickness is much less than the depth span of the perforated zones because the perforations below the Leadville formation were quickly sealed by precipitates. We adjust the \( a \) estimate of \( -1.795 \) from LaForge (1997) for the 8-year time period and the volume of potential activity (2 km radius over 200 m thickness) to obtain a nondimensional estimate of \( a \) of \( -1.67 \). We use information from Paradox Valley on friction angle (40 degrees from Bremkamp and Harr, 1988), \( S = 10^{6} \) m³, and fracture density of 0.5 (Bremkamp and Harr, 1988), the duration of injection considered by Shapiro et al. (2010) of eight years, and the Paradox Valley \( \Sigma = 2.5 \) from Shapiro et al. (2010) to back-calculate \( C_{max} \) for Paradox Valley (Table 1). The estimated Paradox Valley \( C_{max} \) of 16 MPa is close to the well-head fluid injection pressure of ~17 MPa associated with the onset of induced seismicity (Ake et al., 2005).

To check the consistency of the estimated \( C_{max} \) of 16 MPa for Paradox Valley and the \( M_{max} \) prediction approach of Shapiro et al. (2010), we calculate the \( M_{max} \) for the initial 1991 14-day injection sequence that injected a total volume of 11,000 m³. We calculate \( M_{max} \) for a period of 30 days to correspond to the time initially required for pressure to decline after injection was stopped (Ake et al., 2005). During this initial injection the perforated active injection sections extended from the top of the Leadville at about 4.2 km depth into basement at 5.1 km. Consequently, we use a formation thickness of 1 km for the initial injection sequence. We use a hydraulic (or seismogenic) diffusivity of 10 m²/s, calculated using well measurements of permeability in the fractures from initial injection well test (Bremkamp and Harr, 1988), to establish the radius of the potentially affected area around the well. We use the initial injection period \( b \)-value from Ake et al. (2005) \( b=0.82 \) to calculate the \( M_{max} \) probability for the initial 1991 Paradox Injection sequence (Figure 2). The observed \( M_{max}=0.9 \) falls within the 95% confidence region of \( M_{max} < 1.2 \) (Figure 2). This suggests that using a reasonable estimate of \( C_{max} \) and the region recurrence information appropriately scaled for the injection characteristics (volume and time), yields a reasonable estimate of \( M_{max} \) for the total volume injected using a sufficient time for the pressure to decrease after injection stops.

We also calculated \( M_{max} \) for the 1996-2000 injection period at Paradox Valley where injection rates were held at their highest values. This period of induced seismicity culminated with the occurrence of an \( M_{max}=4.3 \) earthquake in May of 2000 (Ake et al., 2005), the largest induced Paradox Valley earthquake to date that represents observed \( M_{max} \) at Paradox Valley through 2012. For this case we use a formation thickness of 200 m because the lower perforations were sealed by precipitate and a time period of 4 years to rescale \( a \) from LaForge (1997) along with the \( C_{max} = 16 \) MPa in (2) and (3) to estimate \( M_{max} \) probability (Figure 2). The observed Paradox Valley \( M_{max}=4.3 \) is close to the median prediction of \( M_{max}=4.39 \) (Figure 3). This result is probably not directly applicable to a closed loop geothermal system, because a total net volume of \( > 2 \times 10^{7} \) m³ of fluid were injected at high pressure at Paradox Valley over four years, so this result may be of more interest for sequestration and sustained waste-water disposal activities than smaller scale injections like closed-loop enhanced geothermal systems (EGS) activities.
However, this analysis of Paradox Valley long-term injection induced seismicity does suggest that the Shapiro et al. (2010) approach may provide a useful assessment of $M_{\text{max}}$ for injection storage applications.

Figure 2: Comparison of Shapiro et al. (2010) predicted $M_{\text{max}}$ and observed $M_{\text{max}}$ for the Paradox Valley 1991 14-day 11,000 m$^3$ initial injection sequence.

Figure 3: Comparison of Shapiro et al. (2010) predicted $M_{\text{max}}$ and observed $M_{\text{max}}$ for the Paradox Valley 1996-2000 >2,000,000 m$^3$ injection sequence.

Figure 4: Parametric sensitivity analysis for the 14-day initial Paradox Valley injection showing that the maximum estimated median $M_{\text{max}}$ is mostly sensitive to low values of $C_{\text{max}}$ or uniaxial storage coefficient and high-values of hydraulic diffusivity for a relatively short injection duration.
PARAMETRIC SENSITIVITY ANALYSIS

To evaluate the sensitivity of estimated median $M_{\text{max}}$ for the initial Paradox 14-day injection, we varied the six input parameters required in the Shapiro et al. (2010) approach where varied over the ranges shown in Figure 4. Sensitivity for each parameter in Figure 4 was calculated by holding all values constant at each value’s preferred value and varying the one parameter being tested over its entire range shown in Figure 4. Figure 5 represents the same analyses for the 4 year Paradox Valley injection period. Since we use median $M_{\text{max}}$ for sensitivity illustration the actual 90th percentile for the median $M_{\text{max}}$ is not represented in some of the parameter ranges for the 14-day injection period case (Figure 4). We use a median hydraulic diffusivity for the 14-day injection of $10^{-5} \text{ km}^2/\text{s}$ consistent with the rate of migration of initial seismicity and a median hydraulic diffusivity of $10^{-6} \text{ km}^2/\text{s}$ for the 4-year injection period since seismicity migrated at a much lower rate away from the vicinity of the well over several km from the well.

These sensitivity analyses demonstrate that there are several important parameters to estimate in order for the Shapiro et al. (2010) approach to provide meaningful estimates of $M_{\text{max}}$. The most crucial is $C_{\text{max}}$, which can be estimated from initial well testing, but can be lower away from the well. The sensitivity tests indicate that the most significant hazard results from a weaker formation (lower $C_{\text{max}}$) with little storage but large hydraulic diffusivity located sufficiently far from a well to avoid detection during well testing. It is not clear if the initial b-value should be retained for $M_{\text{max}}$ appraisal of long-term injection since the triggering of larger magnitude exploits pre-existing tectonic stresses. However, the sensitivity tests (Figure 5) suggest that b-value becomes an important parameter to consider for long-term sustained injection.

DISCUSSION AND CONCLUSIONS

The predicted $M_{\text{max}}$ (1.2 at the 95% exceedance probability) is remarkably close to the observed $M_{\text{max}}$ (0.9 at the 92% exceedance probability) for the Paradox Valley 1991 14-day 11,000 m$^3$ initial injection sequence. The predicted $M_{\text{max}}$ (5.7 at the 95% exceedance probability) and observed $M_{\text{max}}$ (4.3 at the 55% exceedance probability) for the Paradox Valley 1996-2000 >2,000,000 m$^3$ injection sequence is arguably a less robust result in predicting injection related ground motion exceedance probabilities. The long right tail of the $M_{\text{max}}$ distribution for the 1996-2000 injection sequence with $M_{\text{max}}=5.7$ at the 95% exceedance probability is realistic if induced...
seismicity is allowed to include basement below the injection horizon. If induced seismicity can occur in basement then source dimensions are only bounded by the maximum width of the crustal seismogenic zone, which is usually at least 10-15 km. However, at Paradox Valley most of the injection became confined to a relatively narrow region in the Leadville formation and pressure perturbations and induced seismicity appear to be confined primarily to depth interval within and close to the Leadville formation located shallower than basement rocks (Ake et al., 2005). Consequently, the maximum rupture width for induced seismicity above basement was on the order of 250-500 m, which implies a physically constrained \( M_{\text{max}} < 4.32 \) using 4000 m as the maximum rupture length associated with the maximum dimensions of the Wray Mesa fault system (Ake et al., 2005), 500 m for the maximum vertical rupture width, a shear-modulus of \( 3 \times 10^8 \) N/m, an average slip of 0.054 m based on Leonard (2010), and the moment-magnitude relation of Hanks and Kanamori (1979). Thus, the Shapiro et al. (2010) \( M_{\text{max}} = 5.7 \) at the 95% exceedance probability prediction was credible for a generic unconstrained source volume for the Paradox Valley 1996-2000 \( >2,000,000 \) m\(^3\) injection sequence. However, since the actual injected volume appears to remained basically confined to the region within and close to the narrow Leadville formation as intended (Ake et al., 2005; Denlinger et al., 2010). Thus, the actual bounding \( M_{\text{max}} \) appears to be closer to \( M_{\text{max}} = 4.3 \) based on the predominantly near-vertical strike-slip deformation (Ake et al., 2005; Denlinger et al., 2010) and hydraulic constraints on maximum vertical dimensions of pore-pressure perturbations that placed limits on vertical rupture dimensions of \(< 0.5 \) km (Denlinger et al., 2010).

Parametric analysis reveals that \( C_{\text{max}} \) is an important parameter which can be estimated from initial well testing, but can be lower away from the well. The sensitivity tests indicate that the most significant seismic hazard results from a weaker formation (lower \( C_{\text{max}} \)) with little storage but large hydraulic diffusivity located sufficiently far from a well to avoid detection during well testing. It is not clear if the initial b-value should be retained for \( M_{\text{max}} \) appraisal of long-term injection since the triggering of larger magnitude exploits pre-existing tectonic stresses. However, the sensitivity tests (Figure 5) suggest that b-value becomes an important parameter to consider for long-term sustained injection.

These results demonstrate that the Shapiro et al. (2010) methodology may be useful for estimating a probabilistic range of \( M_{\text{max}} \) for proposed injection configurations at other locations. A probabilistic \( M_{\text{max}} \) can be used in a standard probabilistic seismic hazard analysis to estimate ground motion exceedance probabilities for proposed injection configurations. Although low magnitude events result in minimal structural damage, injection-related seismicity has unique ground motion characteristics not adequately accounted for in current ground motion prediction equations (Majer, 2010). Consequently, it will be important to derive and validate ground motion predictions for shallow focus, smaller-magnitude earthquakes associated with proposed injection operations. Lower-magnitude-event probabilistic \( M_{\text{max}} \) and ground motion hazard characterization are likely to be especially useful in public discussions where the tradeoffs between nuisance seismicity and public benefit need to be assessed.

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