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A COUPLED FEM MODEL FOR NUMERICAL SIMULATION OF RECHARGEABLE SHALLOW GEOTHERMAL BHE SYSTEMS

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ABSTRACT

The direct implementation of borehole heat exchangers (BHEs) within a 3D model is a challenging numerical problem caused by the geometry of the boreholes with extreme depth-radius ratio. Since this ratio makes the exact calculation of the pipe flow practically impossible we assume a mean pipe flow velocity and calculate the heat transfer within the BHEs with empirical approaches for the advective heat flux. The simulations include the heat transfer between the BHEs and the subsurface. We use the finite element method based software COMSOL Multiphysics to solve the coupled system of partial differential equations describing the problem. The model has been validated by comparing the results with an existing geothermal BHE array of a triangular arrangement located on a test site at the Institute for Solar Energy Research (ISFH) in Hameln/Germany. This test site includes heat pump systems allowing the emulation of different charging and discharging regimes. Special focus in this study lies on studies regarding geometrical properties and different fundamental parameters of BHEs. Thermal subsurface parameters of the system were determined by thermal response tests (TRTs). The validated model is currently used to enhance synergy effects of coupled geothermal and solar thermal systems. The presented approach can also be applied for long time predictions of BHE systems and for the improvement and optimization of geothermal systems of various types.

INTRODUCTION

The rising demand on renewable energies has led to a rising popularity of residential and commercial ground coupled heat pump systems, among other alternatives. The usage of the shallow subsurface as a renewable heat source (or sink) is a sustainable technique for heat production or cooling. The most common design for the thermal connection of a heat pump to the subsurface is a vertical U-pipe or a double U-pipe respectively. It is usually made of HDPE (High Density Polyethylene), filled with a heat exchange fluid and embedded in a borehole of a typical depth of 50m to 150m. The most common fluid is water, if necessary supplemented by technical substances to gain working temperatures below 0[°C]. The borehole is backfilled with a grout material (i.e. bentonite) providing a proper thermal connection between the subsurface and the borehole heat exchanger. The fact that BHEs are usually sited vertically downwards into the subsurface makes it hard to arrange arbitrary in-situ measurements at test sites. Numerical simulation approaches help to understand fundamental processes that can not be observed in-situ.

One important quality feature of a BHE design is its efficiency in heat exchange with the subsurface. A high efficiency can be equated with a low thermal resistivity. The decrease of the resistivity may lead to the option of less deep and therefore less expensive boreholes. Talking about thermal recharging of shallow geothermal systems, the efficiency becomes even more important.

Concerning modeling several different approaches are used and presented. One of the first studies on the U-pipe system used Fourier expansions to calculate the heat flow between the pipes and the subsurface (CLAESSON et al. 1987). ZENG et al. (2003) worked on analytical solutions that take into account the fluid temperature variation along the pipes and the thermal interference among U-tube pipes. ACUÑA (2010) accomplished experimental and numerical tests regarding the U-pipe and a coaxial approach and found that the distance between the upward and downward branch of a U-pipe BHE has a large influence on the thermal resistivity.

The approach presented here allows a detailed view on the BHE regarding the thermal and geometrical key parameters with regard to efficiency. We set up a numerical full-3D model with only sparse simplifications and access to all supposable process variables.

The work has been done within the project Geo-Solar-WP, which is dealing with the reasonable coupling between geo- and solar thermal devices. One part of the project is a test site located at the

Institute for Solar Energy Research Hameln/Emmerthal, Germany. It comprises three 70m double U-Pipe BHEs in a triangular arrangement and two wells of the same depth in between. The results from TRTs (Thermal Response Tests) made at this test site are used for model validations and comparisons. We performed parametric studies to investigate the potential of changes in the geometrical adjustment and material properties regarding the performance of the state of the art Double-U BHEs.

HEAT TRANSPORT IN BHES

The line source approaches for BHEs in a geological environment do not cover the heat transport processes within a BHE sufficiently because they are only valid in certain distances from the pipes (see also eq. 10). Our model is fully 3D and includes the double U-pipes as three-dimensional objects. Since we focus on heat transport processes in the BHE and the grout, advective heat transport that might appear in the surrounding porous medium is neglected so far and only conductive heat transport outside of the heat pipes is regarded. The heat transport equation

$$\rho c_p \cdot \frac{\partial T}{\partial t} = \nabla \cdot (k \nabla T) + Q_h \tag{1}$$

describes time dependent temperature changes caused by thermal gradients and sinks / sources. Parameters are density ρ, heat capacity c_p and thermal conductivity k. This equation is solved in the grout and the nearby subsurface. The heat transport inside the pipes is mainly governed by advective transport of the fluid. The heat equation (1) is expanded by an advective term and becomes

$$\rho c_p \cdot \left(\frac{\partial T}{\partial t} + \vec{u} \cdot \nabla T \right) = \nabla \cdot (k \nabla T) + Q_h \tag{2}$$

with the velocity field $\vec{u}$. Depending on the injection rate, the flow field may be laminar, transient or even turbulent, which has a major effect on the heat transmission between the fluid and the grout. We can hardly compute the fluid flow inside the heat pipes by solving Navier-Stokes equation because this would exceed our numerical prospects. We solve the problem by assuming a mean fluid velocity in z direction that depends on the injection rate and the cross sectional area of the pipes. Then the convection coefficient for the heat transmission is calculated using common correlations. The fluid flow within the pipes is assumed to be fully developed. The reversal point problem at the bottom of the pipes is solved by measuring the temperature of the down flow pipe and taking this as boundary condition of the up flow pipe. The convection coefficient h is typically quantified as

$$h = \frac{Nu \cdot k_{fluid}}{r_{pipe}}. \tag{3}$$

The dimensionless Nusselt number Nu represents the ratio of convective and conductive heat transfer between the pipe wall and the fluid and depends on the flow regime inside the pipe. Transient and even more turbulent pipe flows tend to have velocity components perpendicular to the walls leading to convective heat transport. There are several empirical correlations to calculate Nu in dependence of the fluid and flow parameters, usually expressed in dimensionless numbers (Reynolds number Re, Prandtl number Pr). The applied Churchill-Bernstein correlation (CHURCHILL and BERNSTEIN 1977),

$$Nu = 0.3 + \frac{0.62\,\mathrm{Re}^{1/2}\,\mathrm{Pr}^{1/3}}{\left(1 + (0.4/\mathrm{Pr})^{2/3}\right)^{1/4}} + \left(1 + \left(\frac{\mathrm{Re}}{28200}\right)^{5/8}\right)^{4/5} \tag{4}$$

provides reasonable values for a wide range of parameters. Re and Pr are given by

$$\mathrm{Re} = \frac{u \cdot 2 \cdot r \cdot \rho}{\eta} \tag{5}$$

and

$$\mathrm{Pr} = \frac{\eta \cdot c_p}{k}. \tag{6}$$

The only restriction here is that $\mathrm{Re} \cdot \mathrm{Pr} > 0.2$ which is always fulfilled in our study because Pr has a magnitude in the range of 1 to 10 and Re between 1e3 and 1e4. The heat transfer from the fluid to the pipe wall can thus be expressed as an effective thermal conductivity of the pipe wall. It is the inverse sum of the wall conductivity and the transitional conductivity:

$$k_{eff}^{-1} = k_{HDPE}^{-1} + k_{transition}^{-1} = \frac{1}{k_{HDPE}} + \frac{1}{h \cdot r_{pipe} \cdot \log\left(\frac{r_o}{r_i}\right)} \tag{7}$$

Numerical Model

We built a finite element model using the commercial software package COMSOL Multiphysics. The model showed its ability by the comparison of different BHE designs (OBERDORFER et al. 2011) and was validated with field data (OBERDORFER et al. 2012). In this study, we focus on parametric variations for thermal resistivity optimization. A top view sketch of a BHE is displayed in Figure 1.

Table 1 shows the essential parameters of the model. All parameters are taken from the field test site. For unknown parameters we chose reasonable estimations.

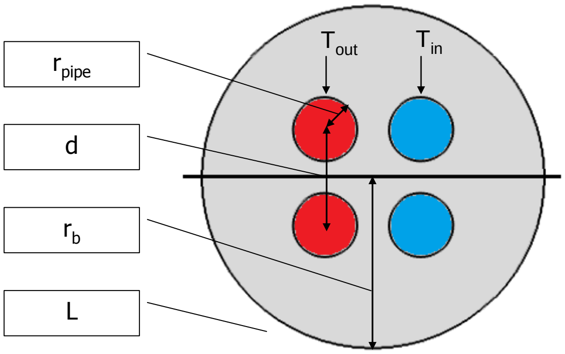

Figure 1: Top view on the Double-U BHE design.

The heat transporting pipe fluid is a brine-water mixture, the grout is a standard filling material (bentonite).
Some assumptions are made that keep the results reasonable but simplify the numerical problem. The most important assumptions are that

- the ground is homogeneous. This is not necessary for computation but to draw conclusions in general.
- at the top surface the temperature of the ground remains unchanged (*Dirichlet condition*) and also in a distance from the boreholes far away enough to have no significant influence on the results. The initial temperature is 11 [°C].
- all material and ground parameters except the heat transmission coefficients are temperature independent.
- the flow in the pipes is full developed.

Table 1: Simulation Parameters

Parameter	Value
L	70 [m]
r_{pipe}	13.1 [mm]
d	61 [mm]
r_B	95 [mm]
ρ_{fluid}	1.0e3 [kgm^{-3}]
ρ_{grout}	7.4e2 [kgm^{-3}]
$\rho_{subsurface}$	2.4e3 [kgm^{-3}]
k_{HDPE}	0.28 [Wm^{-1}K^{-1}]
k_{grout}	2.00 [Wm^{-1}K^{-1}]
$k_{subsurface}$	2.30 [Wm^{-1}K^{-1}]
k_{fluid}	1.71 [Wm^{-1}K^{-1}]
$c_{p,grout}$	2.65e3 [Jkg^{-1}K^{-1}]
$c_{p,subsurface}$	9.17e2 [Jkg^{-1}K^{-1}]
$c_{p,fluid}$	3.81e3 [Jkg^{-1}K^{-1}]
υ_{fluid}	3.9e-6 [m^{2}s^{-1}]

Thermal Resistivity

The efficiency of a BHE can be quantified by the effective borehole thermal resistance

$$R_B = \frac{T_b - T_f}{q} \tag{8}$$

of the BHE. It is defined as the relationship between the heat flow rate transferred by the borehole and the temperature difference between the mean pipe fluid Temperature T_f and the mean temperature T_b of a cylinder around the BHE. The radius of this cylinder determines the included mass that contributes to the thermal resistivity of the BHE. It is here chosen to be exactly the radius of the borehole, hence, the resistivity is only a function of the BHE parameters and not the surrounding subsurface. T_f denotes the arithmetic mean of the inlet and outlet temperatures:

$$T_f = \frac{(T_{in} + T_{out})}{2} \tag{9}$$

Since it is not possible to quantify T_b at in-situ tests, the thermal resistivity (8) can be determined using approximations of the solution of the line source model for the mean fluid temperature, e.g. the approach of INGERSOLL et al. 1954:

$$T_f \cong T_0 + \frac{q}{4\pi k}\left(\ln\left(\frac{4\alpha t}{r^2{}_{BHE}}\right) - \gamma\right) + R_b q \tag{10}$$

This or variations of that approximation is commonly used for the evaluation of TRTs, e.g. GEHLIN (1998) or VDI 4640 (2001). It is valid for times $t > 4r^2/\alpha$. The effective thermal resistance of the BHE system becomes proportional to ln(t) and can easily be derived from (10).

RESULTS

Thermal Response Test Simulation

Initially, we simulated a thermal response test to compare the calculated thermal resistance R_B with field data results. The simulation is done using parameters from

Table 1, a pumping rate of 2000 [l/h] and a heating rate of 4.9 [kW]. The simulation time is 24 [h]. For the evaluation of the test, the knowledge of the mean fluid temperature inside the pipes is essential. Common practice is to use the arithmetic mean of inlet and outlet temperature as in (9). In Figure 2, we compare T_f to the volume integrated mean temperature of the pipes. Both methods differ only slightly in the

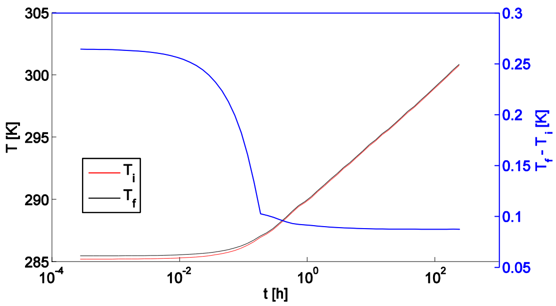

Figure 2: Comparison of mean pipe fluid temperatures T_f (arithmetic mean) and T_i (integrated mean). Blue line shows the temperature difference.

beginning. After the fluid has passed the pipe once (which is after 280 [s]), the difference becomes even lower and stays at less than 0.1 [K].

R_B is calculated according to (8). The results are displayed in Figure 3. The evaluated thermal resistance is about 0.09 [mkW^{-1}]. This value matches to the experimental results at the test site (0.07-0.119 [mkW^{-1}], PÄRISCH et al. 2011). The effect of the different evaluations of the mean fluid temperature is also displayed in Figure 3. $R_B(T_i)$ is about 1.2e-3 [mkW^{-1}] lower at the end of the simulation time.

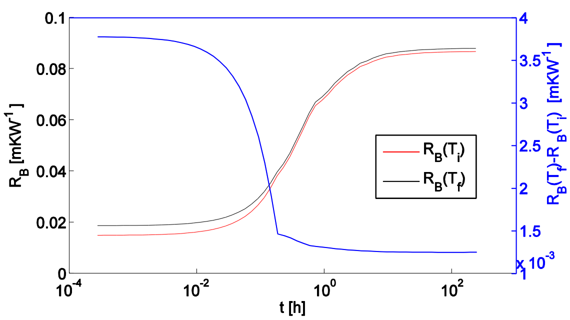

Figure 3: Thermal resistivity R_B of a BHE using T_f (black) and T_i (red) as mean temperatures and the difference between both evaluations (blue).

We conclude that the error made by the arithmetic mean assumption for the mean fluid temperature is small and negligible compared to other errors made by the common evaluation of TRTs. Nevertheless, even this slight difference might have more influence under other conditions.

Parametric Studies

Many studies examine the influence of BHE properties on the thermal resistivity only for certain parametric combinations. Our approach is to compare the effects of relative changes of single parameters. Parameters that are marked by an asterisk (*) are divided by the corresponding test site parameter, i.e. $L^*=L/L_{testsite}$. Thereby, the influences of parameter changes are always illustrated in proportion to defined properties and thus in a more general way.

Thermal Conductivities

Figure 4 shows the results from three parametric studies. We changed the thermal conductivities of the fluid, the HDPE pipe wall and the grout. The conductivities are divided by the test site parameters to make the results comparable. Obviously, the influence of the thermal properties of the grout k^*_{grout} is the most distinct one, followed by k^*_{HDPE} and k^*_{fluid}. Thus, approaches to minimize the resistivity of a BHE should rather focus on a higher grout conductivity than on a higher pipe conductivity.

The resistivity seems to feature a $1/k$ relationship to all three conductivities. The data points were fitted with a function of the form

$$R^*_B(k_i) = a + b \cdot k_i^{*c} \,. \tag{11}$$

The results of the fits are displayed in Table 2. Note that the exponents c of the fits deviate slightly from -1.

The effective resistivity of thermal conductors that are connected in series is the sum of each single partial resistivity and thus the sum of the inverse conductivities. Apparently, this is a good approximation for the grout and BHE conductivities since the deviation from -1 of the exponent c is low in that both cases.

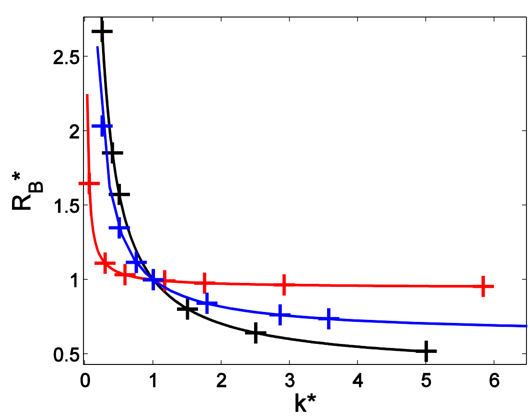

Figure 4: Thermal resistivity changes as functions of the thermal conductivity of the BHE fluid (red), the HDPE (blue) and the grout (black). The solid lines are the corresponding curve fittings.

Table 2: Fitting coefficients of parametric studies in Figure 4

Fitting Function: $R^*_B(k_i) = a + b \cdot k_i{}^{*c}$				
Study	Parameter			MSE
	a	b	c	
k_{fluid}	0.94	0.06	-0.89	1.9e-10
k_{HDPE}	0.63	0.37	-0.96	1.1e-06
k_{grout}	0.38	0.62	-0.94	3.3e-06

Fluid Viscosity

The kinematic viscosity of the heat transfer fluid in the pipes is varied to observe the influence of the variation on the thermal resistance R_B^*. The result of this study in Figure 5 provides a strong increase of R_B^* with increasing ν_{fluid}. A fitting approach gives the empirical relation

$$R^*_B(\nu_{fluid}) = 0.97 + 0.03 \cdot \nu_{fluid}{}^{2.79} \qquad (12)$$

with a *MSE* of 4.48e-4. This nearly cubic dependency of the convection coefficient *h* (3) is due to the dependency on *Nu* which is a function *Re* and *Pr* (4). *Re* is decreasing with increasing viscosity because this inhibits turbulent dynamics. *Pr* is proportional to the viscosity because an increasing ν promotes the viscous diffusion rate. Equation (4) offers that an increasing Reynolds number and a decreasing Prandtl number together decrease *Nu* and therefore decrease the convection coefficient. This relation is also displayed in Figure 5.

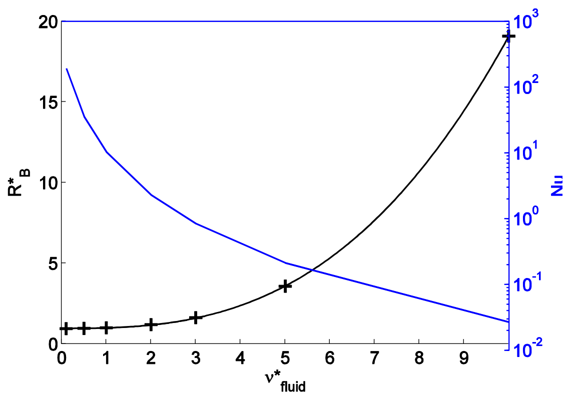

Figure 5: Calculated thermal resistivity changes (black +), fit (black line) and associated Nusselt number as functions of ν_{fluid}.

A lower convection coefficient inhibits the convective heat transfer between the pipes and the grout and yields a higher resistivity of the BHE system.

Geometric Parameters

The influence of geometric parameters on the thermal resistivity of a BHE is examined. Figure 6 shows the results of parametric studies regarding the BHE length L, the borehole radius r_B and the pipe distance *d*. An increasing pipe distance has a decreasing effect on R_B. This conforms to the results of ACUÑA and PALM (2009). An increase of the borehole radius increases the resistivity. This confirms our expectations since it increases the thermally resistive mass surrounding the pipes. The length of the BHE has no significant influence on R_B. This was also expected and proves that the overall heat flux of the BHE is proportional to its length.

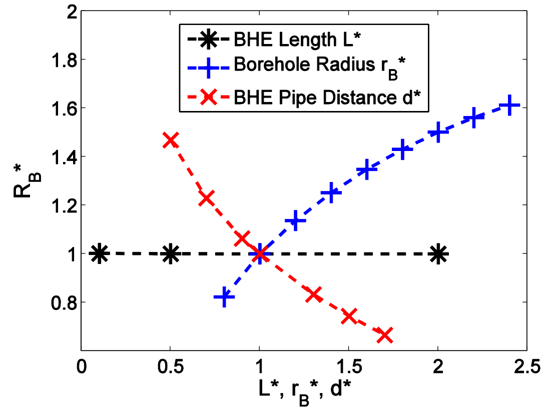

Figure 6: Thermal resistivity changes of the BHE as functions of changes in the length L, the borehole radius r_B and the pipe distance d.

Interestingly, an increase of the pipe radius lowers the resistance to a certain point, before it starts rising again as displayed in Figure 7. Regarding the heat transfer between the fluid and the pipe, a higher pipe radius lowers the velocity, thus it lowers the Nusselt number and the heat transfer coefficient which effects a higher resistivity. Nevertheless, a higher pipe radius also lowers the distance of the pipe walls to the BHE wall which decreases the resistivity analogue to the red curve in Figure 6. At a certain point,

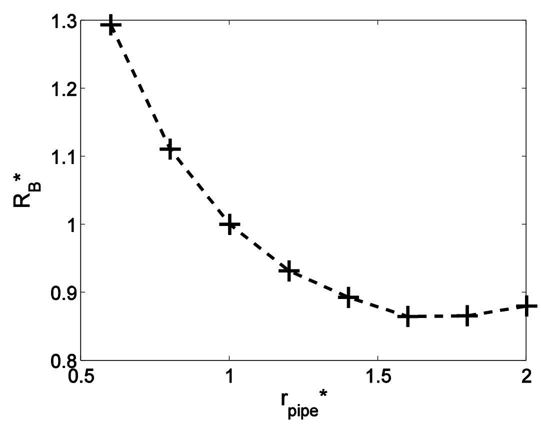

Figure 7: Thermal resistivity changes as a function of r_{pipe}.

it the pipe walls become so close to each other that a thermal short cut occurs. Figure 8 shows the mean heat transfer through the pipes compared to the mean heat transfer through the BHE wall.

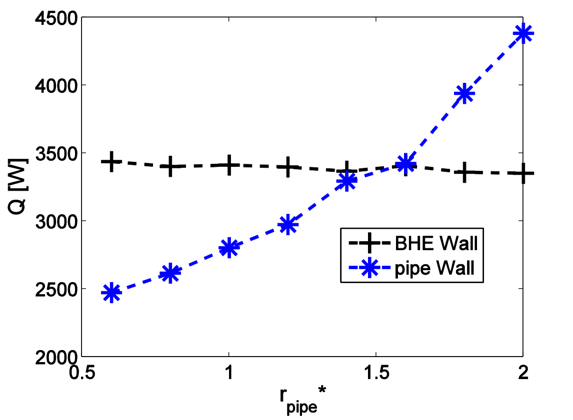

Figure 8: Average heat flux through the pipe walls and the BHE wall as a function of the pipe radius r_{pipe}.

The heat flux through the pipes increases with increasing radius as a consequence of the higher heat exchanging surface. It becomes larger than the BHE wall heat flux. This means that a strong thermal shortcut occurs which increases the thermal resistivity. A comparison shows that the radius at the point of intersection in Figure 8 correlates to the radius of the minimum in Figure 7. Apparently, there is an optimal pipe radius providing much heat exchange between the fluid and the subsurface but not too much thermal short cut between the up and down flow branches.

CONCLUSIONS

In our study, we built reasonable numerical 3D models of BHEs that provide the detailed calculation of inner and outer heat transport processes. The models were validated using test site parameters and experimental results data of thermal response tests.

We use the models for parametric studies of BHE designs. The results show clearances for optimizations of double U-pipe heat exchanger systems. We show that the influences of thermal conductivities of parts of the system on the thermal resistivity provide a $1/k$ correlation in good approximation. Thereby, the highest potential for efficiency enhancement is in increasing the thermal conductivity of the grout material while there is low potential by increasing the thermal conductivity of the BHE fluid.

Computations of geometric BHE parameters show that an increase of the pipe distance and a decrease of the borehole radius have a comparable effect on R_B. We can also conclude that there is an optimal pipe radius leading to a minimal R_B.

Finally, our approach of a 3D BHE simulation gives us a lot of prospects to study fundamental relationships regarding the common heat extraction techniques. Based on the chosen approach in future we will study the influence of ground water flow on the BHE performance.

NOMENCLATURE

α	*thermal diffusivity* $[m^2s^{-1}]$
η	*dynamic viscosity* $[Nsm^{-2}]$
ν	*kinematic viscosity* $[m^2s^{-1}]$
ρ	*density* $[kgm^{-3}]$
d	*pipe distance [m]*
h	*convection coefficient* $[Wm^{-2}K^{-1}]$
k	*thermal conductivity* $[Wm^{-1}K^{-1}]$
k_{eff}	*eff. thermal conductivity* $[Wm^{-1}K^{-1}]$
L	*pipe length [m]*
Nu	*Nusselt number [1]*
Pr	*Prandtl number [1]*
Q_h	*heat source / sink [W]*
q	*heat flow / unit length of pipe* $[Wm^{-1}]$
r_{BHE}	*BHE radius [m]*
r_{pipe}	*inner pipe radius [m]*
R_B	*thermal resistivity of a BHE* $[mKW^{-1}]$
Re	*Reynolds number [1]*
T_f	*arithmetic mean fluid temperature [K]*
T_i	*integrated mean fluid temperature [K]*

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