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THREE-DIMENSIONAL HEAT FLOW MODEL FOR ENHANCED GEOTHERMAL SYSTEMS USING BOUNDARY ELEMENT METHOD

Dharemdra Kumar and Marte Gutierrez

Civil and Environmental Engineering
1500 Illinois Street
Colorado School of Mines
Golden, CO-80401, U.S.A
email:dkumar@mines.edu

ABSTRACT

The paper presents a three-dimensional heat flow model for Enhanced Geothermal Systems (EGS) using the Boundary Element Method (BEM). The heat flow process in the EGS is combination of heat conduction and advection-diffusion. Injection of fracturing fluid during stimulation, with different temperature from the target rock temperature, will induce heat conduction in the surrounding rock mass. Inside the fracture, heat transfer will be mainly by advection-diffusion process due to fluid movement; however, heat process transfer will be a combination of conduction and convection due to fluid leak-off into reservoir. In general, the EGS reservoirs are impermeable or nearly permeable. Hence, zero fluid leak-off condition is assumed for modeling the heat flow. The BEM formulation depends on the fluid flow inside the fracture, which can be modeled either numerically or analytically. The BEM analysis results in a numerical procedure in which discretization of the reservoir geometry is eliminated; hence, only the fracture surface discretization is required. The fracture surface was discretized using 4-node rectangular elements. A point collocation technique with adaptive Gaussian quadrature was used for numerical implementation of boundary integral equation. The weak singularity of integral was removed using singularity transformation technique suggested by Lachat and Watson. Finally, the model was tested with two cases: one with pair of injection and production well and other for the EGS circulation test. Parametric studies were done to quantify the effects injection time on the temperature and on thermal energy extraction rate.

Keywords: Enhanced geothermal system, boundary element method, thermal energy extraction, heat circulation test

INTRODUCTION

Geothermal energy is basically the heat energy stored in the Earth's crust. Geothermal energy can be recovered to generate electricity mainly through two systems; hydrothermal systems or enhanced geothermal systems (EGS). The hydrothermal systems are naturally occurring hot reservoir, which are found at shallow depth. These reservoirs contains sufficient amount of water and steam at high temperature and pressure, which can be recovered to generate electricity. However, the enhanced geothermal or engineered geothermal systems are formed by drilling couple of injection and production wells into hot dry rocks and circulating cool water to fracture network. The thermal energy from the EGS is recovered through production wells in form of hot water and steam. The power plant unit at surface uses the recovered hot water and steam to generate electricity. Finally, the cooled water is recirculated to the reservoir through injection wells to complete the loop. For efficient heat energy production, the injector and production wells need to interconnect either through hydraulically induced or natural fractures. The heat extraction rate of such a system depends on creating permeability so that fluid can contact the rock and extract heat.

Several analytical and numerical models have been developed to estimate Hot Dry Rock (HDR) heat production. Most of these models assume one-dimensional heat condition perpendicular to fracture surface. The analytical schemes can only model case of simple fracture geometry as steady-state flow in a single or in parallel fracture systems (Lauwerier, 1995; Gringarten and Sauty, 1975). Gringanten (1975) presented an analytical solution for cluster of parallel fractures. Elsworth (1989) gave conceptual model for heat extraction based on spherical shape of reservoir. First, coupled three-dimensional heat flow numerical model based on the Finite Element Method was presented by Kolditz (1995b). Kolditz and

Clauser (1998) were able to model the heat circulation test at Rosemanowes (U.K) HDR site. Ghassemi et al. (2003) gave three-dimensional heat extraction model using the BEM approach which eliminated the discretization of the reservoir domain. In this study, the Boundary Integral Equation (BIE) formulation from Ghassemi et al. (2003) was adopted and extended to analyze a 90-day circulation test at the Hijiori (Japan) EGS site.

THEORY AND GOVERNING EQUATIONS

The injection of fracturing fluid during reservoir stimulation with different temperature from the target rock temperature will induce heat conduction and advection-diffusion processes. Inside the fracture, heat transfer will take place mainly due to advection of the fluid mass. However, in direction perpendicular to fracture surface, heat transfer will be due to conduction in the rock formation and convection due to fluid leak-off into the reservoir (if formation is permeable). A schematic of three-dimensional for geothermal reservoir for the heat extraction is shown in Fig. 1. An ellipsoidal shaped reservoir volume is assumed. One single planar fracture of finite length and width is connecting the injection and the production wells. The reservoir rock is assumed homogeneous with constant rock thermal and physical properties. The fluid and the heat flow processes for the heat extraction in the EGS are briefly described in following sections.

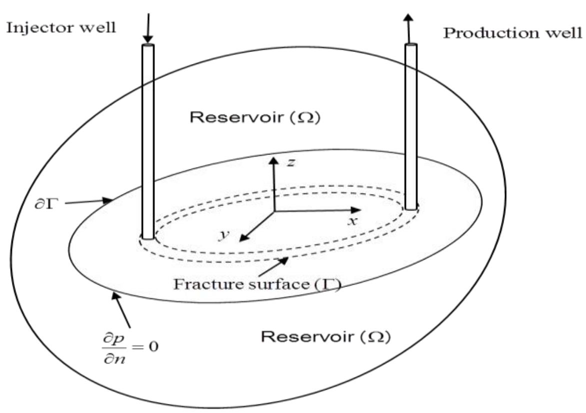

Figure 1. Schematic of geothermal reservoir with planar fracture

Fluid Flow Inside the Fracture

In general, hydraulically driven fractures have very narrow aperture as compare its height and length. Hence, fluid flow inside the fracture can be simulated with flow in large channel with narrow opening. The small fluid injection rate and high fluid viscosity, suggests that laminar fluid flow exist and the lubrication approximation can be applied for flow process. The fracturing fluid is assumed incompressible and follows Newtonian fluid behavior. Assuming impermeable reservoir rock and constant fracture width, the fluid pressure inside the fracture is given as (Ghassemi et al., 2005):

$$\nabla^2 p(\mathbf{x}) = -\frac{\pi^2 \mu}{a^3(\mathbf{x})}\left[Q_I \delta(\mathbf{x}-\mathbf{x_I}) - Q_E \delta(\mathbf{x}-\mathbf{x_E})\right] \qquad (1)$$

where ∇^2 is the second order divergence operator, a is fracture width, μ is the fluid viscosity , p is the fluid pressure, Q_I is the fluid injection rate and Q_E is the fluid extraction, δ is Dirac delta function, $\mathbf{x} = (x, y)$ is any given point inside the fracture, and $\mathbf{x_I} = (x_i, y_i)$ and $\mathbf{x_E} = (x_e, y_e)$ are location of the injection and the production wells, respectively.

Equation (1) can be solved numerically for the fluid pressure using no flux boundary condition on the fracture boundary as shown in Fig. 1. Due to impermeable nature of reservoir rocks Q_I=Q_E is assumed; in other words no fluid loss condition is considered. Once, the fluid pressure is known from the solution of Eq. (1), the fluid flow vector can be estimated using lubrication equation as:

$$\mathbf{q}(\mathbf{x}) = -\frac{a^3(\mathbf{x})}{\pi^2 \mu}\frac{\partial p}{\partial n} \qquad (2)$$

where n is out ward normal to the fracture surface.

Heat Flow Inside The Fracture

The heat flow process inside the fracture is governed by advection-diffusion process. Cheng et al., (2001) described in their work that the fluid diffusion can be neglected in case of continuous injection of cooling fluid and large advection velocities. The heat supply in the fracture occurs due to heat conduction through the fracture walls and heat advection inside the fracture occurs through fracturing fluid. Based on these assumptions, the governing equation for heat flow inside the fracture can be written as (Ghassemi et al., 2003):

$$\mathbf{q}(\mathbf{x}) \cdot \nabla T_N(\mathbf{x}, 0, t) = \frac{2K_r}{\rho_f c_f}\frac{\partial T_N(\mathbf{x}, z, t)}{\partial z} \;;\; \mathbf{x}, z \in \Gamma \qquad (3)$$

where ρ_f is the water density, c_f is the specific heat of water, and $\mathbf{q}(\mathbf{x})$ is the fluid flow vector obtained by solving the fluid flow equation, $T_N(\mathbf{x}, z, t)$ is normalized temperature defined as:

$$T_N(\mathbf{x}, z, t) = \frac{T_R - T(\mathbf{x}, z, t)}{T_R - T_f} \qquad (4)$$

where T_R is reservoir temperature, T_f is injected fluid temperature, $T(\mathbf{x}, z, t)$ is temperature at any location at of the reservoir. The temperature continuity condition is assumed across the fracture surface; hence, same temperature $T(\mathbf{x}, z, t)$ is used for both the reservoir rocks and the fracture fluid.

Heat Conduction between the Fracture and Reservoir Rocks

The heat conduction process between the fracture and the reservoir rocks can be represented using three-dimensional diffusing equation as (Minkowycz et al., 2006):

$$\nabla^2 T_N(\mathbf{x},z,t) = \frac{1}{\alpha}\frac{\partial T_N(\mathbf{x},z,t)}{\partial t}; \quad \mathbf{x}, z \in \Gamma \tag{5}$$

where t is time, $\alpha = K_r/\rho_r c_r$, ρ_r and c_r are the rock density and the specific heat, respectively, K_r is rock thermal conductivity, and Γ is the fracture domain.

Equations (3) and (5) are subject to initial and boundary conditions as: (i) at time t=0 the fracture temperature is equal to reservoir temperature ($T_N(\mathbf{x}, z, 0) = 0$), and (ii) at given any time t, the injection well temperature remains constant and equal to the injection temperature ($T_N(\mathbf{x}_I, 0, t) = 1$).

Boundary Integral Formulation (BIE)

The time dependency of Eqs. (3) and (5) can be removed temporarily using the Laplace transform as (Greenberg, 1998):

$$\tilde{T}_N(\mathbf{x},z,s) = \int_0^\infty T_N(\mathbf{x},z,t)\exp(-st)dt \tag{6}$$

where s is the Laplace transform parameter, $\tilde{T}_N$ is normalized temperature in the Laplace space. The numerical solution in the Laplace space is non-transient and depends explicitly on the transform parameter. This parameter can now simply be treated as a constant for further derivations. The Laplace transform of Eq. (5) results in modified Helmholtz equation as (Divo and Kassab, 2003):

$$\nabla^2 \tilde{T}_N(\mathbf{x},z,s) = \frac{s}{\alpha}\tilde{T}_N(\mathbf{x},z,s) - \frac{1}{\alpha}\tilde{T}_N(\mathbf{x},z,0) \tag{7}$$

and Eq. (3) is given as:

$$\mathbf{q}(\mathbf{x})\cdot\nabla\tilde{T}_N(\mathbf{x},0,s) = \frac{2K_r}{\rho_f c_f}\frac{\tilde{T}_N(\mathbf{x},z,s)}{\partial z} \tag{8}$$

Assuming that the heat lost by the reservoir is equal to heat gain by fracture. Using the Green's function fundamental solution for transient heat diffusion equation (Banerjee, 1994), the BIE for temperature variation is given as (Ghassemi et al., 2005):

$$\tilde{T}_N(\mathbf{x},z,s) = -\frac{\rho_f c_f}{4\pi K_r}\cdot \int_\Gamma \left[\mathbf{q}(\mathbf{x}')\cdot\nabla\tilde{T}_N(\mathbf{x}',0,s)\right]\frac{1}{R}\exp\left(-\sqrt{\frac{\rho_r c_r s}{K_r}}R\right)d\mathbf{x}' \tag{9}$$

where $\tilde{T}_N$ is normalized temperature in the Laplace space, $R = |\mathbf{x} - \mathbf{x}'|$ is distance between field point $\mathbf{x}$ and source point $\mathbf{x'}$ as shown in Fig. 2. The initial condition is absorbed in transform of Eq. (5); however, the boundary condition is changed to $\tilde{T}_N(\mathbf{x}_I, 0, t) = 1/s$.

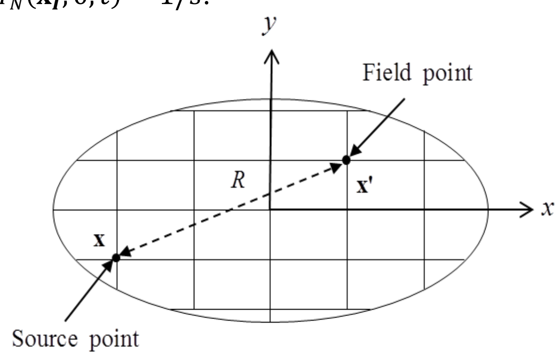

Figure 2. Discretized fracture surface

NUMERICAL IMPLEMENTATION OF THE BIE

Numerical integration of Eq. (9) was performed over the fracture in intrinsic coordinate systems $\{\xi, \eta \mid -1 \le \xi, \eta \le 1\}$ using adaptive Gaussian quadrature. The point collocation technique which says that certain points can be selected to satisfy the BIE in the domain is used (Becker, 1992). In general, numbers of collocation points are equal to total number of nodes in the grid. The nodal fluid flow components were estimated using the FEM technique as described by Ghassemi et al. (2005). The temperature gradients in Eq. (9) were estimated using a combination of backward and forward Finite Difference approximation depending upon nodal fluid flow direction (Ghassemi et al., 2003). The main steps involved in the numerical implementation of the BIE in Eq. (9) are described briefly in following sections.

Domain Discretization

The fracture surface is discretized using E connected 4-node rectangular elements with total N_e nodes.

Over each element the linear variation of the geometry and variables (temperature gradients and fluid flow components) is approximated using shape functions as (Kwon and Bang, 1997):

$$x(\xi,\eta)=\sum_{j=1}^{e}N_j(\xi,\eta)\cdot x_j$$
$$\frac{\partial T_N}{\partial \xi}=\sum_{j=1}^{e}N_j(\xi,\eta)\cdot\left(\frac{\partial T_N}{\partial x}\right)_j$$
$$q_x(\xi,\eta)=\sum_{j=1}^{e}N_j(\xi,\eta)\cdot q_{x,j} \tag{10}$$

where e is number of nodes in an element, N are shape functions for isoparametric quadrilateral elements as defined in Kwon and Bang (1997).

Singular Integration

In point collocation method, when the source $\mathbf{x}$ is near to or coincident with one of the integration point $\mathbf{x}'$, the regular Gaussian integration scheme can't be used for singular integration due to weak singularity of order R^{-1}. A special treatment of integration is required in this situation. A regularization technique proposed by Lachat and Watson (1976) is used to deal with weak singularity issue. This regularization technique suggests a transformation of a triangular domain into a square domain centered about singular node as shown in Fig. 3. As a result of transformation a new Jacobian is introduced which cancels out the singularity of the integrand. The number of triangular divisions depends on the location of the singular node. When the collocation point is on the corner, the original square is into divided into 2 triangular sub-elements as shown in Fig. 3. When the collocation point is on the sides or inside the element, the original square is into divided into 3 or 4 triangular sub-elements, respectively.

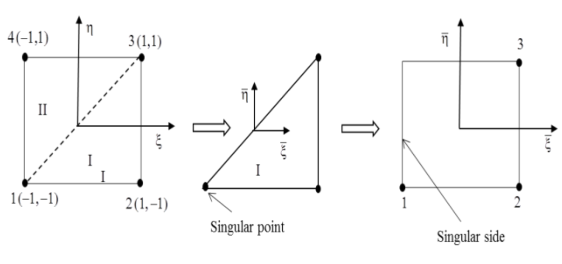

Figure 3. Regularization for weak singular integration (Beer, 2001)

The transformation which maps a triangle with a singularity at node-1 to a square with a degenerated singular side is given as (Hall, 1994):

$$\tilde{\xi}=\xi;\quad \tilde{\eta}=\frac{(1-\xi+2\eta)}{1+\xi}$$
$$J_r=\frac{(1+\xi)}{2} \tag{11}$$

where $\{\bar{\xi},\bar{\eta}\,|\,-1\le\bar{\xi},\bar{\eta}\le 1\}$ are transformed intrinsic coordinates and J_r is Jacobian of transformation. Other cases of singularity can be handled in similar manner.

Numerical Inverse of the Laplace Transform

In application of the Laplace transform approach for time variable, the numerical inversion in real time domain is an important issue. Small truncation errors may greatly influence the results due to an ill-posed nature of the Laplace inversion formulation (Sutradhar et al., 2001). Stehfest (1970) transform technique is mostly used for heat conduction problems due to its stability and accuracy and simplicity for implementation (Erhart et al., 2006). The Stehfest inversion of a Laplace transform $F(s_n)$ of a function $f(t)$ is represented as (Sutradhar et al., 2001):

$$f(t)=\frac{\ln 2}{t}\sum_{n=1}^{N}V_nF(s_n) \tag{12}$$

where $F(s_n)$ value of function in transform parameter , t is time. Values of of s parameter and coefficient V_n are givens as:

$$s_n=n\frac{\ln 2}{t}$$
$$V_n=(-1)^{\frac{N}{2}+1}\sum_{k=\frac{n+1}{2}}^{N}\frac{k^{\left(\frac{N}{2}+2\right)}(2k)!}{\left(\frac{N}{2}-k\right)!(n-k)!(2k-1)!} \tag{13}$$

The parameter N is the number of terms used in summation. Stehfest (1970) suggested for stability of results N=12-14 should be taken. The results from different values of N should be checked to verify its accuracy and smoothness.

RESULTS AND DISCUSSION

The three-dimensional heat flow model was tested for two different systems: (i) a system of injection and production well pair, and (ii) an EGS circulation test field history. The reservoir rock and the fluid data as listed in Table 1 are taken form the Hijiori EGS site;

Table 1. Input Parameters for three-dimensional heat flow model (Yamaguchi, 1992)

No.	Parameter	Symbol	Unit	Value
1	Young's modulus	E	MPa	6000
2	Poisson's ratio	ν		0.25
3	Injection rate	Q	kg/sec	16.7
4	Fluid density	ρ_w	kg/m^3	1000
5	Rock density	ρ_r	kg/m^3	2700
6	Specific heat of fluid	c_w	$\mathrm{J/kg\,^oK}$	4200
7	Specific heat of rock	c_r	$\mathrm{J/kg\,^oK}$	900
8	Rock thermal conductivity	K_r	$\mathrm{W/m\,^oK}$	3.00
9	Injection temperature	T_{inj}	^{o}C	55
10	Reservoir temperature	T_r	^{o}C	250

(1) <u>Injection Production Well Pair</u>

An injection and production well system model was tested using input data form Table 1. The injection well is located at (-25, 0) and the production well at (+25, 0). A surface plot of temperature distribution after 10 years of injection is shown Fig. 4. As seen from the Fig. 4, as we move away from the injection well, the fracture temperature is increasing. The production well temperature after 10 years of injection is recorded around 200°C. A temperature change between the injection and the production well with time is shown in Fig. 5, which shows reservoir cooling with time. At time t=25 years, the production well was cooled down up to 190 °C.

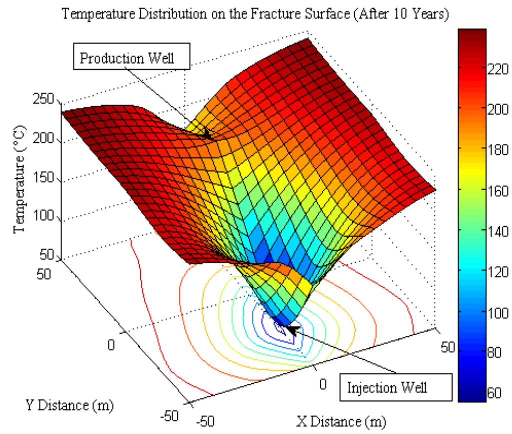

Figure 4. Surface plot of temperature distribution on the fracture surface (after10 years)

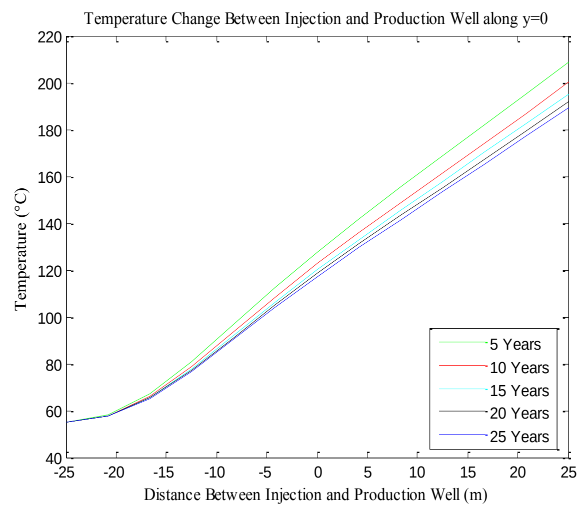

Figure 5. Change of temperature with time between injection and production well

<u>Thermal Induced Stresses</u>

The temperature change of reservoir rocks due to injection induced cooling or temperature uplift will change the state of stress. This process will induce additional stress component in the horizontal plane. Using uniaxial strain assumption, the thermo-elastic stresses changes in the rock mass is given as (Prats, 1981):

$$\Delta\sigma_{ij}^{T} = \frac{E\alpha_T}{1-\nu}\Delta T \cdot \delta_{ij} \qquad (14)$$

where E is the Young's modulus, ν is Poisson's ratio, and α_T is the coefficient of linear thermal expansion of the rock mass(=$8.5 \cdot 10^{-5}$ /^{o}C for Granite), ΔT is change in temperature, and δ_{ij} is the Kronecker delta function. Thermal induced stresses between the injection productions well were estimated and its variation with time is shown in Fig. 6. From Fig. 6, it is obvious that thermal induced stresses are increasing with time and decreasing with distance from the injection well.

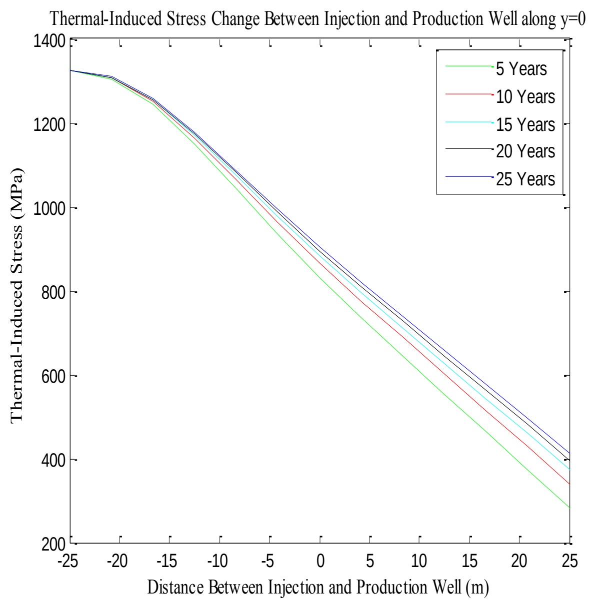

Figure 6. Change of thermal-induced stresses with time between injection and production well

(2) <u>Field Circulation Test</u>

For the field validation of heat flow model, 90-day circulation test form the Hijiori EGS site was selected. A layout of the Hijiori EGS site which was developed by the New Energy and Industrial Development Organization (NEDO) is shown in Fig. 7 This EGS site was developed in two levels: shallow reservoir at 1800 m depth and deep reservoir at 2200 m. A 90-days circulation test which involves 4 well systems in shallow reservoir in 1991 was reported by Yamaguchi (1992). The water was injected in well SKG-2 and produced from well HDR-1, HDR-2 and HDR-3, respectively. The distances from well SKG-2 to well HDR-1, HDR-2, and HDR-3 were about 40, 50, and 55 m, respectively, at a depth of 1800 m. The reservoir temperature was about 250°C and injection water temperature was about 55°C. This test is a unique circulation test in development of the EGS reservoirs, which involved multiple wells and multiple production zones for heat recovery. The data from 90-days circulation test are listed in Table 2, which were used for the heat flow simulation model.

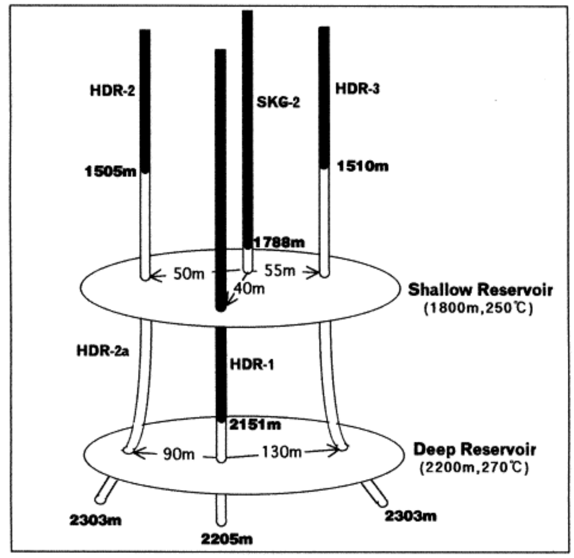

Figure 7. Schematic of position of wells and reservoirs at Hijiori EGS site (Kuriyagawa and Tenma, 1999)

Table 2. 90-day circulation test data (Yamaguchi, 1992)

No.	Parameter	Unit	Value
1	Injection flow rate	Kg/s	16.7
2	Injection pressure	MPa	3
3	Injection temperature	°C	55
4	Production temperature	°C	160-190
5	Pumping duration	days	90
6	Fluid recovery	%	77
7	Thermal output	MW	8.5 MW

Three dimensional heat flow model assuming 100% fluid recovery was developed for the 90-days circulation test using wells geometrical location as shown in Fig. 7. The location of wells coordinates were SKG-2 (0, 20), HDR-1(0,-20), HDR-2 (-24,-16), and HDR-3 (56,-4), respectively. A surface plot of temperature at end of circulation test is shown. The reported some higher range production wells temperature as 182-224°C.

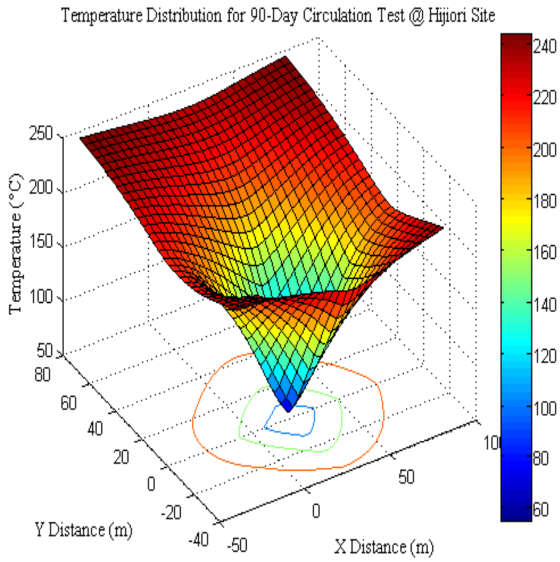

Figure 8. Surface plot of temperature distribution for 90-day circulation test

Heat Extraction

The heat extraction rate or geothermal power output by circulation of fluid between the injection and the production well is given as (Gringarten et al., 1975):

$$W = \rho_f c_f Q(T - T_f) \tag{15}$$

The cumulative heat extraction over a time period t is given as:

$$W = \int_0^t \rho_f c_f Q(T_R - T_f) \cdot \left(1 - \frac{T_R - T}{T_R - T_f}\right) dt \tag{16}$$

The calculated heat extraction form model result is about 10.13 MW, which is higher than actual heat produced during circulation test.

SUMMARY AND CONCLUSION

The three-dimensional heat flow model in this study presents an efficient method for heat extraction modeling both in terms of accuracy and computational efficiency. Since, only the fracture surface discretization is required. It over comes the tradition one-dimensional conduction approach in the heat extraction models. A single fracture model produced higher range of production temperature as compare to field circulation test. The possible reason might be in actual EGS field, there might be a network of fractures connecting the wells. Incorporating fluid-leak in the model, better accuracy of circulation tests can be predicted. Since, the temperature of the fracturing fluid will be lower than that of the formation; the temperature changes will create contractive strains. As a result, the thermo-elastic stress changes will increase the stress state in the reservoir rocks, which eventually will help in hydraulic stimulation process.

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