A NEW LUMPED PARAMETER (TANK) MODEL FOR RESERVOIRS CONTAINING CARBON DIOXIDE

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ABSTRACT

Two of the common characteristics of Turkey’s geothermal fields are that they are initially all liquid dominated and almost all contain some amounts of carbon dioxide. However, such small quantities of carbon dioxide have profound effects on the reservoir pressure behavior. Carbon dioxide has the tendency to shift the flashing point of the reservoir fluid to a considerably higher value. This causes a gas phase to form in the reservoir. Due to the gas phase formed during production, reservoir pressure can be maintained better. When modeling such a reservoir, it is crucial that the effects of carbon dioxide be included in the model. In this study we present a new lumped parameter (tank) model capable of considering the effects of carbon dioxide. The model is based on three conservation equations; mass balances on water and carbon dioxide and an overall energy balance. By doing so, we are able to keep track of average reservoir pressure, average reservoir temperature and the amount of carbon dioxide. A Synthetic field example is given to illustrate the affects of CO2 on reservoir behavior. It can be utilized to better understand the behavior of hot water systems that contain carbon dioxide and to forecast future performance.

INTRODUCTION

Many geothermal reservoirs contain some amount of CO2. For liquid dominated geothermal reservoirs, mass fractions of CO2 dissolved in liquid water can be as much as 5%. The Kizildere field in Turkey, for example, contains around 1.5% CO2 dissolved in the liquid water (Alkan and Satman, 1990).

When modeling geothermal reservoirs (either using numerical models or lumped parameter models) it is crucial to account for the effects of CO2. When production starts in a geothermal field, CO2 dominates the thermodynamic properties of flow. Particularly the flashing point of the water CO2 mixture changes significantly with changing mass fraction of CO2 in the mixture. Geothermal reservoirs with CO2 content have a higher flashing point mixture than reservoirs without it. Increasing amounts of CO2 increase the flashing point pressure.

The effects of CO2 in modeling geothermal systems have been considered by many authors in the literature. Zyvoloski and O’Sullivan (1980) give a very detailed description of the conservation equations to be used in numerical simulation of geothermal reservoirs. In summary the authors use three conservation equations; a mass balance equation for water, an overall energy balance equation and a mass balance equation for CO2. In this study the primary variables are taken to be pressure, enthalpy and temperature.

Atkinson et. al. (1980) present a lumped parameter model for a vapor dominated reservoirs to be used in modeling the Bagnores geothermal reservoir which contains considerable amounts of CO2. However it is important to note that the initial conditions of the Bagnores field are reported to be two phase. Hence the authors have adopted a model that is composed of two tanks. One for modeling the liquid region and the other for modeling the vapor region. Mass transfer is allowed between the two tanks.

O’Sullivan et. al. (1985) give a detailed description of how primary variables should be adjusted during the numerical simulation of a geothermal reservoir based on if the fluid is under a compressed liquid state, two phase state or single phase gas state. The approach given by the authors is still used in many of the numerical models today.

Alkan and Satman (1990), improved the lumped parameter model of Whiting and Ramey (1969)
originally developed for pure water systems, by introducing a thermodynamic package that describes the behavior of water-CO\textsubscript{2} systems. This thermodynamic package simply replaces the thermodynamic package describing pure water systems.

**WATER-CO\textsubscript{2} SYSTEMS**

In this section we briefly describe the behavior of water-CO\textsubscript{2} systems. In the following subsection review of the equations that represent the thermodynamic package of water-CO\textsubscript{2} systems used in this study is given. Then this is followed by an illustration of the most profound effect of CO\textsubscript{2} on water-CO\textsubscript{2} mixtures.

**The thermodynamic package**

The thermodynamic package described in this section is actually a collection of correlations and relationships that are given in the literature previously. The partial pressure of CO\textsubscript{2} is linked with the mass fraction of CO\textsubscript{2} in the liquid water through Henry’s law given in Eq. 1.

\[ f_{CL} = p_{CO_2}H(T) \]

Here \( p_{CO_2} \) is the partial pressure of CO\textsubscript{2} (Pa), \( f_{CL} \) is the mass fraction of CO\textsubscript{2} in liquid water, \( H(T) \) is Henry’s constant (Pa \( \cdot \) s/m\textsuperscript{3}), and \( T \) is temperature (K). As shown in Eq. 1, Henry’s constant is given as a function of temperature. An explicit relation for Henry’s constant is given by Sutton (1976) and is shown in Eq. 2.

\[
H(T) = \frac{5.4 - 3.5(T - 273.15)}{100} + 1.2(T - 273.15)^2 \times 10^{-9}
\]

For simplicity the liquid phase density and the liquid phase viscosity of a water-CO\textsubscript{2} mixture will be taken equal to that of the density and viscosity of liquid water. For the enthalpy of a liquid phase of a water-CO\textsubscript{2} system we use the relationship given by O’Sullivan et. al. (1990) shown in Eq. 3.

\[ h_L = h_w(1 - f_{CL}) + h_{CO_2} + h_{sol}f_{CL} \]

Here \( h_L \) is the enthalpy of the liquid phase (J/kg), \( h_w \) is the enthalpy of liquid water (J/kg), \( h_{CO_2} \) is the enthalpy of the gaseous CO\textsubscript{2} (J/kg), and \( h_{sol} \) is the enthalpy of solution. \( h_{CO_2} \) is calculated based on Eq. 4 given by Sutton (1976):

\[ h_{CO_2} = -2.18 \times 10^5 + 732T + 0.252T^2 - 2.63 \times 10^{-5}T^3 \]  

(4)

The solution enthalpy can be determined using Eq. 5 given by Ellis and Golding (1963).

\[
h_{sol} = \begin{bmatrix}
-1.351 + 0.01692(T - 273.15) \\
-7.5524 \times 10^{-2}(T - 273.15)^2 \\
+1.318 \times 10^{-7}(T - 273.15)^3
\end{bmatrix} \times 10^6
\]

(5)

The pressure of the gas phase can be computed by simply adding the vapor pressure of steam and the partial pressure of CO\textsubscript{2} as shown in Eq. 6.

\[ p = p_s + p_{CO_2} \]

(6)

Here \( p \) is the pressure of the gas (Pa) and \( p_s \) is the partial pressure of steam (Pa). \( p_s \) in this study is determined from IAPWS (2007). The gas phase density in the system can be computed using Eq. 7.

\[ \rho_G = \rho_s + \rho_{CO_2} \]

(7)

Where \( \rho_G \) is the gas phase density (kg/m\textsuperscript{3}), \( \rho_s \) is the steam density (kg/m\textsuperscript{3}), and \( \rho_{CO_2} \) (kg/m\textsuperscript{3}) is the density of gaseous CO\textsubscript{2}. The gas phase viscosity can be computed using Eq. 8.

\[ \mu_G = \mu_s(1 - f_{CG}) + \mu_{CO_2}f_{CG} \]

(8)

Where \( \mu_G \) is the viscosity of the gas phase (Pa\( \cdot \)s), \( \mu_s \) is the viscosity of steam (Pa\( \cdot \)s), \( \mu_{CO_2} \) is the viscosity of gaseous CO\textsubscript{2} and \( f_{CG} \) is the mass fraction of CO\textsubscript{2} in the gas phase. The enthalpy of the gas phase can be determined using Eq. 9.

\[ h_G = h_s(1 - f_{CG}) + h_{CO_2}f_{CG} \]

(9)

Where \( h_G \) is the enthalpy of the gas phase (J/kg), \( h_s \) is the enthalpy of steam (J/kg), and \( h_{CO_2} \) is the enthalpy of gaseous CO\textsubscript{2}. Finally at a given pressure and temperature, the mass fraction of CO\textsubscript{2} in the gas phase can be determined using Eq. 10.

\[ f_{CG} = \frac{\rho_{CO_2}}{\rho_G} \]

(10)

Eq’s. 6, 7, 8, 9 and 10 are taken from O’Sullivan et. al. (1985).
The effects of CO₂ on water properties

In this subsection we mainly illustrate the effects of dissolved CO₂ on the phase behavior of water. This illustration is performed through a pressure-temperature diagram for various mass fractions of dissolved CO₂. The flashing point pressure of the water-CO₂ mixture is obtained using Eq. 6 for various temperatures. The vapor pressure of steam is obtained using IAPWS (2007). The partial pressure of CO₂ is obtained using Eq. 1. Figure 1 illustrates the results.

As mentioned earlier, the most profound effect of CO₂ on the behavior of water - CO₂ mixtures is the shift it causes on the flashing point pressures. For example, at 200°C pure water starts to boil at around 1.5×10⁶ Pa. If dissolved CO₂ exists in the water phase with a mass fraction of 0.5% then the mixture would boil at around 3.1×10⁶ Pa. At a 2.5% mass fraction, the mixture boils at around 8.9×10⁶ Pa. Small amounts of CO₂ dissolved in water considerably changes the bubble point pressure of water.

During depletion, if the flashing point is to move into the reservoir, then a gas phase would start to form. This would have the effect such that, the decline rate in pressures due to production would be decreased significantly. This is due to the fact that below the flashing point pressure a gas phase starts to evolve. Since gas has much higher compressibility when compared with liquids it can compensate for production simply by expanding more than liquids hence causing a decrease in the pressure decline rate.

DESCRIPTION OF THE MODEL

In the model presented here, each component of a geothermal system is represented using a tank that is composed of fluid and rock. The tanks represent either the reservoir, the aquifer, the heating source or the atmospheric block to which natural discharge occurs. In some cases more than one tank can be used to represent the reservoir or the aquifer. Here we will consider that any tank can make an arbitrary number of connections with any other tank. This generalized approach is discussed by Tureyen and Akayapi (2011). Figure 1 illustrates any tank i and the connections it makes to neighboring tanks.

The overall model will be assumed to be composed of a total of N_t number of tanks. Tank i in the system is assumed to make an N_ci number of connections to other tanks. Note that N_ci can vary from tank to tank because each tank in the model can make a different number of connections. Liquid water may be injected into the tank at a specified temperature T_{inj}. Production is specified at a total production rate which is the sum of gas and liquid rates. The individual amounts are determined based on their mobility. The fluid is produced at the tank temperature T_i.

The liquid mass flow rate between any tank is given in Eq. 11.

\[ W_{L,i,j} = \alpha_{L,i,j} (p_j - p_i) \]  

(11)
Here \( W_{L,i,j,i} \) is the mass flow rate of the liquid phase transferred between tank \( i \) and tank \( j \) (kg/s), \( p_{j,i} \) is the pressure of tank \( j \) (Pa), \( p_i \) is the pressure of tank \( i \) (Pa) and \( \alpha_{L,i,j} \) is the recharge index (kg/(bar.s)) which represents the mass flow rate for a given unit pressure drop between the tanks. At this point it is important to note that the recharge index is composed of two parts; a rock part (which is assumed to be independent of pressure and temperature) and a fluid part (which is assumed to be a strong function of pressure and temperature). Hence the recharge index can be written as follows:

\[
\alpha_{L,i,j} = \psi_{i,j} \lambda_L \tag{12}
\]

Where \( \psi_{i,j} \) is the rock part of the recharge index (m³) and \( \lambda_L \) is the fluid part of the recharge index (kg/(Pa.s.m³)). The fluid part is given in Eq. 13.

\[
\lambda_L = \frac{k_{r,L} \rho_L}{\mu_L} \tag{13}
\]

Where \( k_{r,L} \) is the relative permeability of the liquid. The rock part of the recharge index is given in Eq. 14.

\[
\psi_{i,j} \propto \frac{kA}{d} \tag{14}
\]

Here \( k \) represents the permeability of the medium which tanks are assumed to be composed of (m²), \( A \) is the cross-sectional area that the fluid passes through when being transferred between the tanks (m²) and \( d \) is some characteristic length which is a measure of the distance the fluid takes when it is being transferred from one tank to the other (m). It is important to note that the individual values of \( k \), \( A \) and \( d \) need not be known. They are all lumped in \( \psi_{i,j} \) which is treated as an input parameter or can be treated as a parameter to be adjusted during history matching. The fluid part of the recharge index on the other hand is computed for a given pressure, temperature and saturation.

The gas mass flow rate is determined in the same fashion using Eq. 11, except with the subscript \( L \) replaced with \( G \).

The mass balance for tank \( i \) can be written as shown in Eq. 15.

\[
V_i \frac{d}{dt} \left( \rho_L S_L \phi + \rho_G S_G \phi \right) - \sum_{i=1}^{N} \alpha_{L,i,j} \left( p_{j,i} - p_i \right) - \sum_{i=1}^{N} \alpha_{G,i,j} \left( p_{j,i} - p_i \right) + W_{p,L,i} + W_{p,G,i} + W_{inj,L,i} = 0 \tag{15}
\]

Where \( S \) represents saturation (fraction), and \( \phi \) represents the porosity (fraction). The first term represents the accumulation of mass in the tank, the second term represents the liquid mass contribution from other tanks and the third term represents the gas mass contribution from other tanks.

The energy flux due to conduction between any tank \( j \) and tank \( i \) is given by Eq. 16.

\[
Q = \gamma_{i,j} \left( T_j - T_i \right) \tag{16}
\]

Where \( Q \) is the energy rate (J/s) and \( \gamma_{i,j} \) is the conduction index (J/K.s)).

We assume that there is local thermal equilibrium in tank \( i \) between the gas phase, the liquid phase and the rock. Under this assumption the energy balance can be given as:

\[
\frac{d}{dt} \left[ \left( 1 - \phi \right) \rho_m C_m T + V \left( \rho_L S_L \phi + \rho_G S_G \phi \right) \right] + W_{p,L,i} h_L + W_{p,G,i} h_G + W_{inj,L,i} h_{inj,i} - \sum_{i=1}^{N} \alpha_{L,i,j} \left( p_{j,i} - p_i \right) h_L \tag{17}
\]

\[
- \sum_{i=1}^{N} \alpha_{G,i,j} \left( p_{j,i} - p_i \right) h_G
\]

\[
- \sum_{i=1}^{N} \gamma_{i,j} \left( T_j - T_i \right) = 0
\]

Where \( \rho_m \) represents the rock matrix density (kg/m³), \( C_m \) represents the specific heat capacity of the rock (J/(kg.K)), \( u \) represents the internal energy (J/kg) and \( h \) represents the enthalpy (J/kg). When considering the energy contribution from other tanks, we perform an upwinding scheme on the enthalpy as given in Eq. 17.

\[
h_z = \begin{cases} 
    h_i & \text{if } p_i > p_{j,i} \\
    h_{j,i} & \text{if } p_i < p_{j,i}
\end{cases} \tag{18}
\]
Finally the mass balance on the CO$_2$ component is given in Eq. 18.

$$V_i \frac{d}{dt} \left( \rho_L S_L f_{CL} + \rho_G S_G f_{CG} \right)_i - \sum_{l=1}^{N_{\rho L}} \alpha_{L,j,i} (p_j - p_i) f_{CL,j} - \sum_{l=1}^{N_{\rho G}} \alpha_{G,j,i} (p_j - p_i) f_{CG,j} + W_{p,i,j} f_{CL,j} + W_{p,G,i} f_{CG,j} = 0$$

Here $f$ represents the mass fraction of CO$_2$ either in the liquid or the gas phase. We perform an upwinding approach similar to that given in Eq. 18. The approach is given in Eq. 20.

$$f_{\xi} = \begin{cases} f_i & \text{if } p_i > p_j \\ f_j & \text{if } p_i < p_j \end{cases}$$

Eq's. 15, 17 and 19 are non-linear equations and are solved in a fully implicit manner using the Newton-Raphson technique. For the selection of primary variables we use the approach given by O’Sullivan et al. (1985). If the tank contains a single phase fluid, then the primary variables are chosen as pressure, temperature and partial pressure of CO$_2$. If the tank contains gas and liquid phases at the same time, the gas saturation is used as a primary variable instead of the partial pressure of CO$_2$.

**SYNTHETIC EXAMPLE**

In this section we provide a synthetic example to illustrate the effects of CO$_2$ on the performance of geothermal reservoirs. For simplicity a single tank model is chosen. It is assumed that the outer boundaries of the model are closed to fluid flow. The properties of the model are given in Table 1. Constant production at 2 kg/s is assumed for a duration of 10000 days. The pressure behavior of such a system is given in Figure 3. The pressure behavior is given for three different mass fractions of CO$_2$ (0%, 0.5% and 1%).

If no CO$_2$ were present in the water, then production is maintained in a compressed liquid state until 5000 days. After 5000 days, steam and water co-exist in the reservoir. However, it is important to note that once the reservoir fluid becomes two-phase, the decline rate of pressure is decreased due to the much higher compressibility (when compared with liquid compressibility) of the gas phase that co-exists with the liquid.

When 0.5% CO$_2$ is dissolved in water, then two-phase conditions are reached much more quickly (at around 2500 days). The pressure for the remaining 7500 days remains fairly constant maintained by gas compressibility. As expected, even further increasing the CO$_2$ content results in pressure maintenance at even smaller times.

Figure 4 illustrates how the gas saturation changes with time for the same amounts of dissolved CO$_2$. As expected, the gas saturation starts increasing as soon as the flashing point pressures are reached in the reservoir.

At this point it is important to note that, the computed pressures and saturations of the model reflect the average pressure and saturations of the reservoir. During production, gas saturations would be varying with position and would be at a maximum around the well in a case where the bottomhole pressures of wells have dropped below the flashing point pressure.

<table>
<thead>
<tr>
<th>Table 1: Reservoir properties.</th>
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<tbody>
<tr>
<td>Bulk volume, m$^3$</td>
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<tr>
<td>Porosity, fraction</td>
</tr>
<tr>
<td>Initial pressure, Pa</td>
</tr>
<tr>
<td>Initial Temperature, K</td>
</tr>
<tr>
<td>Rock compressibility, Pa$^{-1}$</td>
</tr>
<tr>
<td>Rock thermal expansion coefficient, K$^{-1}$</td>
</tr>
<tr>
<td>Density of rock, kg/m$^3$</td>
</tr>
<tr>
<td>Heat capacity of rock, J/(kg.K)</td>
</tr>
</tbody>
</table>

Figure 3: Pressure behavior for various amounts of CO$_2$ dissolved in water.
Figure 4: Gas saturation behavior for various amounts of CO₂ dissolved in water.

Figure 5 gives the evolution of the mass fraction of the CO₂ dissolved in the water. For both initial mass fractions of 1% and 0.5%, the mass fractions tend to decrease. However as expected this decrease is very small.

Finally Figure 6 gives the evolution of the mass fraction of CO₂ in the gas phase. At first the mass fractions are zero since no gas phase is present. Then when the bubble point pressure is reached and gas phase starts to form, we observe that the gas phase is made up of mostly CO₂. For a mass fraction of 1% CO₂ dissolved in water, the gas phase is composed 85% of CO₂. For a mass fraction of 0.5% CO₂ dissolved in water, the gas phase is composed 80% of CO₂.

Discussion

In the model developed so far, the link between the partial pressure of CO₂ and the mass fraction of CO₂ in water is provided by Eq. 1. The Henry’s constant given in Eq. 1 may be obtained from Eq. 2. However other correlations for Henry’s constant linking the mass fraction with the partial pressure can be found in the literature. One such correlation is given by Upton and Santoyo (2002) as shown in Eq. 21.

\[ \ln K_H = a + b(T + 273.15) + c(T + 273.15)^2 + d(T + 273.15)^3 \]  

(21)

where the constants \( a, b, \) and \( c \) are provided in Table 2.

Table 2: Regression coefficients for Eq. 21.

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
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<tbody>
<tr>
<td>( a )</td>
<td>4.517428673</td>
</tr>
<tr>
<td>( b )</td>
<td>2.5554535 × 10^4</td>
</tr>
<tr>
<td>( c )</td>
<td>-1.02213 × 10^-4</td>
</tr>
<tr>
<td>( d )</td>
<td>9.30689 × 10^-8</td>
</tr>
</tbody>
</table>

Then the relation between the constant given in Eq. 21 and the partial pressure of CO₂ is given in Eq. 22.

\[ p_{CO_2} = \frac{18}{44} K_H f_{CL} \]  

(22)

The differences between the above correlation and the correlations used for the model could be significant. In the future the model will also be updated to use the correlations given in Eq’s. 21 and...
22 and the effects on the reservoir behavior will be studied.

CONCLUSIONS AND FUTURE WORK
This study presents the results of an ongoing study. The following conclusions are obtained and necessary future work is listed:

- A lumped parameter model capable of modeling the pressure and temperature behavior of geothermal systems that contain CO$_2$ is developed.
- The effects of CO$_2$ are most profound on the flashing point pressure. A small amount of CO$_2$ dissolved in the liquid water phase can significantly increase the flashing point pressure for any given temperature.
- Due to the increase in flashing point pressure, two phases can form in the reservoir at relatively higher pressures.
- Due to the gas phase that forms in the reservoir, the pressure decline rate is slowed down. This is because of the much higher compressibility of the gas phase.
- The model has not yet been validated by methods present in the literature. Hence, the next step in the study is the validation of the model.

REFERENCES


