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DETERMINATION OF RESISTIVITY INDEX, CAPILLARY PRESSURE, AND RELATIVE PERMEABILITY

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ABSTRACT

It is known that the three important parameters: resistivity, capillary pressure, and relative permeability, are all a function of fluid saturation in a porous medium. This implies that there may be a correlation among the three parameters. There have been many papers on the approach to inferring relative permeability from capillary pressure data. However the literature on the interrelationship between resistivity index, capillary pressure and relative permeability has been few. The models representing such relationships have been proposed in this study. It has been shown that the other two parameters could be determined using these models if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known. Using this approach, it would be possible to quickly obtain a distribution of capillary pressure and relative permeability characteristics as a function of depth and location across an entire reservoir.

INTRODUCTION

Capillary pressure and relative permeability are the key parameters that govern fluid flow in porous media and have been adopted in petroleum reservoir engineering, geothermal reservoir engineering, soil science, and many other industries. Determination of capillary pressure and relative permeability are traditionally conducted in the laboratory. However it is expensive, difficult, and time-consuming to measure capillary pressure and relative permeability in many cases, especially in cases in which phase transformation and mass transfer exist between the two phases as pressure changes. These include steam-water flow (Li and Horne, 2001 and 2004), gas-condensate flow (App and Burger, 2009; Kumar, et al., 2006; Li and Firoozabadi, 2000), CO_2-oil flow (Dria, et al., 1993), CO_2-water flow (Bennion and Bachu, 2008), etc. It is also difficult to maintain exact reservoir conditions in taking a core sample out from reservoirs and bringing it to the surface, and it is almost impossible either to obtain capillary pressure and relative permeability in real time. On the other hand, it is easier to measure resistivity in the reservoir; a large number of resistivity measurements are available from well logging, even in real time.

Literature on the relationship between capillary pressure and resistivity index has been scarce. A brief discussion on this is presented as follows. Szabo (1974) proposed a linear model to correlate capillary pressure with resistivity by assuming the exponent of the relationship between capillary pressure and water saturation is equal to that of the relationship between resistivity and water saturation. This assumption may not be reasonable in many cases. The linear model proposed by Szabo (1974) can be expressed as follows:

$$\frac{R_t}{R_0} = I = a + bP_c \qquad (1)$$

where R_o is the resistivity of rock at a water saturation of 100%, R_t is the resistivity at a specific water saturation of S_w, I is the resistivity index, P_c is the capillary pressure, a and b are two constants.

The results from Szabo (1974) demonstrated that a single straight line, as predicted by the model (Equation 1), could not be obtained for the relationship between capillary pressure and resistivity index. Longeron et al. (1989) measured the resistivity index and capillary pressure under reservoir conditions simultaneously. Longeron et al. (1989) didn’t attempt to correlate the two parameters. Li and Williams (2006) developed a correlation between resistivity and capillary pressure theoretically. The model was derived according to the fractal modeling of porous media.

As mentioned previously, it is difficulty to measure both capillary pressure and relative permeability. But it is relatively easier to measure capillary pressure, especially when mercury-intrusion approach is applied. It may be because of this that several mathematical models have been proposed to infer relative permeability from capillary pressure data. In

1949, Purcell (1949) developed a method to calculate the permeability using capillary pressure curves measured by mercury-injection. Later, Burdine (1953) introduced a tortuosity factor in the model. Corey (1954) and Brooks and Corey (1966) summarized the previous work and modified the method by representing capillary pressure curve as a power law function of the wetting-phase saturation. The modified model was known as the Brooks and Corey relative permeability model. Li and Horne (2005, 2006) reported that steam-water relative permeability could be calculated from capillary pressure data.

It would be helpful to establish the relationship between relative permeability and resistivity index. However, literature on the relationship between relative permeability and resistivity index has been scarce as well (Pirson et al., 1964; Li, 2007). Routine well testing can only provide the effective permeability of the rock at one specific value of water saturation (usually at irreducible water saturation). Most of the existing approaches to evaluating absolute permeability from resistivity well logging are based on empirical relationships between porosity and permeability. Note that absolute permeability is a concept in single phase fluid flow but resistivity well loggings are usually conducted in rock in which multi-phase flow exists near wellbores. It is not surprised if the absolute permeability data estimated from resistivity well logging (conducted in multi-phase flow) are not consistent with those determined in single-phase flow using other techniques.

In this study, analytical mathematical models correlating resistivity index, capillary pressure, and relative permeability were proposed. It is shown that capillary pressure and relative permeability can be inferred from resistivity data. Actually the other two could be inferred using these models if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known. Experimental data of capillary pressure, relative permeability, and resistivity index were used to test these models.

MATHEMATICS

Resistivity, capillary pressure, and relative permeability have similar features. For example, all are a function of fluid saturation in a porous medium. This implies that there should be a correlation among the three parameters. The models representing such relationships are discussed in this section.

Relationship between Wetting-phase Relative Permeability and Resistivity Index

Li (2007) derived the relationship between relative permeability and resistivity index:

$$k_{rw} = S_w^* \frac{1}{I} \qquad (2)$$

k_{rw} is the relative permeability of the wetting phase. S_w^* is the normalized saturation of the wetting-phase and is expressed as follows:

$$S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \qquad (3)$$

where S_{wr} is the residual saturation of the wetting phase.

The resistivity index, as a function of the wetting-phase saturation, can be represented using the Archie's equation (1942):

$$I = \frac{R_t}{R_0} = S_w^{-n} \qquad (4)$$

where n is the Archie's saturation exponent.

Relative permeability of the wetting-phase can be calculated using Eq. 2 from resistivity index data once the residual saturation of the wetting-phase is available. Note that the residual saturation of the wetting-phase can be obtained from the experimental measurement of resistivity in the porous medium.

Calculation of Nonwetting-phase Relative Permeability

The wetting-phase relative permeability can be inferred from the resistivity data based on Eq. 2. However the relationship between nonwetting-phase relative permeability and resistivity has not been established. The computation of nonwetting-phase relative permeability will be described as follows.

According to Li and Horne (2006), the wetting-phase relative permeability can be calculated using the Purcell approach (1949):

$$k_{rw} = (S_w^*)^{\frac{2+\lambda}{\lambda}} \qquad (5)$$

where λ is the pore size distribution index and can be calculated from capillary pressure data. After the relative permeability curve of the wetting-phase is obtained using Eq.2, the value of λ can be inferred using Eq. 5.

According to the Brooks-Corey model (1966) and the study by Li and Horne (2006), the relative permeability of the nonwetting-phase can be calculated once the value of λ is available. The equation is expressed as follows:

$$k_{rnw} = (1 - S_w^*)^2 [1 - (S_w^*)^{\frac{2+\lambda}{\lambda}}] \qquad (6)$$

One can see that the entire relative permeability set (both wetting and nonwetting phases) can be inferred from resistivity index data using Eqs. 2 and 6.

Relationship between Capillary Pressure and Resistivity Index

There are two approaches to determining capillary pressure once resistivity index data are available. The first approach is to calculate capillary pressure using the Brooks and Corey capillary pressure model (Brooks and Corey, 1966):

$$P_{cD} = (S_w^*)^{-1/\lambda} \quad (7)$$

where P_{cD} is the dimensionless capillary pressure (P_c/p_e); p_e is the entry capillary pressure and λ is the pore size distribution index.

As pointed out previously, the value of λ could be inferred once resistivity index data are available. Therefore the dimensionless capillary pressure can be determined using Eq. 7 with the value of λ.

The second approach to determining capillary pressure is the application of the model developed by Li and Williams (2006):

$$P_{cD} = (I)^{\beta} \quad (8)$$

where β is the exponent in the relation between disjoining pressures and film thickness. One can see from Equation 8 that the dimensionless capillary pressure can be calculated from the resistivity index once the value of β is known.

Relationship between Wetting-phase Relative Permeability and Capillary Pressure

According to Eq. 7:

$$S_w^* = P_{cD}{}^{-\lambda} \quad (9)$$

Substituting Eq. 9 into Eq. 5, one can obtain:

$$k_{rw} = P_{cD}{}^{-(\lambda+2)} \quad (10)$$

According to the above description, Eqs. 2, 5, 6, 7, 8, and 10 constitute the interrelationship among resistivity index, capillary pressure, and relative permeability. This implies that if one of the three parameters (capillary pressure, relative permeability, and resistivity) is known, the other two could be inferred using these models (also see Figure 1). As shown in Figure 1, assuming that the resistivity index data are available, the wetting-phase relative permeability can be calculated using Eq. 2. Then the value of λ can be estimated using Eq. 5. Finally the nonwetting-phase relative permeability can be determined using Eq. 6. Capillary pressure can be estimated using Eq. 7 or 8 when resistivity index data are known. Relative permeability and capillary pressure can be inferred from each other based on Eq. 10.

Note that the end points of relative permeability at initial water saturation or at residual fluid saturation and the entry capillary pressure can not be inferred using the above models. Those values need to be determined using different approaches. For example, the relative permeability of oil phase at the initial water saturation may be estimated from well testing data.

RESULTS

In this section, the results of relative permeability calculated using resistivity index and capillary pressure data are analyzed and compared with experimental data. Also discussed are the relationships between resistivity index and capillary pressure.

Verification of the relationship between relative permeability and resistivity index

As shown in Figures 2 and 3, Li (2007) demonstrated that the values of the relative permeability calculated using resistivity index were consistent with those calculated using capillary pressure data measured by Sanyal (1972) in different rocks (Berea, Boise sandstone and limestone) with different permeability. Figure 2 shows the oil/water relative permeability data obtained from resistivity index and capillary pressure in Berea sandstone sample with a porosity of 0.204 and a permeability of 300 md at 175°F and 300°F. The relative permeability data inferred from the resistivity index data are close to those calculated using the capillary pressure data. The oil relative permeabilities inferred from the resistivity index data are almost equal to those calculated from the experimental capillary pressure data. Figure 3 shows the relative permeability data calculated from both the resistivity index and the capillary pressure data measured in the Boise sandstone core sample with a porosity of 0.32 and a permeability of 960 md at different temperatures (175°F and 300°F). The oil relative permeabilities calculated from the resistivity index data are almost the same to those calculated from the capillary pressure data at both temperatures. However the water relative permeability calculated from resistivity index data is smaller than those inferred from capillary pressure at the temperature of 300°F.

In Figures 2 and 3, however, the relative permeability data calculated from resistivity index are not compared with the experimental data of relative permeability. It would be better to verify Eq. 2 by comparing the relative permeability data inferred from resistivity index with the experimental data of relative permeability directly (Li, 2006). The experimental data of resistivity and gas/water relative permeability measured by Pirson et al. (1964) in eight core samples (sandstone) with different permeability were used to test the models (Eqs. 2 and 6). The permeability of the core samples ranged from 10 to 280 md. The values of porosity, permeability, and initial water saturation (S_{wi}) are listed in Table 1.

Nitrogen was the nonwetting-phase and brine with a concentration of 5% NaCl was the wetting phase. The resistivity and relative permeability were measured simultaneously at an ambient temperature. The results of relative permeability inferred from resistivity index data using Eqs. 2 and 6 were compared with the experimental data. Figure 4 shows the comparison of gas and water relative permeability calculated from resistivity index with experimental data in core sample No.1. Both the gas and water relative permeability data calculated from resistivity index data using the mathematical models (Eqs. 2 and 6) were almost equal to the experimental data at the same water saturation.

For core sample No. 2, the results are plotted in Figure 5. The water relative permeability data calculated using Eq. 2. are approximately equal to the experimental data. But for the gas phase, the calculated relative permeability is smaller than the experimental data.

For all of the rest six core samples, the results are shown in Figs. 6 to 11. One can see that the models (Eqs. 2 and 6) work better in core samples with greater permeabilities than in those with lower permeabilities. The calculated gas phase relative permeability is smaller than experimental data in core samples with low permeabilities. One of the possible reasons may be due to gas slippage in two phase flow (Li and Horne, 2004). The gas slip effect in two phase flow was not considered in the experimental data of relative permeability. Note that the gas slippage is greater in core samples with low permeabilities than in those with high permeabilities.

Verification of relationship between capillary pressure and resistivity index with experimental data

Li and Williams (2006) verified, to some extent, the relationship between capillary pressure and resistivity index (Eq. 8) using the experimental data of gas-water capillary pressure and resistivity measured simultaneously. The experiments were conducted at a room temperature. All of the core samples were sandstones and were obtained from one oil reservoir but different formations. Core samples in Group 1 were from one formation with a high permeability ranged from 437 to 3680 md and those in Group 2 were from another formation with a low permeability ranged from 0.028 to 387 md. To further verify Eq. 8, both the experimental data of capillary pressure and resistivity index measured by Sanyal (1972) in different rocks (Berea, Boise sandstone and limestone) were used. The results are presented as follows.

Figures 12 and 13 show the relationship between dimensionless capillary pressure and resistivity index measured in Berea and Boise sandstone at different temperatures. Figure 14 plots the experimental data of dimensionless capillary pressure and resistivity index in different rocks at a temperature of 300°F. One can see that Eq. 8 can fit the experimental data of dimensionless capillary pressure and resistivity index in different rocks and at different temperatures of 175, 250, and 300°F. Note that most of the fitting curves go through the point of (1, 1) in Figures 12-14, as foreseen by Eq. 8. The values of β and the regression coefficient are listed in Table 2.

Verification of the Relationship between Wetting-phase Relative Permeability and Capillary Pressure

The experimental data of relative permeability and capillary pressure measured by Richardson et al. (1952) and Li and Horne (2004) respectively were used to verify Eq. 10. Richardson et al. (1952) measured oil-gas relative permeability and capillary pressure in Berea sandstone with permeability and porosity of 107 md and 0.177. Li and Horne (2004) measured the nitrogen-water relative permeabilities using a steady-state method and measured the capillary pressure using semiporous plate technique in a fired Berea core sample with a porosity of 0.244 and a permeability of 1200 md. All the experimental data of the wetting-phase (oil phase in Richardson et al., 1952 and water phase in Li and Horne, 2004) relative permeability and capillary pressure are plotted in Figure 15. As one can see, Eq. 10 matches the experimental data very well. The value of regression coefficient (R^2), i.e., the goodness of fitting, of Eq. 10 to the experimental data oil relative permeability vs. capillary pressure is 0.982 and is 0.986 in the case of water relative permeability vs. capillary pressure.

CONCLUSIONS

The following conclusions may be drawn according to the present study. The three saturation functions, resistivity index, capillary pressure and relative permeability, are coupled and can be inferred from each other when one of the three parameters is known. The relationship between wetting-phase relative permeability and dimensionless capillary pressure has been derived and is a power law function. The mathematical models have been verified using experimental data of resistivity index, capillary pressure, and relative permeability.

ACKNOWLEDGEMENTS

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NOMENCLATURE

k =permeability
k_{rnw} =relative permeability of nonwetting phase
k_{rw} = relative permeability of wetting phase
k_{rg} = relative permeability of gas phase
k^*_{rw} = end-point relative permeability of wetting phase
k_{ro} = relative permeability of oil phase
P_c = capillary pressure
p_e =entry capillary pressure
P_{cD} =the dimensionless capillary pressure (P_c/p_e)
S_w = wetting phase saturation
S_{wi} = initial water saturation
S^*_w = normalized wetting phase saturation
S_{wr} = residual wetting phase saturation
λ = pore size distribution index
ϕ = porosity
β = the exponent in the relation between disjoining pressures and film thickness
n= the Archie's saturation exponent
R_o = the resistivity of rock at a water saturation of 100%
R_t = the resistivity at a specific water saturation of S_w

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Table 1: Properties of core samples

Core #	Porosity, f	Permeability, md	S_{wi}, f
1	0.25	280	0.37
2	0.26	250	0.39
3	0.19	70	0.35
4	0.20	15	0.48
5	0.21	10	0.46
6	0.27	100	0.50
7	0.17	40	0.28
8	0.26	75	0.40

Table 2: Rock properties and the values of regression coefficient

Rock	Porosity, f	Permeability, md	T, °F	β	R^2
Berea	0.204	300	175	2.27	0.966
			250	2.46	0.986
			300	1.37	0.989
Boise	0.32	960	175	1.49	0.997
			250	2.10	0.972
			300	2.12	0.987
Limestone	0.19	410	300	2.08	0.969

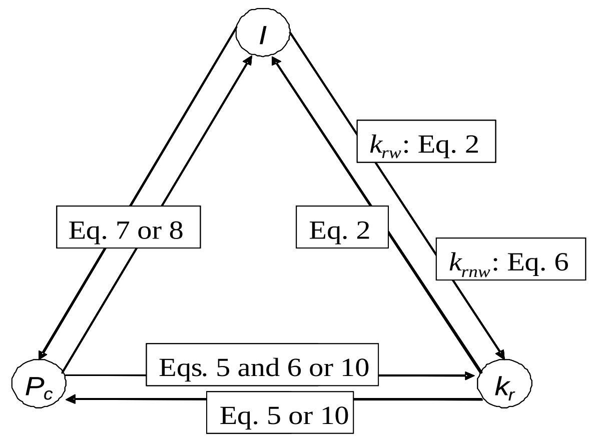

Figure 1: Procedure of inferring saturation function from each other

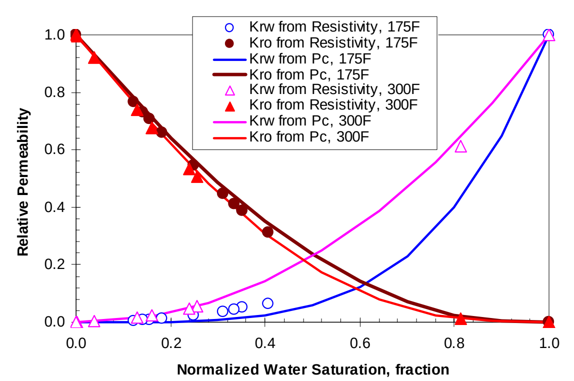

Figure 2: Relative permeability calculated from resistivity and capillary pressure data in Berea sandstone at 175°F and 300°F

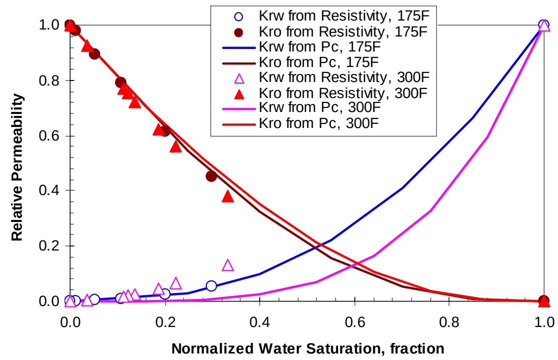

Figure 3: Relative permeability calculated from resistivity and capillary pressure data in Boise sandstone at 175°F and 300°F

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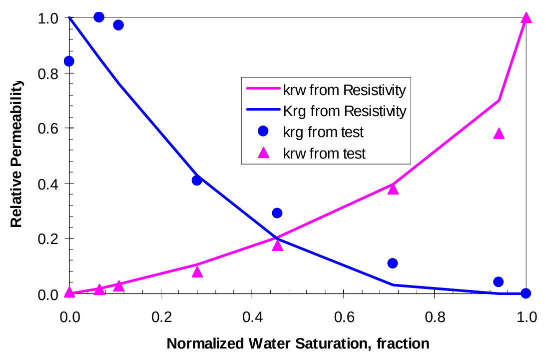

Figure 4: Comparison of relative permeability calculated from resistivity with experimental data (core No.1, ϕ=0.25, k=280 md, Swi=0.37)

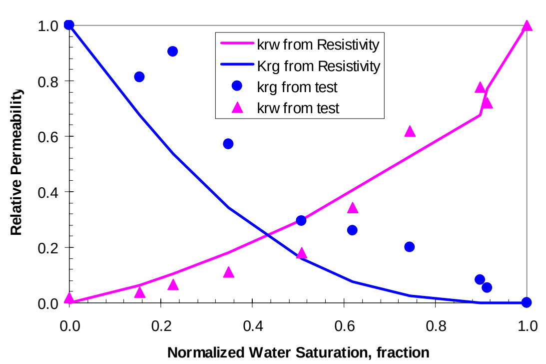

Figure 7: Comparison of relative permeability calculated from resistivity with experimental data (core No.4, ϕ=0.20, k=15 md, Swi=0.48)

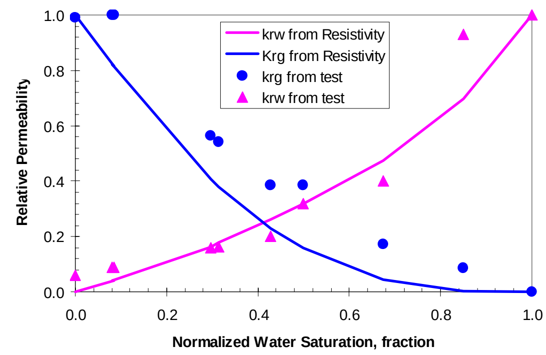

Figure 5: Comparison of relative permeability calculated from resistivity with experimental data (core No.2, ϕ=0.26, k=250 md, Swi=0.39)

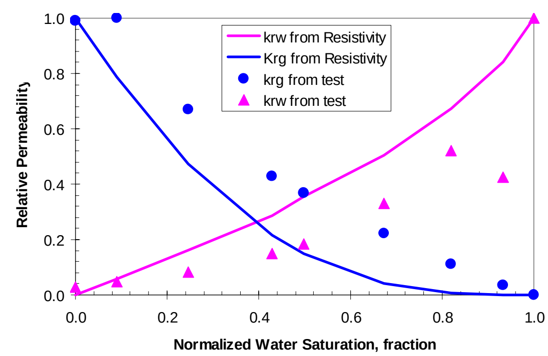

Figure 6: Comparison of relative permeability calculated from resistivity with experimental data (core No.3, ϕ=0.19, k=70 md, Swi=0.35)

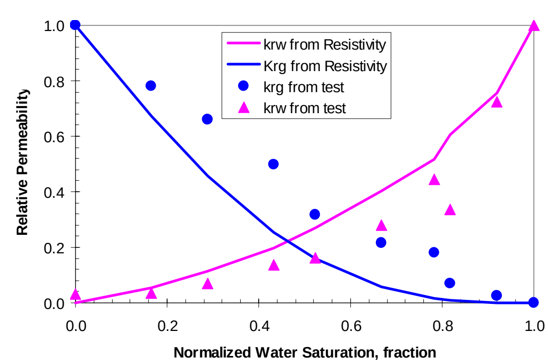

Figure 8: Comparison of relative permeability calculated from resistivity with experimental data (core No.5, ϕ=0.21, k=10 md, Swi=0.46)

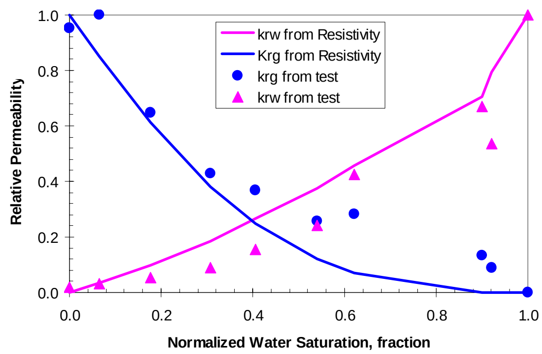

Figure 9: Comparison of relative permeability calculated from resistivity with experimental data (core No.6, ϕ=0.27, k=100 md, Swi=0.50)

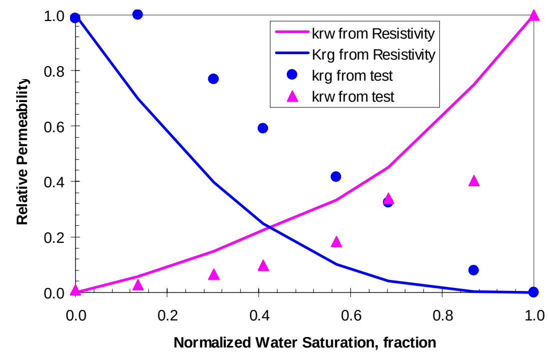

Figure 10: Comparison of relative permeability calculated from resistivity with experimental data (core No.7, ϕ=0.17, k=40 md, Swi=0.28)

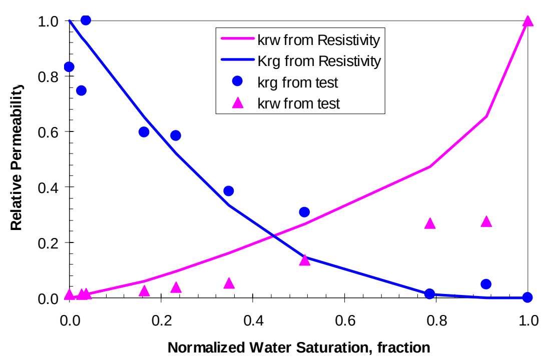

Figure 11: Comparison of relative permeability calculated from resistivity with experimental data (core No.8, ϕ=0.26, k=75 md, Swi=0.40)

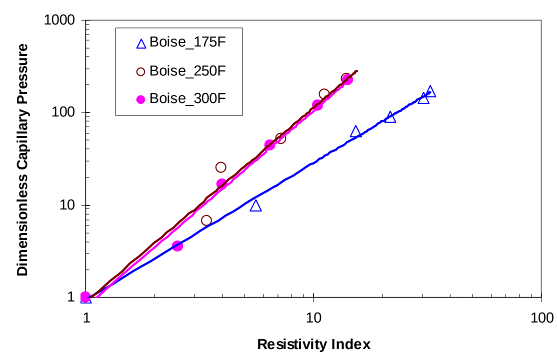

Figure 13: Relationship between dimensionless capillary pressure and resistivity index in Boise sandstone at different temperatures

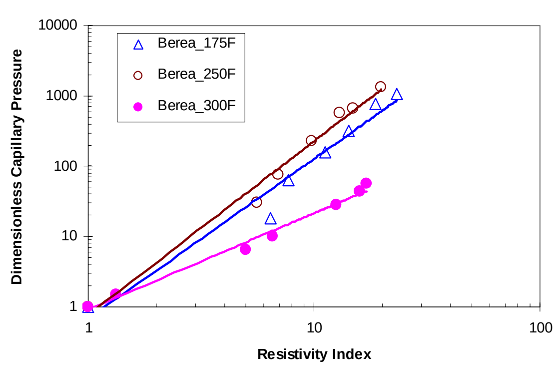

Figure 12: Relationship between dimensionless capillary pressure and resistivity index in Berea sandstone at different temperatures

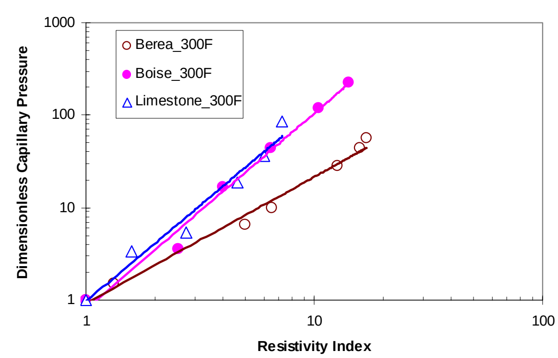

Figure 14: Relationship between dimensionless capillary pressure and resistivity index in different rocks at a temperature of 300^{o}F

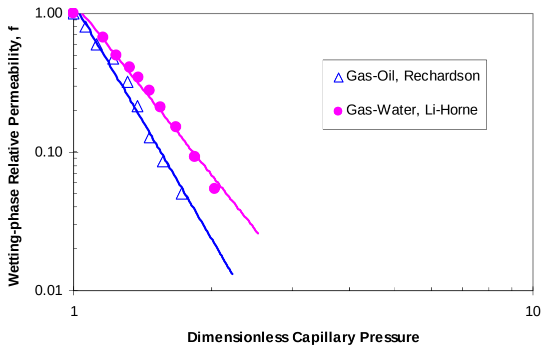

Figure 15: Relationship between wetting-phase relative permeability and dimensionless capillary pressure