COMPARISON OF AN ANALYTIC AND A NUMERICAL APPROACH FOR TRACER TRANSPORT IN A FRACTURED GEOTHERMAL RESERVOIR

Aniko Toth, Peter Szucs and Elemer Bobok
University of Miskolc
Miskolc-Égyetemváros, 3515 Hungary
e-mail: toth.aniko@uni-miskolc.hu

ABSTRACT
Between two wells in a fractured limestone geothermal reservoir the flow was investigated by two different methods in order to characterize tracer transport phenomena. The knowledge of reliable tracer transport simulation is a key issue to derive the petrophysical properties of geothermal reservoirs. In the first simulation the flow was considered in an equivalent plane fracture by the Hele-Show flow approach. The second method replaced the fractured system by an equivalent porous layer in which a Darcy flow develops. The Hele-Show flow is described by analytic complex variable functions, while the Darcy flow is determined applying finite difference simulation method. The obtained results are similar adequately. Especially the coincidence is very strong between the injection and production wells..

1. INTRODUCTION
A pre-feasibility study was elaborated to prepare the implementation of the first Hungarian geothermal pilot power plant. The hydraulic and thermodynamic performance of the fractured geothermal reservoir was modeled for this purpose. The flow pattern in the fracture and the heat transfer in the adjacent rock was determined by the method of hydrodynamical singularities and the transient heat conduction equation. The same phenomenon was modeled from another aspect: the simulation of tracer transport in the fractured reservoir. The hydraulic and tracer transport modeling is based on a finite difference method applying the MODFLOW 2000 and MT3DMS simulation softwares. If the results obtained by two independent approach are similar, the adequacy of both method is confirmed.

2. DESCRIPTION OF THE SOFTWARE USED FOR MODEL CONSTRUCTION
The hydraulic and contaminant modeling using the MODFLOW-2000 and MT3DMS simulation software are resorted for evaluation of the available options of tracer transport in a fractured geothermal reservoir. An equivalent porous media (EPM) was assumed with the Darcy-flow approach during the numerical modeling activity. This modeling study facilitates a hydraulic and transport modeling approach for efficient investigation and management of fractured geothermal reservoirs. Flow and transport modeling and simulations based on the all available information can help the decision makers to find the most effective and environment friendly solution for injection strategy. Here, the Processing MODFLOW for Windows 7.0 (PMWIN Pro) software (Chiang and Kinzelbach 2001) was used for different simulations. The finite-difference MODFLOW 2000 module (Harbaugh et al. 2000) is an industrial standard used to create accurate and reliable 3-dimensional groundwater flow models. Besides the groundwater quantity issues, the PMWIN Pro can handle tracer or contamination transport processes using its well-known MT3DMS program. In most cases, transient-state simulations are required to follow up the consequences of the time dependent processes. The PMWIN PRO modelling environment provides the experts with convenient input options, where boundary conditions, hydrogeological parameters, and all other necessary information can be given.

MODFLOW is a U.S. Geological Survey modular finite-difference flow model. This computer code can solve the groundwater flow equation. The governing partial differential equation solved numerically in MODFLOW is given in the following form:

\[
\frac{\partial}{\partial x} \left( K_x \frac{\partial h}{\partial x} \right) + \frac{\partial}{\partial y} \left( K_y \frac{\partial h}{\partial y} \right) + \frac{\partial}{\partial z} \left( K_z \frac{\partial h}{\partial z} \right) + W = S_{s} \frac{\partial h}{\partial t} \tag{1}
\]

Where \( K_x, K_y, \) and \( K_z \) are the values of the hydraulic conductivity along the x, y, and z coordinate axes (L/T), \( h \) is the hydraulic head (L), \( W \) is the volumetric flux per unit volume representing the sources and sinks of groundwater, for which the negative values denote extractions while the positive
values denote injections (T⁻¹), \( S_\phi \) is the specific storage of the investigated aquifer (L⁻¹), and \( t \) is time (T). This program is widely used throughout the world by hydrogeologists to simulate the flow of groundwater through aquifers. The code is free software, written in the FORTRAN language, and can be compiled and run on the DOS, Windows, or UNIX operating systems. Since its original development in the early 1980s, the USGS have released four major versions of this code, and is now considered to be the de facto standard industrial code among the groundwater specialists for aquifer simulation. Currently, there are many actively developed commercial and non-commercial graphical user interfaces for MODFLOW.

The MODFLOW-2000 version was released on July 20, 2000. Many new packages and enhancements were also released, including new solvers, and stream and saturated flow packages. The following packages of MODFLOW-2000 were used to describe the different source and sink terms during the above-mentioned simulation activity of the present studies: General-Head Boundary, Drain, Evapotranspiration, Reservoir, Lake, and Well and Recharge.

There are several graphical interfaces to MODFLOW, which often include the compiled MODFLOW code. These programs provide convenient means of supplying the input data for creating MODFLOW models. Commercial MODFLOW programs are typically used by governments and consultants for practical applications of MODFLOW to real-world groundwater problems. The applied PMWIN-Pro may be considered as a professional commercial version of MODFLOW.

A three-dimensional flow model considering three-layers was implemented with the help of the MODFLOW-2000 module in the present study. This model was used to characterize the main hydrostratigraphic units of the investigated area, namely the lower Pleistocene "waterworks" aquifer, the middle Pleistocene aquifer, and the unconfined upper aquifer that also contains a shallow groundwater aquifer. The input data required for the flow model was readily available from an earlier geological and hydrogeological prospecting activity.

Besides the flow model, a transport model was also built to investigate the several groundwater quality issues. The transport movement investigations were carried out in the field-study by the help of the MT3DMS model (Zheng and Wang 1999), where MT3D stands for the Modular 3-dimensional transport model, and MS denotes the multi-species structure for accommodating add-on reaction packages. MT3DMS has a comprehensive set of options and capabilities for simulating the advection, dispersion, diffusion, and chemical reactions of contaminants in groundwater flow systems under the general hydrogeologic conditions. The MT3DMS was developed for use with any finite-difference flow model such as MODFLOW, and is based on the assumption that changes in the concentration field will not affect the flow field appreciably.

The partial differential equation describing the fate and transport of contaminants of the species \( k \) in a three-dimensional space in transient groundwater flow systems can be written as follows:

\[
\frac{\partial (\phi C^k)}{\partial t} = \frac{\partial}{\partial x_i} \left( \rho D_{ij} \frac{\partial C^k}{\partial x_j} \right) - \frac{\partial}{\partial x_i} (\phi v_i C^k) + q_i C^k + \sum R_i
\]

Where \( \phi \) denotes porosity of the aquifer, dimensionless, \( (\text{fraction}) \), \( C^k \) is the dissolved concentration of species \( k \), \( (\text{M/L}^3) \), \( t \) is time \( (\text{T}) \), \( x_{ij} \) is distance along the respective Cartesian coordinate axis \( (\text{L}) \), \( D_{ij} \) is the hydrodynamic dispersion coefficient tensor \( (\text{L}^2/\text{T}) \), \( v_i \) is the seepage or linear pore water velocity based on the Darcy equation \( (\text{L/T}) \), \( q_i \) is the volumetric flow rate per unit volume of aquifer representing fluid sources and sinks \( (1/\text{T}) \), \( C_i^k \) is the concentration of the source or sink flux for species \( k \) \( (\text{M/L}^3) \), and \( \sum R_i \) is the chemical reaction term \( (\text{M/L}^3/\text{T}) \).

Based on the available geological and tracer material information, a tree-dimensional steady-state flow model was created. Of course, the time variable (up to 400 days) was involved into the transport modeling phase to describe the tracer concentration changes in the simulated formation. In the model, the thickness is 10 m, the length is 2000 m and the width is 1200. A finite difference grid was applied for simulations. The basic grid size is 25 m. A pair of wells (a doublet) was placed into the model symmetrically. A finer grid mash was applied around the wells to increase the numerical accuracy of the simulation calculations. The basic properties of this 10 meter thick limestone aquifer were given. The bottom elevation of the model is -2900 m, and the top elevation is -2890 m. Hydrostatic pressure (300 bar or 30 MPa) distribution was assumed. This means that the initial hydraulic head of the model is 100 m. The natural hydraulic gradient (l) was zero. No-flow boundary conditions were applied. The natural replenishment rate is zero. Concerning the water balance, the change of the stored water resource is zero, as the assumed production and injection wells have the same productions rates \( Q = 50 \text{l/s} \). This also means that only horizontal flow occurred in the investigated aquifer. Estimation was applied to derive the hydraulic conductivity and porosity values. The permeability was 1000 mDarcy, which corresponded 0.864 m/day hydraulic conductivity. The porosity
was 0.05. Based on these data, the hydrodynamic or flow model was built. Figs. 1-5 describe the most important information connected to the flow model.

Then, the MT3DMS module of the PMWIN Pro package was applied to simulate the transport processes of investigated fluorescent tracer in the function of time and space. Besides advection, the phenomena of hydrodynamic dispersion were also taken into account. The adsorption and the decay of the tracer were neglected in the investigated limestone environment. The length of the simulation was 400 days. Tracer concentration maps were drawn in different times. The tracer and dispersion properties were estimated from the special literature.

It was assumed that the tracer (1200 l) is pumped into the formation in the injection well for 15 minutes at the beginning of the simulations. When the tracer injection is finished, the tracer concentration was about 5 g/l or 5000 mg/l, or 5.000.000 µg/l in a 50 m³ fracture space.

Then, the tracer transport was simulated in space and time. Figs. 6-12 give the main results about the transport modeling concerning concentration distributions and breakthrough curves.

---

**Figure 1:** The hydraulic head distribution in the flow model (1 injection and 1 production well, Q=50 l/s).

**Figure 2:** The position of the path-lines from the injection well after 20-day-long simulation in the flow model (1 injection and 1 production well, Q=50 l/s).

**Figure 3:** The position of the path-lines from the injection well after 365-day-long simulation in the flow model (1 injection and 1 production well, Q=50 l/s).

**Figure 4:** The position of the injected tracer plume with a higher concentration than 10 µg/l in the transport model after 20 days.

**Figure 5:** The position of the injected tracer plume with a higher concentration than 10 µg/l in the transport model after 100 days.
5. SUMMARY

The flow between and around the two wells was investigated by two different methods. The streamlines were determined using an analytical method of hydrodynamical singularities. The path lines to follow the tracer motion were calculated by a numerical procedure based on finite differences. The same phenomenon was approached from two different aspects.

Streamlines and path lines are congruent in a steady flow. Thus, in spite of some different details of the applied mathematical tools considerable similarity can be recognized especially between the wells. Far from the wells, close to the arbitrarily taken boundaries of the reservoir, streamlines and path lines may have different patterns. Naturally there are some differences between two models in the used approximations, but the high degree of similarity indicates the adequacy of both models.

REFERENCES


