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COLD FRONT PROPAGATION IN A FRACTURED GEOTHERMAL RESERVOIR

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ABSTRACT

Hungary has decided to implement its first geothermal pilot power plant for electricity production. The site chosen is in South-West-Hungary and is positioned on a fractured limestone reservoir. The reservoir's depth is about 3000m and the temperature is 142^{o}C. Two existing petroleum-prospecting dry holes were chosen to implement a doublet for the geothermal energy supply with a separation of approximately 1000m. The projected flow rate of the circulating water is 5000 m^{3}/d. The lifespan of this production-injection system is one of critical concern to the industry. During a recent pre-feasibility study, an analytic model was developed to determine the propagation of the cooled region between the two wells. In this model, the reservoir is replaced by a plain equivalent fracture with a Hele-Shaw flow. The Hele-Show flow can be viewed as a quasi-potential motion. After applying complex variable functions, a theoretical flow pattern can be determined. The heat transfer between the water and the rock is calculated with a transient overall heat transfer coefficient. The cold front is not obtained as an abrupt temperature drop but, as an extended moving region with a very slight temperature gradient.

INTRODUCTION

The implementation of the first Hungarian geothermal pilot power plant occurred in 2004. After a comprehensive site investigation, a fractured limestone reservoir was selected in Southwestern Hungary, close to the Slovenian border. It is located at a depth of 3000m. The reservoir temperature is 142^{o}C. There are two unsuccessful petroleum prospecting boreholes with a good mechanical integrity. The distance between the wells is 1000m. After some work a doublet can be found to be a production and an injection well. A pre-feasibility study investigated the hydraulic and thermal behavior of the reservoir within production and injection. The most important questions of this study were:

- What kind of flow system will be developed between and around the wells?
- How will injection affect to the temperature distribution in the reservoir and the adjacent rock mass?
- How will the cooled region propagate from the injection well toward the production well?
- How will the produced water temperature decrease with the time?

And armed with the answers to these questions we can predict the lifetime of the system.

This problem has been investigated first by Bodvarsson (1975) and later Bodvarson, Preuss and O'Sullivan (1985) and Ghassemi and Tarason (2004).

THE CONCEPTUAL MODEL

The existing large, horizontal, fractured Triassic limestone reservoir is replaced by a single equivalent fracture bounded parallel plane walls. The primary reason for this simplification is to get a preliminary result without suitable or reliable input data. Hot water fills the fracture and its compression characteristics are considered. Thus mechanical and thermal processes can be treated separately. There is no overpressure in the reservoir. The pressure distribution is hydrostatic along the depth. Horizontal extension of the equivalent fracture is much greater than the distance between the two boreholes. The injection well occurs in the plane fracture as a source, the production well is a sink. The flow in the fracture is steady, laminar and two dimensional. This is the so-called Hele-Show flow. The bottomhole temperature in the injection well is constant. The injected cold water occurs in the fracture as an abrupt thermal inhomogeneity, thus, toward the fracture, a transient heat conduction is generated within the adjacent rock. Thus, the hot rock heats the injected cold water. The water temperature increases, as it flows toward the production well, while the rock temperature decreases. As the result of this heat transfer, the produced water temperature decreases. If the produced water temperature drops under a given limit the doublet will not operate efficiently and its operation terminated.

THE MATHEMATICAL MODEL

An orthogonal coordinate system is chosen. The xy plane is parallel to the fracture walls at halfway between them. Z is the transverse direction. 2b is the gap between the plates. The x-axes is fitted to the source and the sink. It is directed toward the sink. Because of the incompressibility of water, the flow and the heat transfer can be determined separately.

The governing equations of the Hele-show flow (Polubarinova-Kotschina, 1952) are

$$\frac{\partial p}{\partial x} = \rho\nu \frac{\partial^2 v_x}{\partial z^2}; \frac{\partial p}{\partial y} = \rho\nu \frac{\partial^2 v_y}{\partial z^2}; \frac{\partial p}{\partial x} = 0 \qquad (1)$$

It can be proven, that

$$\frac{\partial^2 p}{\partial x^2} + \frac{\partial^2 p}{\partial^2 y} = 0 \qquad (2)$$

Solving Eq. (1) using the no-slip boundary condition, we have

$$v_x = \frac{1}{2\rho\nu}\left(z^2 - b^2\right)\frac{\partial p}{\partial x};$$
$$v_y = \frac{1}{2\rho\nu}\left(z^2 - b^2\right)\frac{\partial p}{\partial y} \qquad (3)$$

Their integral mean between the planes

$$c_x = \frac{b^2}{3\nu\rho}\frac{\partial p}{\partial x}; \quad c_y = \frac{b^2}{3\nu\rho}\frac{\partial p}{\partial y} \qquad (4)$$

The pressure p is a harmonic function fulfilling Eq. (2) c_x and c_y can be derived from a scalar potential.

$$c_x = \frac{\partial \Phi}{\partial x}; \qquad c_y = \frac{\partial \Phi}{\partial y} \qquad (5)$$

where

$$\Phi = \frac{b^2 p}{3\rho\nu} \qquad (6)$$

Because of the fluid is incompressible

$$\frac{\partial c_x}{\partial x} + \frac{\partial c_y}{\partial y} = 0 \qquad (7)$$

which became an identity, if

$$c_x = \frac{\partial \Psi}{\partial y} \qquad (8)$$

Eqs. (5) and (8) are the Cauchy-Riemann equations

$$\frac{\partial \Phi}{\partial x} = \frac{\partial \Psi}{\partial y}; \qquad \frac{\partial \Phi}{\partial y} = -\frac{\partial \Psi}{\partial x} \qquad (9)$$

Fulfillment of Eq. (9) is equivalent with the existence of an analytic complex variable function $W(\xi)$, the so-called complex potential

$$W(\xi) = \Phi(x, y) + i \cdot \Psi(x, y) \qquad (10)$$

of which real part is the velocity potential Φ and the imaginary part is the stream function Ψ. The $\Psi = \text{const}$ curves are the streamlines.

The complex potential of the Hele-Show flow between the source and the sink can be written applying the method of hydrodynamic singularities:

$$W = \frac{Q}{2\pi}\ln\frac{\xi + a}{\xi - a} \qquad (11)$$

In Fig.1. is shown the two singularities at the point $x = -a$ the source of Q and at the point $x = a$ the sink of capacity of -Q. Using the exponential form of ξ the real and imaginary parts of W can be separated easily.

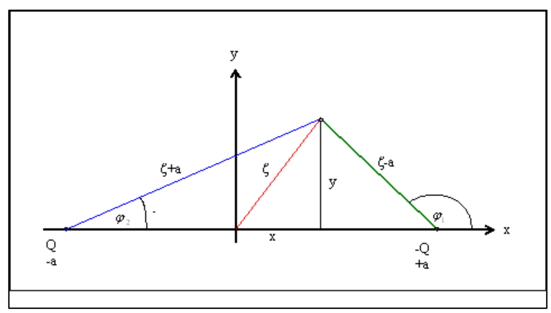

Figure. 1. The two singularities.

$$W = \Phi + i \cdot \Psi = \frac{Q}{2\pi}\ln\frac{r_1}{r_2} + i \cdot \frac{Q}{2\pi}\left(\varphi_1 - \varphi_2\right) \qquad (12)$$

Thus the equation of the streamlines is

$$\varphi_1 - \varphi_2 = \frac{2\pi k}{Q} = \text{const} \qquad (13)$$

Considering Fig.1, we see that

$$\operatorname{tg}\left(\varphi_1 - \varphi_2\right) = \frac{\frac{y}{x-a} - \frac{y}{x+a}}{1 + \frac{y}{x-a} - \frac{y}{x+a}} = C \qquad (14)$$

After some manipulation we obtain

$$x^2+\left(y-\frac{a}{C}\right)^2=a^2\left(1+\frac{1}{C^2}\right) \qquad (15)$$

The shape of the streamlines are a family of circles between the source and the sink, with centers at $x=0$ and $y=\frac{a}{C}$. It's shown in Fig.2.

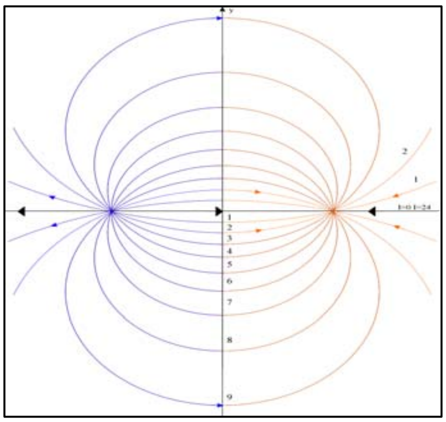

Figure. 2. Sub-Dividing intoPart-Channels .

In the case of $C=0$ the circle becomes a straight line: the x axis. If $C=\infty$, $tg\frac{2\pi}{Q}k=\infty$, $k=\frac{Q}{4}$ and the radius of the circle is a. Thus, half of the flow rate runs inside an origin-centered circle of radius a.

The plane of the flow can be splinted decomposed into part-channels in each the flow rate are the same. In these part-channels the flow is one-dimensional in a curve-linear coordinate system. Thus the two-dimensional plane flow is replaced by a finite set of one-dimensional flows. In this way common differential equation are obtained along the streamlines, while finite differences are obtained perpendicular to them. The heat transfer in the fracture can be solved by this complex method, simultaneously using finite differences and common differential equations.

HEAT TRANSFER IN THE FRACTURE

At the beginning of the injection the fracture is filled hot water. Its temperature is the same of the natural geothermal temperature at the given depth. As the injected water flows along the streamlines, it will warm up, while the rock temperature decreases. The whole heat transfer process can be separate into two sub-processes: advection in the water and transient one-dimensional heat conduction toward the fracture in the rock mass. Since the injected water mass is much smaller than the rock, the slow transient heat transfer is followed by the fluid instantaneously. The internal energy balance for an infinitesimal volume element is written, as it is shown in Fig.3.

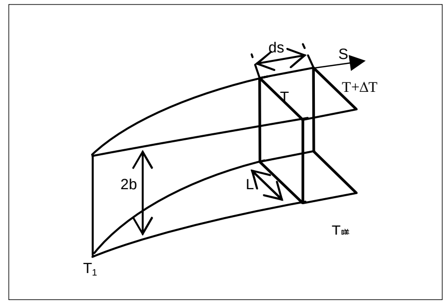

Figure. 3. Infinitesimal volume element schema.

$$\dot{m}c[(T+dT)-T]=2UL(T_\infty-T)ds \qquad (16)$$

where $\dot{m}$ is the injected mass flow rate, c is the heat capacity, U is the transient overall heat transfer coefficient, T_∞ is the initial geothermal rock temperature, L is the width of the part-channel. Solving Eq. (16) we can get

$$T=T_\infty-(T_\infty-T_1)\cdot e^{\frac{2ULs}{\dot{m}c}} \qquad (17)$$

where T_1 is the temperature of the injected water at the bottom hole, s is the actual length along the streamline in question.

The transient overall heat transfer coefficient can be calculated as

$$\frac{1}{U}=\frac{1}{h}+\sqrt{\frac{\pi\cdot t}{(\rho ck)_r}} \qquad (18)$$

In which ρ is the density, c is the heat capacity and k is the heat conductivity of the rock. In point of view of computation, the very favorable condition exists, in that, the Nusselt number is constant for the heat transfer on a flat plate between the solid and the laminar flow. The experimentally determined value is $Nu=5{,}12$ (Lundberg, Mc Cuen, Reynolds, 1963). Thus the heat transfer coefficient is obtained as

$$h=\frac{5{,}12\cdot k_w}{2b} \qquad (19)$$

Where k_w is the waters heat conductivity. Consequently h is independent of the changing velocity along the streamlines, heat transfer coefficient can be determined without the knowledge of the velocities.

Knowing the value U, water temperature can be calculated in any part-channel at the function of the length and time. Note, that Eq. (19) is valid only for that region of the fracture which is filled the injected water yet.

Propagation of the cooled region along the streamlines lags behind compared to the motion of the injected fluid. This is an important difference in comparison to the oil displacement by water. The boundary surface between the water and the oil phase moves together to the flowing fluids, and the material properties suffer an abrupt jump on this strong material singular surface. In the geothermal reservoir the injected water temperature increases gradually there is no a sharp contour of the cooled region.

In the bottomhole of the production well the cooled region will arrive first along the straight streamline between the two wells. All other part-channels carry hot water of undisturbed reservoir temperature still. The parallel connected part-channels carry water of different temperatures. The bottom of the production side of the well have a result of calorimetric temperature. This homogenous temperature is shown in Fig. 4. depending on time. These temperatures characterize the sustainability of the system.

Increase of the mass flow rate results in a temperature drop of the injected water and in effect, a smaller temperatures decrease. Choosing the best and most economic temperature limit of this produced water enables one to estimate the lifetime of the doublet.

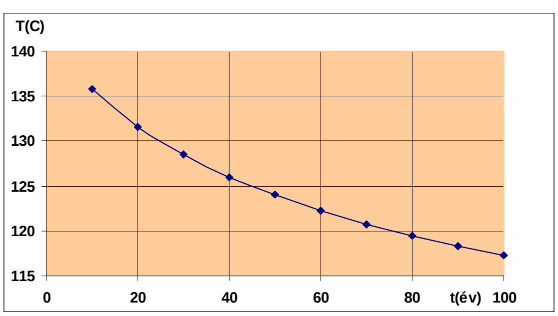

Figure. 4. Temperature is depending on time.

SUMMARY

This paper is a part of a pre-feasibility study, made for the implementation of the first Hungarian geothermal pilot power plant. The site chosen is a fractured limestone reservoir in the Southeastern part of Hungary. A doublet was planned to supply geothermal energy for the plant. For this preliminary investigation the fracture system is replaced by an equivalent fracture in which a Hele-Show flow is observed. The flow pattern and the streamline system is treated by methods of hydrodynamic singularities. The heat transfer in the fracture is advection and conduction in the adjacent rock. The cooled region propagates from the injection well towards the production well and it lags behind the motion of the fluid. The decrease in the produced water temperature is not a sudden drop, but it suffers from a gradual change. This is determined as the function of time. If we use appropriate heat temperature limits, an accurate, effective operational lifetime of the doublet can be determined as outlined in this work.

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