INJECTION OF SALTY WATER INTO HOT ROCK, 
SATURATED WITH SUPERHEATED VAPOUR  

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ABSTRACT  
We consider salty water injection into depleted geothermal reservoir saturated with superheated vapour. If the water temperature and pressure reach a boiling point then liquid evaporates and boiling front migrates away from the injection well. Water vaporization increases salt concentration in the liquid zone and excess salt may precipitate at the vaporization front. Behind the vaporization front salt in solid state occupies some part of the volume of the porous space and an extended domain arises in which solid salt coexists with the saturated solution. As initial salt concentration of the salty water is less than the solubility value, then the dissolution front develops between the vaporization front and injection well. The dissolution front also migrates in the same direction as the vaporization front. A family of self-similar solutions that describes salty water boiling, salt precipitation and dissolution of solid salt was derived. It was found that this family of self-similar solutions is multi-valued. At some critical values of parameters these solutions converge. Beyond the critical point, there is no similarity solution and this behavior of the solutions means that the rock becomes fully sealed through precipitation. 

INTRODUCTION  
There are many different situations in which salty water moves through a superheated permeable rock. Important examples arise in geothermal reservoirs. Liquid may migrates along fractures or permeable strata due to cold water injection into depleted geothermal reservoir or due to hot water motion from a high pressure domain towards a law pressure domain saturated with superheated vapour. The vaporization increases salt concentration in the liquid zone and excess salt may precipitate. In this paper we use similarity solution to analyze the precipitation process. There are a lot of publications in which an analytical method was applied to the problem of injection of pure cold water (Bodvarsson, 1972; O'Sullivan & Pruess, 1980; Woods & Fitzgerald, 1993, 1997; Woods, 1999; Barmin & Tsypkin, 1996 etc.). If water in geothermal reservoir contents salt in dissolved form then the system acquires some new features. Battistelli, Calore & Pruess (1995, 1997) used the reservoir simulator TOUGH2 to study the effect of permeability reduction due to salinity in the depletion of tight matrix blocks. Tsypkin & Woods (2005) investigated the vapour production from a hot rock saturated with salty water. The family of self-similar solutions that describes water boiling and salt precipitation was obtained. It was found that this family of self-similar solutions is multi-valued. At some critical values of parameters these solutions converge. Beyond the critical point, there is no similarity solution and this behavior of the solutions means that the rock becomes fully sealed through precipitation. 

PROBLEM FORMULATION  
Consider salty water injection into a hot permeable rock saturated with superheated vapour, assuming for simplicity that initial temperature of injected water is equal to the initial reservoir temperature and ignore change in temperature due to phase transition. When water is injected into hot rock, water saturated region develops adjacent to the injection well. The brine and vapour migrate away from the well. 

As fluid contents the salt in the dissolved form and vaporizes at the boiling front, then the salt concentration of the fluid behind the front increases. If the salt concentration increases beyond the saturation limit $c=c_{sat}(T_{b})$, then the vaporization leads to a precipitate formation behind the boiling front.
We assume that behind the front the solid salt and saturated solution coexist at the thermodynamic equilibrium state. As the salt concentration the injected brine is less than the saturation value \( c_0 < c_{sat} \), then the second dissolution front forms between the well-bore and the boiling-precipitation interface. The pure liquid zone occupies region between the injection well and the dissolution front. Respectively, the zone of the heterogeneous mixture solid salt – saturated solution occupies region between the moving fronts (Fig. 1).

![Diagram showing the configuration of two moving fronts: vaporization - precipitation front and dissolution front.](image)

*Figure. 1. Sketch of the problem showing typical configuration of two moving fronts: vaporization-precipitation front and dissolution front.*

In the liquid zones 2 and 3 the pressure evolves according to the Darcy’s law and the equations for mass conservation. The systems of the equations can be expressed as equations for the pressure

\[
\frac{\partial P}{\partial t} = \kappa_{2,3} \Delta P - \frac{k_{2,3}}{\phi \mu_v} (\nabla P)^2 + \frac{k_{2,3}}{\phi \mu_v} P \Delta P
\]

where \( \kappa \) is the permeability, \( \phi \) the porosity, \( \mu \) the viscosity, \( \alpha \) the water compressibility coefficient. The subscripts 2,3 refer to the region 2 and region 3 respectively.

In the vapour region from the conservation of mass, Darcy’s law and the equation of state we also obtain equation for the pressure

\[
\frac{\partial P}{\partial t} = \frac{k_{2,3}}{\phi \mu_v} (\nabla P)^2 - \frac{k_{2,3}}{\phi \mu_v} P \Delta P
\]

The subscript \( v \) refers to the property of vapour. Notice that we ignore salt diffusion as the diffusive flux for a high permeability rock is much less than the advective flux of salt (Tsypkin & Woods, 2005). Therefore in the salty water region 3 the salt concentration is equal to the initial salt concentration \( c = c_0 \). In the middle region 2 salt concentration value coincides with the solubility \( c = c_{sat} \), where

\[
c_{sat}(T_0) = 0.3486 - 0.2527 \frac{T_0}{373.15} + 0.1847 \left( \frac{T_0}{373.15} \right)^2
\]

In this region salt in a solid state occupies some part of the volume of the porous space, which we denote as \( S_{pr} \). The solid salt reduces the permeability of the rock and to describe this change we use a parametric relation (Tsypkin & Woods, 2005)

\[
k_2 = k_0 \frac{1 - \exp(\theta \phi (1 - S_{pr}))}{1 - \exp(\theta \phi)} \equiv k_0 K(S_{pr})
\]

where \( \theta \) is a coefficient which accounts for different changes in the permeability as a result of precipitation and \( k_0 \) is the initial permeability of the rock.

The boundary conditions across the boiling front may be found using the conservation of mass for \( H_2O \) the salt including the reduction in the porosity and permeability of the middle region 2 due to the formation of a salt deposit with density \( \rho_{salt} \). Meanwhile the permeability in the region is the known function \( S_{pr} \).

The boundary conditions for the mass of \( H_2O \) salt have the forms

\[
\phi \left( 1 - \frac{\rho_v}{\rho_w} \right)_n = \frac{k_0}{\mu_v} \frac{\rho_v}{\rho_w} \left( \nabla P \right)_{n1+} - \frac{k_0 K(S_{pr})}{\mu_2} \left( \nabla P \right)_{n1-} \left( 1 - S_{pr} \right) \phi \rho_{salt} S_{pr} \nabla V_{n1} = - \rho_v c_0 \frac{k_0 K(S_{pr})}{\mu_2} \left( \nabla P \right)_{n1-}
\]

Analogously the boundary conditions at the dissolution front have the form

\[
\left( \frac{\rho_w - 1 + S_{pr}}{\rho_w} \right)_n = \frac{k_0}{\mu_3} \frac{\rho_w}{\phi \mu_v} \left( \nabla P \right)_{n2-} - \frac{k_0}{\rho_w} \frac{k_0}{\phi \mu_3} \left( \nabla P \right)_{n2-} \left( \frac{\rho_{salt}}{c_{sat}} S_{pr} + 1 - S_{pr} - \frac{c_0 \rho_{v}}{c_{sat}} \rho_{sol2} \right) \nabla V_{n2} = \frac{c_0 \rho_{v}}{c_{sat}} \rho_{sol2} \frac{k_0}{\phi \mu_3} \left( \nabla P \right)_{n2-} - \frac{k_0 K(S_{pr})}{\phi \mu_2} \left( \nabla P \right)_{n2-}
\]

Here \( \rho_{salt} \) is the density of solid salt, \( \rho_{w,2,3} \) the effective density of \( H_2O \) (mass of \( H_2O \) per unit volume of the brine), \( \rho_{sol} \) is the brine density, which satisfies the relation \( \rho_{sol} (1-c) = \rho_w \), \( V_{n1} \) and \( V_{n2} \) are the normal components of the vaporization and dissolution fronts respectively.

\( c_0 \) is the salt concentration at the boiling – precipitation front.

**SIMILARITY SOLUTION OF THE PROBLEM**

Consider a similarity solution of one-dimensional problem

\[
P = P(\xi), \xi = x/t^{1/2}, V_{1,2} = dX_{1,2}(t)/dx, V_{1,2} = \beta_{1,2} t^{1/2}
\]
when initial pressure \( P_0 \) and injection pressure \( P^0 \) are constants

\[ x=0: \quad P= P^0, \quad t=0 \quad P= P_0. \]

As we ignore the salt diffusivity in the third region the similarity solution for salt concentration has the trivial form \( c=c_0 \) and \( c_s=c_0 \).

In the vapour-saturated domain we use a linear approach, assuming that the initial and boiling pressure have the same order. Then in the vapour domain 1 from the diffusion equation we obtain the pressure distribution

\[
P(\xi) = \left(P_* - P_0\right) \frac{\text{erfc}(\xi / 2 \sqrt{\kappa_2}) + P_0}{\text{erfc}(\beta_1 / 2 \sqrt{\kappa_1})}.
\]

The boiling front migrates slowly with respect to the pressure redistribution in both liquid zones and the pressure distribution can be derived within the quasi-stationary approximation (Tsypkin & Calore, 2007).

As a result a simple equation for the pressure is valid in the regions 2 and 3.

\[
\Delta P = 0
\]

Thus, for the similarity solution we obtain linear distributions for the pressure in the liquid zones

\[
\beta_2 < \xi < \beta_1:\quad P(\xi) = \frac{P_* - P_0}{\beta_2 - \beta_1} \xi + \frac{\beta_2 P_1^* - \beta_1 P_2^*}{\beta_2 - \beta_1},
\]

\[
0 < \xi < \beta_2: \quad P(\xi) = P^0 + \left(P_*^0 - P^0\right) \frac{\xi}{\beta_2}
\]

Here \( P_1^* \) is the boiling pressure, which can be found as a function of the reservoir temperature from the Clausius-Clapeyron relation

\[
\ln \frac{P_*}{P_0} = A + \frac{B}{T_0},
\]

\[
A = 12.512, \quad B = -4611.73 \quad K, \quad P_a = 10^5 Pa
\]

while \( P_2^* \) is the unknown parameter of the problem.

Introducing dimensionless velocities \( \gamma_i, \) where

\[
\beta_{1,2} = 2 \sqrt{\kappa_2 \gamma_{1,2}} \quad \text{где} \quad \kappa_1 = \frac{k_1 P_0}{\phi_1}, \quad \kappa_2 = \frac{k_2 P_0}{\phi_2},
\]

and substituting the obtained solutions for the pressure, we obtain four transcendental equations for the quantities \( \gamma_1, \gamma_2, P_2^* \) and \( S_{pr} \)

\[
\left[1 + S_{pr} \frac{P_{sat}}{\rho_{sat2} \rho_s} - 1\right] \gamma_1 + \frac{1}{2} \left(P_*^0 - P_*^0\right) \frac{K(S_{pr})}{\gamma_2 - \gamma_1} = 0,
\]

\[
\frac{\rho_{sat2} - 1 + S_{pr}}{\rho_{sat2}^2} \gamma_2 + \frac{1}{2} \frac{P_{sat2}^0 \left(P_*^0 - P_*^0\right)}{\gamma_2 - \gamma_1} - \frac{1}{2} \frac{P_*^0 - P_*^0}{\gamma_2 - \gamma_1} \frac{K(S_{pr})}{\gamma_2 - \gamma_1} = 0,
\]

\[
\left\{1 + S_{pr} \frac{P_{sat}}{\rho_{sat2} \rho_s} - 1\right\} \gamma_1 + \frac{1}{2} \left(P_*^0 - P_*^0\right) \frac{K(S_{pr})}{\gamma_2 - \gamma_1} = 0.
\]

The system of was solved numerically for typical values of the parameters.

**CALCULATION RESULTS**

Figure 2 shows typical variation of the pressure and solid salt saturation with distance from the injection well. Between the boiling– precipitation front and the dissolution front the layer of brine at the saturation state and salt precipitate is formed.

In the case of a sea water (\( c_0 = 0.03 \)) injection a very thin intermediate zone arises (figure 3). The length of the region increases with the initial salt concentration.

If the initial salt concentration tends to the saturation value \( c_0 \rightarrow c_{sat} \) the velocity of the dissolution front tends to zero \( \gamma \rightarrow 0 \) and dimension of the intermediate region reaches the maximal value.
Figure 3. Distribution of the pressure and solid salt saturation as a function of dimensionless coordinate. $\phi=0.1$, $T_0=580$ K, $P^0 = 16$ MPa, $c_0= 0.03$

Figure 4 illustrates the rates of propagation of the boiling and dissolution fronts as a function of the injection pressure. There are two similarity solutions for a large injection pressure. As the injection pressure decreases these two solutions come together and then coincide at the critical value of the pressure.

Beyond the threshold there is no similarity solution as the rock becomes fully sealed through precipitation. This situation is analogous to the case of vapour extraction from a geothermal reservoir saturated with salty water (Tsypkin & Woods, 2005). In conjunction with figure 4, in figure 5 the variation of the solid salt saturation as a function of the injection pressure is shown. We identify that similarity solution ceases to exist at enough small value of the solid salt saturation. As in previous figure the solid line represents solution, which may arise in practice.

Figure 5. Variation of volume of the solid salt as a function of the well pressure. $\phi=0.1$, $T_0=580$ K, $P_0=2\cdot10^6$ MPa.

Figures 6a, 6b summarize the results of calculation as a regime diagrams, showing the domains of existing of different regime of salty water injection into geothermal reservoir.

Figure 6a illustrates the impact of the reservoir pressure on the critical injection pressure. Three regimes of salty water injection may realize. If the reservoir pressure and injection pressure are small (region III) then intensive boiling process leads to the arising of considerable amount of salt in a solid state that is enough for the reservoir sealing.

Beyond the threshold there is no similarity solution as the rock becomes fully sealed through precipitation. This situation is analogous to the case of vapour extraction from a geothermal reservoir saturated with salty water (Tsypkin & Woods, 2005). In conjunction with figure 4, in figure 5 the variation of the solid salt saturation as a function of the injection pressure is shown. We identify that similarity solution ceases to exist at enough small value of the solid salt saturation. As in previous
substantial mass of precipitate accumulates at the vaporization front. As a result solid salt occupies the porous space and solution, which describes a fluid motion through the rock, ceases to exit.

\[ P_0 = 2 \text{ MPa.} \]

**Figure. 6b. Critical diagram.**

**SUMMERY**

In this work we have considered salt precipitation process as a result of the percolation of salty water into hot rock. Our analysis has based on isothermal approximation because the pressure plays the role of driving force for the process. Thus, we suppose that the model based on the approximation allows us to detect the main features of the precipitate formation in geothermal reservoirs due to the evaporation of saline water.

We have derived a class of similarity solutions for the injection problem and have found that the solution is multi-valued. Our analysis has established that for the large reservoir temperature or low injection pressure the solution reaches a threshold and above the critical values of the parameters the solution ceases to exist that corresponds to the regime when the rock becomes sealed with solid salt.

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**REFERENCES**


