PROCEEDINGS, Thirty-Third Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 28-30, 2008
SGP-TR-185

LIMITS OF HEAT EXTRACTION FROM DRY HOLE

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ABSTRACT

Deep borehole heat exchangers are an environmentally favorable way for geothermal energy production. The weakness of this proposed production technology is the moderate temperature of the outflowing water and the relatively low thermal power. The impact of the bottom-hole depth and the heat conductivity of the surrounding rock are also considered.
The primary limit of the extractable thermal power is the restricted heat replenishment by conduction towards the well. In order to the temperature difference is negligible, between the tubing and the annulus heat insulation is necessary. During the initial period of the operation the temperature and the power has a higher value, but they tend a lower equilibrium value. This is the sustainable production of the system. Semi-numerical simulation is used to determine this limit. Electricity production from dry holes cannot be an economic way.
But the direct heat utilization would be possible in confirmation with a heat pump.

INTRODUCTION

The present practice of geothermal energy production in Hungary is hot water production mainly from the so-called Upper Pannonian sedimentary aquifer. Most geothermal wells operate by the elastic expansion of this system without any artificial lift method. The mineralized hot water represents a major problem for geothermal energy utilizers. An obvious solution is to reinject the utilized thermal water into the aquifer where it is originated, which likewise allows the reservoir pressure to be maintained, assures the continuous supply of water and limits the surface subsidence. Furthermore, water reinjected at a lower temperature than the aquifer's allows higher recovery of the energy contained in the reservoir rocks by cooling the aquifer. In spite of these benefits, an adverse phenomenon occurs: the increasing reinjection pressure because of the decreasing permeability of the aquifer around the reinjection well.

To avoid this problem some useful ideas were suggested by HORNE (1980), ARMSTEAD (1983), MORITA, et. al. (1985, 2005). Their recommendations to circulate water is in a closed casing well. The water flows downward through the annulus between the casing and the tubing while it warms up, and it returns at the bottomhole and flows upward through the tubing. The upward flowing water cools to a certain extent because of the heat transfer across the tubing wall.

Such an experimental production unit has been installed in 1989 in Szolnok, central Hungary. The results, as expected, were rather modest because the insufficient heat transfer area around the well, and the low heat conductivity of the surrounding rocks. The circumstances were analyzed by BOBOK et. al. (1991) and BOBOK and TÓTH (2002). In the following we shall introduce a more sophisticated mathematical model to describe the heat transfer mechanics of such a system, to predict its thermal behavior in order to avoid further inefficient and expensive experiments, and to show the range of the dry hole geothermal utilization. Our attention is focused to the annular heat transfer phenomenon.

THE MATHEMATICAL MODEL

The simplified model of a closed geothermal well is shown in the following. The casing is closed at the bottom without any perforations. The water flows downward through the annulus between the coaxial casing and tubing. Since the adjacent rock is warmer than the circulating water, the water temperature increases in the direction of the flow. An axisymmetric thermal inhomogeneity is developed around the well together with radial heat conduction toward the well. This is the heat supply of the system. The warmed up water flows upward through the tubing while its temperature slightly decreases, depending mainly on the heat conduction coefficient of the tubing. The system is analogous to a countercurrent heat exchanger. The main difference is the increasing adjacent rock temperature distribution with the depth. Thus the familiar methods for design of heat exchangers are not sufficient for this case.

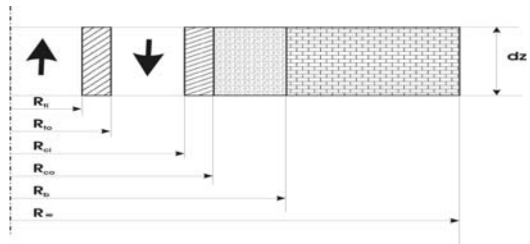

Figure. 1. Schematic drawing of control volume.

Let's consider the schematic drawing of the system in Figure 1. The geometric parameters are defined as shown in the figure. It is convenient to separate the system into two subsystems. One of them is the flowing fluid, in which the convective heat transfer is dominant. The other is the adjacent rock mass around the well, with a radial conductive heat flux. Thus the internal energy balance can be written for the two subsystems in a simplified form. Cylindrical coordinates are chosen. The radial coordinate r is measured from the axis of the coaxial cylinders, while z lies along the axis directed downward. The steady, axisymmetric turbulent flow is taken to be uniform at a cross-section, the velocity v and temperature T are cross-sectional average values. The t, c, and a indices refer to the tubing the casing and the annulus. Thus the balance equation of the internal energy for the flow across the tubing is:

$$R_{ti}^2\pi\rho c v_t dT_t = 2R_{ti}\pi U_{ti}\left(T_t - T_a\right)dz \qquad (1)$$

in which ρ is the density, c is the heat capacity of the fluid, U_{ti} is the overall heat transfer coefficient referring the inner radius of the tubing. For the annular flow we get:

$$\left(R_{ci}^2 - R_{t0}^2\right)\pi\rho c v_a dT_a = 2R_{ci}\pi U_{ci}\left(T_b - T_a\right)dz + \\ + 2R_{ti}\pi U_{ti}\left(T_t - T_a\right)dz \qquad (2)$$

where T_b is the temperature at the borehole radius R_b, U_{ci} is the overall heat transfer coefficient referring to the radius R_{ci}.

The unsteady axisymmetric heat flux around the well is equal to the heat flux through the casing. It can be expressed as:

$$\dot{Q} = 2\pi k_R \frac{T_\infty - T_b}{f(t)} = 2\pi R_{ci} U_{ci}\left(T_0 - T_a\right) \qquad (3)$$

in which $\dot{Q}$ is the heat flux over the unit length cylinder, k_R the heat conductivity of the rock. The undisturbed natural rock temperature is T_∞, its distribution linear with depth

$$T_\infty = T_s + \gamma z \qquad (4)$$

where T_s the annual mean temperature at the surface γ is the geothermal gradient. The parameter f(t) is the transient heat conduction time function (RAMEY, 1962).

SOLUTION

Text To solve the differential equation system it is necessary to know the overall heat transfer coefficients U_{ti} and U_{ai}. The determination of U_{1B} and U_{ci} needs the knowledge of the heat transfer coefficients h_{to} and h_{ci} referring to the inner and outer surface of the annulus.

The downflowing fluid in the annulus of the closed-loop geothermal system is heated from two directions independently: across the tubing and across the casing. These heat fluxes can be varied independently. It is obvious, that two independent heat transfer coefficients and two different Nusselt numbers are obtained on the inner and the outer wall of the annulus. To determine this heat transfer mechanism is more difficult than in a simple circular tube.

LUNDBERG, et. al. (1963) have shown that (1)
it is possible to reduce the problem to four fundamental solutions in accordance to the different boundary conditions. These can be combined using superposition techniques to yield a solution for any desired boundary conditions. The present case can be interpreted as the superposition of two fundamental solutions. One of them is perfectly insulated. Another particular solution is obtained interchanging the two surfaces. (2) The two particular solutions can be superimposed providing solution for the two-sided heating.

Following the familiar semi-analytical treatment, we will employ the subscript t_1 to designate conditions on the tubing surface when this surface alone is heated. The subscript c_1 designates conditions on the casing surface when this surface alone is heated. The opposite surface in either case is insulated. The single subscript t or c refers to the conditions on the tubing or casing surfaces (3) respectively under any conditions of simultaneous heating at both surfaces.

For the case of constant heat rate per unit tube length it is possible to express Nu_T and Nu_c for any heat flux ratio on the two surfaces in terms of Nu_{t1} and Nu_{c1}. Similarly can be derived two influence coefficients θ_t^* and θ_c^*. Finally the following expressions are obtained:

$$Nu_t = \frac{Nu_{t1}}{1-\frac{q_{c1}}{q_{t1}}\theta_t^*} \qquad (5)$$

$$Nu_c = \frac{Nu_{c1}}{1-\frac{q_{t1}}{q_{c1}}\theta_c^*} \qquad (6)$$

KAYS and LEUNG (1963) carried out experiments in annuli using air for various values of radius ratio with constant heat rate per unit tube length but with various heat flux ratios at the inner and outer surfaces including the two limited cases of only one side is heated. They then obtained a solution in tabulated form for constant heat rate under fully developed turbulent flow based upon empirical data. The results are presented in the form Nu_{t1} and Nu_{c1} and the two influence coefficients θ_t^* and θ_c^*. These are given in tables for a wide range of Reynolds and Prandtl numbers and for radius ratios. These results are then directly applicable to Eqs. (5) and (6). Data is widened by some of our experimental results obtaining with water for 4 ½" and 7" tubing and casing diameters. The heat flux q_{c1} can be obtained from the temperature distribution of an injection well. As it is known (Ramey, 1962) the bottomhole temperature of the injected water is

$$T_{bh} = T_s + \gamma(H-A) + (T_i - T_s + \gamma A)\cdot e^{\frac{-H}{A}} \qquad (7)$$

The overall heat flow into the downflowing water is

$$\dot{Q} = \dot{m}c(T_{bh} - T_i) \qquad (8)$$

Thus the integral main of the heat flux per unit length of casing

$$q_{c1} = \frac{\dot{m}c(T_{bh} - T_i)}{2R_{ci}\pi H} \qquad (9)$$

In the second case the casing is perfectly insulated, the annular flow is heated across the tubing only. The mass flow rates in the tubing and in the annulus are the same. Equations (1) and (2) lead to the relation

$$\frac{dT_t}{dz} = \frac{dT_a}{dz} \qquad (10)$$

Derivating Eq.(1) by z, we obtain

$$\dot{m}c\frac{d^2T_t}{dz^2} = 2\pi R_{ti}U_{ti}\left(\frac{dT_t}{dz} - \frac{dT_a}{dz}\right) \qquad (11)$$

comparing Eqs. (1) and (2) it is obtained

(6)

$$\frac{d^2T_t}{dz^2} = 0 \text{ and } \frac{d^2T_a}{dz^2} = 0 \qquad (12a,b)$$

Thus the general solutions of Eqs. (12a and 12b)

$$T_t = K_1z + K_2 \qquad (13)$$

$$T_a = K_1z + K_3 \qquad (14)$$

The boundary conditions are the following:

if $z = 0$ $T_a = T_i$,

if $z = H$ $T_t = T_a$ and

$K_1 = 2\pi R_{ti}U_{ti}(T_t - T_a)$

Solving the obtained equation system, finally the heat flux across the tubing wall

$$q_{T1} = \frac{\dot{m}c(T_{bh} - T_i)}{B + H} \qquad (15)$$

The expressions (9) and (15) can be substituted can be substituted into (5) and (6). Based on experimental data the particular Nusselt numbers can be calculated by the following formulas:

$$Nu_{t1} = 0{,}016\cdot Re^{0,8}\, Pr^{0,5}$$
$$Nu_{c1} = 0{,}018\cdot Re^{0,8}\cdot Pr^{0,5} \qquad (16)$$

$$\theta_T^* = 0{,}410\cdot Re^{-0,078}\cdot Pr^{-0,58}$$

$$\theta_T^* = 0{,}325\cdot Re^{-0,078}\cdot Pr^{-0,58} \qquad (17)$$

Determining the Nusselt numbers on both surfaces the heat transfer coefficients on the walls of the annulus are

$$h_t = \frac{k \cdot Nu_t}{2R_{t0}} \quad \text{and} \quad h_c = \frac{kNu_c}{2R_{ci}} \quad (18)$$

Knowing the h_{t0} and h_{ci} values, the overall heat transfer coefficients U_{ti} and U_{ci} can be determined as

$$\frac{1}{U_{ti}} = \frac{1}{h_{ti}} + \frac{R_{ti}}{k_{ins}} \cdot \ln\frac{R_{t0}}{R_{ti}} + \frac{R_{ti}}{R_{t0}h_{t0}} \quad (19)$$

and

$$\frac{1}{U_{ci}} = \frac{1}{h_{ci}} + \frac{R_{ci}}{k_s} \cdot \ln\frac{R_{c0}}{R_{ci}} + \frac{R_{ci}}{k_c}\ln\frac{R_b}{R_{co}} \quad (20)$$

Combining the equations (1), (2) and (3) we obtain two simple differential equations are obtained:

$$A\frac{d(T_t - T_a)}{dz} = T_\infty - T_a \quad (21)$$

in which

$$A = \frac{\dot{m} \cdot c \cdot (k_r + R_{ci}U_{ci}f)}{2\pi R_{ci}U_{ci}k_r} \quad (22)$$

and

$$B\frac{dT_t}{dz} = T_t - T_a \quad (23)$$

where

$$B = \frac{\dot{m} \cdot c}{2\pi R_{ti}U_{ti}} \quad (24)$$

Combining the equations (21) and (23), a second-order inhomogeneous differential equation is obtained.

$$AB\frac{d^2T_a}{dz^2} + B\frac{dT_a}{dz} - T_a + T_s + \gamma(z - B) = 0 \quad (25)$$

In a similar way we can obtain for the flow through the tubing:

$$AB\frac{d^2T_t}{dz^2} + B\frac{dT_t}{dz} - T_t + T_s - \gamma z = 0 \quad (26)$$

These equations can be solved easily in the form

$$T_t = T_s + \gamma(z + B) + K_1e^{x_1 z} + K_2e^{x_2 z} \quad (27)$$

and

$$T_a = T_s + \gamma z - T_i + C_1e^{x_1 z} + C_2e^{x_2 z} \quad (28)$$

where x_1 and x_2 are the roots of the characteristic equations of (38) and (39), i.e.

$$x_1 = -\frac{1}{2A}\left(1 - \sqrt{1 + \frac{4A}{B}}\right) \quad \text{and}$$

$$x_2 = -\frac{1}{2A}\left(1 + \sqrt{1 + \frac{4A}{B}}\right) \quad (29)$$

The constants of integration in (40) and (41) can be determined satisfying the following boundary conditions

1. At z =0, $T_a = T_i$, where T_i is the temperature of the cooled injected water
2. At z = H, $T_a = T_t$, the bottomhole temperatures in the annulus and in the tubing are the same.
3. At z = H, $\frac{dT_t}{dz} = 0$, the depth derivative of the tubing temperature at the bottomhole is zero. It is the consequence of the equation (23).
4. The energy increase of the circulating fluid is equal to the integral of the heat flux across the borehole wall between the bottom and the surface:

$$\dot{m}c(T_{out} - T_i) = \int_0^H q(z)dz \quad (30)$$

where T_{out} is the temperature of the outflowing water at the wellhead.

The obtained equations from the boundary conditions are

$$T_i - T_s = C_1 + C_2 \quad (31)$$

$$K_1e^{x_1 H} + K_2e^{x_2 H} + \gamma B = c_1e^{x_, H} + c_2e^{x_2 H} \quad (32)$$

$$K_1 x_1 e^{x_1 H} + K_2 x_2 e^{x_2 H} = -\gamma \qquad (33)$$

$$A\left(T_i - T_x + \gamma B + K_1 K_2\right) = = -\frac{c_1}{x_1}\left(e^{x_1 H} - 1\right) - \frac{c_2}{x_2}\left(e^{x_2 H} - 1\right) \qquad (34)$$

After solving the equation system for the constants C_1, C_2, K_2, K_2, the temperature distributions in the annulus and in the tubing can be determined by the equations (40) and (41).

The thermal power can be calculated using the equation

$$P = \dot{m} \cdot c\left(T_{wh} - T_i\right) \qquad (35)$$

where T_{wh} is the temperature of the outflowing water.

RESULTS

The temperature distribution at the annulus and the tubing flow is determined by Eq.(27) and (28). The equations show that many variables influence the temperature distributions and the attainable exit temperature at the wellhead. **The solution makes possible to take into consideration the variable influencing the temperature distribution, the exit temperature and the thermal power of the system.** The variables significantly impact temperature distribution are: the depth of the well, mass flow rate, time of operation, inlet temperature, the geometry of the well completion, thermal insulation of the tubing, the overall heat transfer coefficient, thermal conductivities of the surrounding rocks, and the geothermal gradient. It is obvious as deep the well, as high the bottomhole temperature.

It can be recognized, that the bottomhole temperature depends strongly on the mass flow rate. Our first example is the temperature distribution of a closed-loop well having the main data:

The depth of the well is 2000m, the casing is 7”, the tubing is 4 ½”. The tubing is a steel pipe with polypropylene heat insulation by 0,2 W/m^{o}C. The heat conductivity of the rock is 850 J/kg^{o}C. The inlet water temperature is 20 ^{o}C. The surface earth temperature is 10,5^{o}C. Geothermal gradient is 0,05^{o}C/m. The math flow rates are 5, 10 and 15 kg/s. The average heat conductivity of the rock is 2,5 W/m^{o}C.

Temperature distributions obtained different mass flow rates is shown Figure 2. Temperature difference between the upflowing and the downflowing water is decreasing as the mass flow rate is increasing.

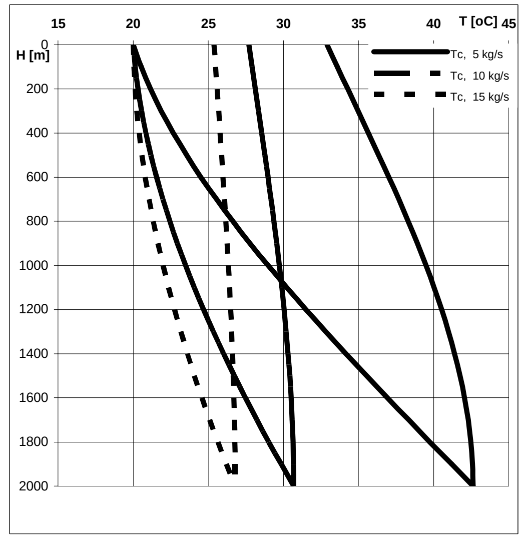

Figure.2. Temperature distributions.

The quality of the heat insulation of the tubing has a very important role. Applying a vacuum-insulated tubing (VIT) with an extreme low heat conductivity (k=0,006W/m^{o}C) the upflowing water temperature is almost constant. In this case the casing diameter is 9 5/8”, the inner tube diameter is 4 ½”, the outer is 5 ½”. Effect of insulation is shown in Figure 3.

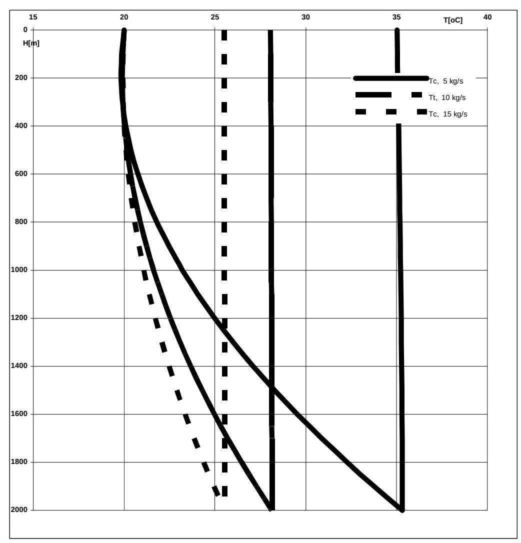

Figure.3. Temperature distributions at Vacuum Insulated Tubing k_{VIT}=0,006 W/m^{o}C.

The temperature distribution both in the annulus and in the tubing depends strongly on operation time of the system. As the rate of heat transfer between the wellbore and the surrounding rock diminishes with increasing operation time, tamcrature profiles both in the annulus and the tubing tend to an equilibrium-distribution.

One very interesting aspect considering the exit temperatures at the wellhead depending on the operation time , while mass flow rate is the parameter of the different curves. There is of a short initial period of important temperature decrease. Later the rate of change will be smaller. Finally each outflowing water temperatures tend to an equilibrium steady walue, as it is shown in Figure 4.

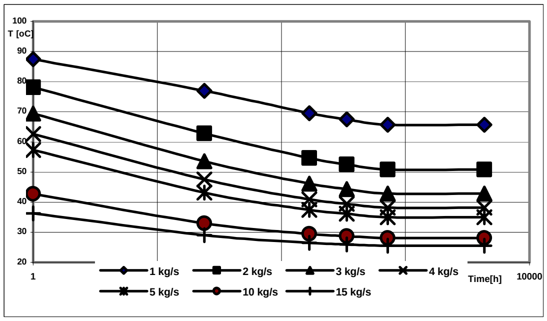

Figure. 4. Outflowing water temperature devrease vs. time

Consider two uniform borehole heat exchangers. The only difference between them is in their different tubing material. One of them is made of polypropylene, while the other is a vacuum insulated tube (VIT). Commonly the downflowing fluid in the annulus is heated both across the casing and the tubing wall. The heat loss of the upflowing fluid in the tubing appears as a heat source for the annular flow. The almost perfectly insulated VIT terminates the heat transfer across the tubing wall. In this case the annular flow gains heat across the casing only. Thus the bottomhole temperature is lower than that is obtained using polypropylene tubing. In the other hand the outflowing water temperature at the upper end of the VIT is higher than the wellhead temperature of the polypropylene tubing (PPT). The thermal power is slightly greater applying VIT, because the annular temperature is lower than PPT. The temperature difference between the undisturbed rock and the annulus is greater using VIT. The greater temperature difference induces a greater heat flux toward the well, hence the produced thermal power also increases. It is shown in Figure 5. It seems unnecessary to apply the expensive VIT.

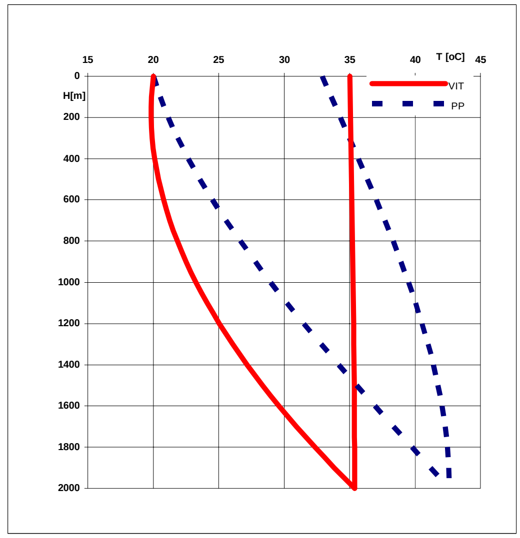

Figure. 5. Comparison of temperature distributions

The influence of the mass flow rate and the operation time on the thermal power of the system can be seen in Figure 6. The thermal power is plotted against the production time. The mass flow rate is the parameter of the family of the curves. For small mass flow rates the effect of the time is very small. For higher flow rates the change with time is important. While the growth of the mass flow rate has a substantial influence on the thermal power in the initial stage of production, as the operation time increases the differences of the power curves caused by the different mass flow rates decrease. The curves converge, especially for higher flow rates as they tend to an equilibrium state. It can be recognized that there exist an upper limit of the flow rate over which the equilibrium thermal power is not increasable. This thermal power determines the sustainability of the system. It is shown in Figure 6. Heat conduction toward the well cannot carry more heat than this upper limit. Heat conductivity of the adjacent rock mass restricts the exploitable thermal power by a single closed-loop geothermal well. The sustainable power production of such a system can be determined knowing the depth and the completion of the well, the way of heat insulation of the tubing, the local geothermal gradient, and the material properties of the rocks around the well.

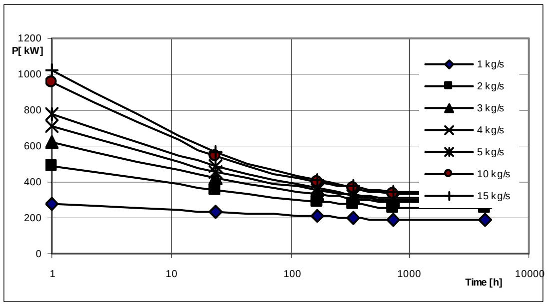

Figure. 6. Thermal power decrease vs. time

It can be seen that the temperature and the thermal power of such a closed-loop system is rather moderate. The cause of this is, the small heat transfer area and the low heat conductivity of the rocks. It seems only small-scale utilizations can be based on this clean technology, even applying heat pumps as well.

ACKNOWLEDGEMENT

The financial support of the National Scientific Research Foundation of Hungary in funding the Project OTKA-A07 69223 is gratefully acknowledged.

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