

## TIME-DEPENDENT SEISMIC TOMOGRAPHY AND ITS APPLICATION TO THE COSO GEOTHERMAL AREA, 1996-2006

Bruce R. Julian<sup>1</sup>, Gillian R. Foulger<sup>2</sup>, Francis C. Monastero<sup>3</sup>

<sup>1</sup>U.S. Geological Survey, 345 Middlefield Rd., Menlo Park, CA 94306, e-mail: julian@usgs.gov

<sup>2</sup>Dept. Earth Sciences, University of Durham, Science Laboratories, South Rd., Durham, DH1 3LE, U.K.,  
e-mail: g.r.foulger@durham.ac.uk

<sup>3</sup>Geothermal Program Office, U.S. Navy, China Lake, CA 93555-6001, e-mail:  
francis.monastero@navy.mil

### **ABSTRACT**

Measurements of temporal changes in Earth structure are commonly determined using local-earthquake tomography computer programs that invert multiple seismic-wave arrival time data sets separately and assume that any differences in the structural results arise from real temporal variations. This assumption is dangerous because the results of repeated tomography experiments would differ even if the structure did not change, simply because of variation in the seismic ray distribution caused by the natural variation in earthquake locations. Even if the source locations did not change (if only explosion data were used, for example), derived structures would inevitably differ because of observational errors. A better approach is to invert multiple data sets simultaneously, which makes it possible to determine what changes are truly required by the data. This problem is similar to that of seeking models consistent with initial assumptions, and techniques similar to the “damped least squares” method can solve it. We have developed a computer program, **dtomo**, that inverts multiple epochs of arrival-time measurements to determine hypocentral parameters and structural changes between epochs. We shall apply this program to data from the seismically active Coso geothermal area, California, in the near future.

The permanent network operated there by the US Navy, supplemented by temporary stations, has provided excellent earthquake arrival-time data covering a span of more than a decade. Furthermore, structural change is expected in the

area as a result of geothermal exploitation of the resource. We have studied the period 1996 through 2006. Our results to date using the traditional method show, for a 2-km horizontal grid spacing, an irregular strengthening with time of a negative  $V_p/V_s$  anomaly in the upper ~ 2 km of the reservoir. This progressive reduction in  $V_p/V_s$  results predominately from an increase of  $V_s$  with respect to  $V_p$ . Such a change is expected to result from effects of geothermal operations such as decreasing fluid pressure and the drying of argillaceous minerals such as illite.

### **INTRODUCTION**

Many geothermal reservoirs exhibit strong anomalies in the seismic-wave speeds  $V_p$  and  $V_s$ , and particularly clearly in the ratio  $V_p/V_s$  (Julian et al., 1996). Moreover, intensively exploited systems, such as The Geysers in northern California, show rapid and easily detectable temporal changes in the  $V_p/V_s$  ratio (Foulger et al., 1997; Gunasekera et al., 2003). Careful measurement of these changes is a promising technique for monitoring changes in the physical state of geothermal reservoirs for the purpose of optimizing exploitation strategies.

Studies of temporal changes in seismic-wave speeds such as those referenced above have until now been based on seismic tomography methods that analyze a single set of seismic-wave arrival-time data and produce a single structural model, assuming that the structure does not vary with time. Seismologists have applied such programs to invert data sets from different epochs independently, and assumed that differences in the derived models represent real temporal

variations. Figure 1 shows an example of the results of this approach. This approach can, however, produce spurious variations caused by differences in the sampling of the structure by seismic rays in different epochs caused by differences in the distributions of earthquakes and seismometers. This difficulty has been addressed by using complicated *ad hoc* iterative strategies. A model derived for one epoch is used as a starting model for analyzing data from another epoch, in an attempt to eliminate model differences that are not required by the data. However, such approaches are not entirely satisfactory. For example, they do not address the problem of apparent variations that are inevitably produced by random measurement errors that are present in all data.

### INVERSION FOR SECULAR CHANGE

A better approach is to invert multiple data sets jointly, imposing constraints to make the derived models as similar as possible, allowing differences only when the data require them. We have developed a method to do this, summarized

in the Appendix, which requires little more computational effort than would independent inversions of the data sets. The new computer program **dtomo** (Julian, 2008), based on this method, incorporates several other advantages over previous programs. It is written in the C language, and hence can dynamically allocate memory, so that it is never necessary to re-compile the program to handle different sizes of data sets. It uses the “bending” method of true three-dimensional ray tracing (Julian and Gubbins, 1977), rather than “Approximate Ray Tracing” (ART), “pseudo-bending”, or other approximate methods. It uses smooth tri-cubic functions to represent the three-dimensional wave-speed distribution, removing a major cause of erratic numerical behavior in tomography codes based, for example, on tri-linear functions. It corrects bugs such as one in the Earth-flattening transformation that makes some codes unusable for large regions at high latitudes. Nevertheless, **dtomo** is efficient, able to trace more than 10,000 rays per second on a typical laptop computer.

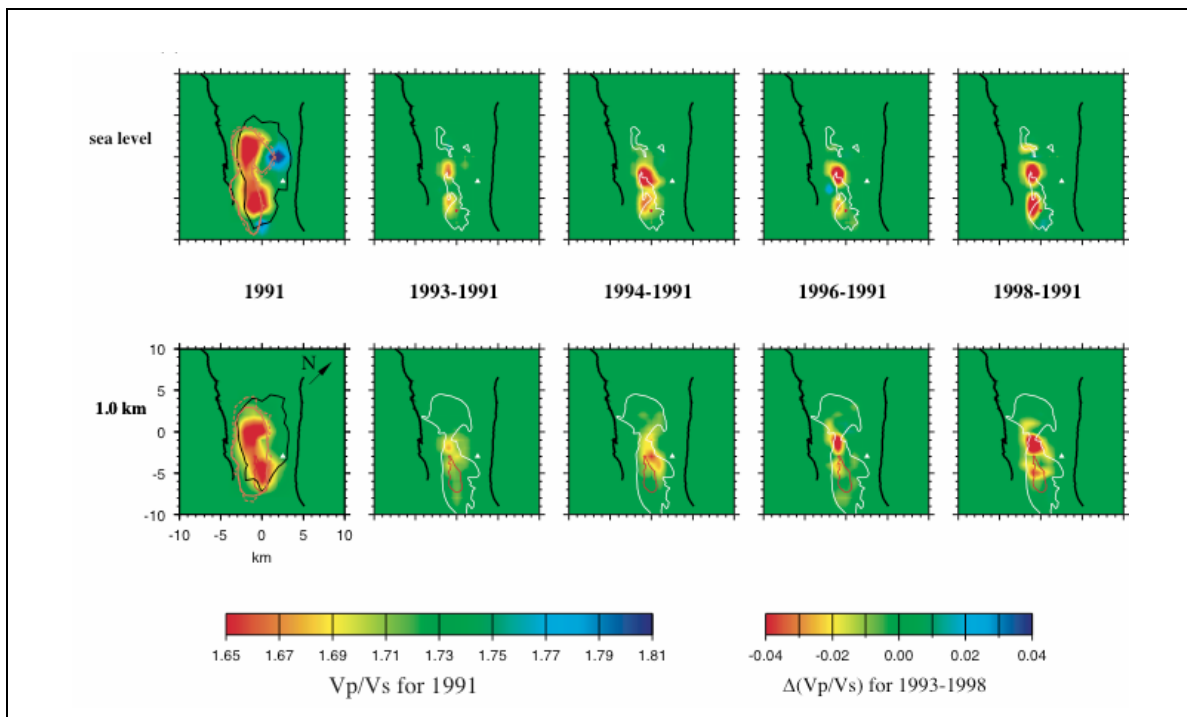


Figure 1. Maps showing changes in the  $V_p/V_s$  ratio at two depths at The Geysers geothermal area, California, between 1991 and 1998, as determined by Gunasekera et al. (2003) by inverting five data sets independently using the local-earthquake tomography computer program *simulps12* (Evans et al., 1994).

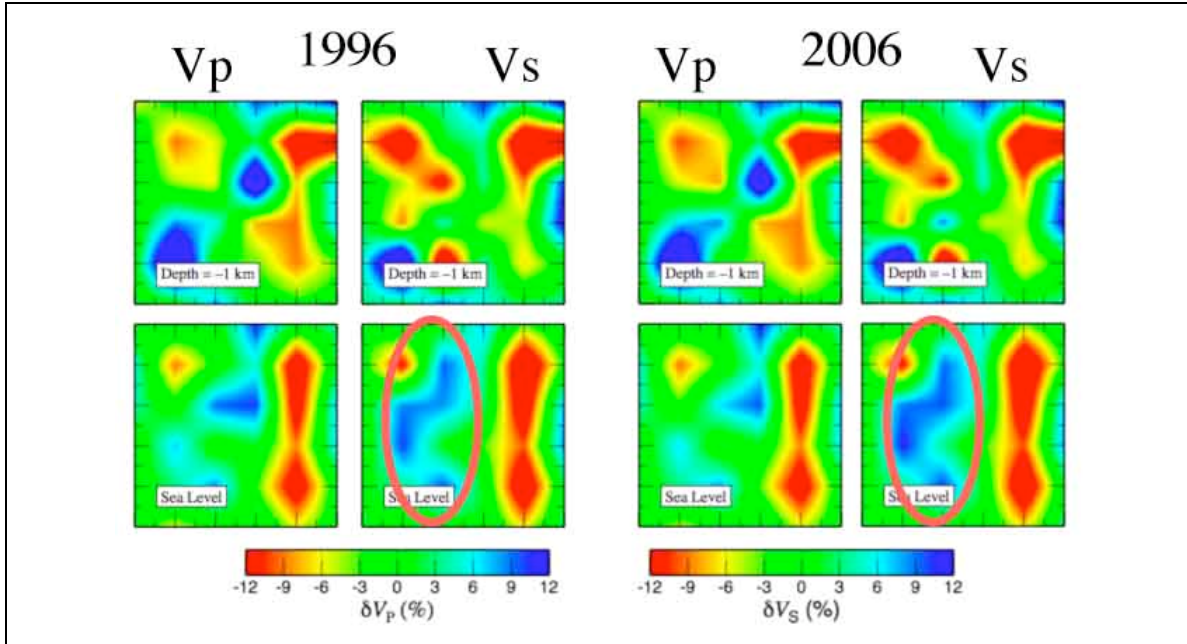


Figure 2. Maps of  $V_p$  and  $V_s$  at shallow depths at the Coso geothermal area, eastern California, in 1996 and 2006 as computed by *dtomo*. The region shown is  $10 \times 10$  km in area and the grid spacing is 2 km horizontally and 1 km vertically. The average surface elevation is about 1.3 km.

### EXAMPLE RESULT

Figure 2 shows a typical early result of applying *dtomo* to data from the Coso geothermal area. The strongest temporal change found is an increase in  $V_s$  near sea level in the central part of the area, possibly caused by reservoir pressure decrease and the drying of argillaceous minerals. There is a weak decrease in  $V_p$  in the same region, which could reflect an increase in pore-fluid compressibility caused by conversion of liquid water to steam.

### CONCLUSIONS

Secular changes in seismic wave speeds in geothermal reservoirs provide valuable evidence about changes in the pore-fluid compressibility and the elasticity of the host-rock matrix and the effects of energy exploitation. Accurate measurement of such changes, however, is complicated by biases caused by changes in seismic-ray distribution and random measurement errors, which can be greatly reduced by the use of modified tomographic methods such as the one described herein.

### APPENDIX – INDIRECT MEASUREMENT OF SECULAR CHANGES

Let the Earth model at an epoch be described by  $n$  parameters (for example, wave-speed values at the nodes of a spatial grid) arranged in a column vector, let the model vectors for two epochs be  $\mathbf{x}$  and  $\mathbf{y}$ , and let the observed arrival-time residuals (observed minus predicted times) similarly be arranged in column vectors  $\mathbf{b}$  and  $\mathbf{d}$ , of dimensions  $m_b$  and  $m_d$ . The functional relation between a model and the corresponding predicted arrival times is nonlinear, because of the dependence of the ray paths on the model, and the only practical solution method consists of iteratively solving linearized problems in which small changes in the models,  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$ , are related to predicted changes  $\delta\mathbf{b}$  and  $\delta\mathbf{d}$  in the residuals by linear operators given by the first terms in Taylor-series expansions. These operators take the form of “design” matrices  $\mathbf{A}$  and  $\mathbf{C}$ , of dimensions  $m_b \times n$  and  $m_d \times n$ , such that, to first order, predicted changes in the residual vectors are given by  $\delta\mathbf{b} = -\mathbf{A}\delta\mathbf{x}$  and  $\delta\mathbf{d} = -\mathbf{C}\delta\mathbf{y}$  (the “design equations”).

The conventional least-squares method seeks at each iteration to optimize the fit between the

predicted and observed arrival times by minimizing the quantity chi-squared, defined as

$$\chi^2 = (\mathbf{A}\delta\mathbf{x} - \mathbf{b})^2 + (\mathbf{C}\delta\mathbf{y} - \mathbf{d})^2. \quad (1)$$

The solution to this minimization problem is found by differentiating the right-hand side of this equation with respect to each of the components of the unknown vectors  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$  and setting the resulting expressions to zero. In the case when the problems for the two epochs are independent, we obtain two independent  $n \times n$  systems of linear “normal” equations,

$$\mathbf{A}^T \mathbf{A} \delta\mathbf{x} = \mathbf{A}^T \mathbf{b} \text{ and } \mathbf{C}^T \mathbf{C} \delta\mathbf{y} = \mathbf{C}^T \mathbf{d}, \quad (2)$$

to solve for  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$ . Here the superscript  $T$  indicates matrix transposition.

We wish to suppress any tendency of the derived models to differ unless doing so significantly improves the fit to the data. We therefore modify the least-squares method to seek to minimize the quantity

$$\chi^2 + \beta(\delta\mathbf{y} - \delta\mathbf{x})^2, \quad (3)$$

where the adjustable parameter  $\beta$  controls the tendency of the models to be similar. This modification leads, by a similar process of differentiation with respect to the unknown elements of  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$ , to a  $2n \times 2n$  system of normal equations that can be written in partitioned form as

$$\begin{bmatrix} \mathbf{A}^T \mathbf{A} + \beta \mathbf{I} & -\beta \mathbf{I} \\ -\beta \mathbf{I} & \mathbf{C}^T \mathbf{C} + \beta \mathbf{I} \end{bmatrix} \begin{bmatrix} \delta\mathbf{x} \\ \delta\mathbf{y} \end{bmatrix} = \begin{bmatrix} \mathbf{A}^T \mathbf{b} \\ \mathbf{C}^T \mathbf{d} \end{bmatrix}, \quad (4)$$

where  $\mathbf{I}$  is the  $n \times n$  identity matrix. Although the order of this system is twice that of the two original normal-equation systems (2), most of the new coefficients are zero, and it is possible to take advantage of this fact to solve for  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$  with essentially the same amount of computer memory and computational effort.

In realistic seismic-tomography problems, several practical complications arise. The normal-equation matrices  $\mathbf{A}^T \mathbf{A}$  and  $\mathbf{C}^T \mathbf{C}$  are almost always singular, because the data are inadequate to determine all the model parameters. This difficulty is usually dealt with by modifying (“regularizing”) the matrices  $\mathbf{A}$  and  $\mathbf{C}$  in some way, for example to make the model perturbations  $\delta\mathbf{x}$  and  $\delta\mathbf{y}$  small or spatially smooth at each iteration step.

Furthermore, most of the seismic data usually come from earthquakes, whose locations and origin times are not known *a priori* and must be estimated along with the model parameters. The approach of simply incorporating the extra unknown location parameters into the problems would multiply the sizes of the normal-equation systems by factors of fifty or more, and the computational effort required to solve them by factors of hundreds of thousands, and is completely impractical. Again, however, most of the coefficients that would need to be added to the equations are zero (because one earthquake’s location effects only its own arrival times), and efficient methods exist to separate and solve the event-location and Earth-structure problems (Spencer and Gubbins, 1980). Both of these additional complications are easily incorporated into the solution method represented by equations (4).

## REFERENCES

- Evans, J.R., Eberhart-Phillips, D. and Thurber, C.H., 1994. User’s manual for SIMULPS12 for imaging  $V_p$  and  $V_p/V_s$ , a derivative of the Thurber tomographic inversion SIMUL3 for local earthquakes and explosions, USGS.
- Foulger, G.R., Grant, C.C., Ross, A. and Julian, B.R., 1997. Industrially induced changes in Earth structure at The Geysers geothermal area, California. *Geophys. Res. Lett.*, 24(2): 135-137.
- Gunasekera, R.C., Foulger, G.R. and Julian, B.R., 2003. Reservoir depletion at The Geysers geothermal area, California, shown by four-dimensional seismic tomography. *J. Geophys. Res.*, 108(B3): 2134, doi:10.1029/2001JB000638.
- Julian, B.R., 2008. User’s Guide to Secular Seismic Tomography using dtomo. Open-File Report (in preparation), USGS, Menlo Park, California.
- Julian, B.R. and Gubbins, D., 1977. Three-dimensional seismic ray tracing. *J. Geophys.*, 43(1/2): 95-113.
- Julian, B.R., Ross, A., Foulger, G.R. and Evans, J.R., 1996. Three-dimensional seismic image of a geothermal reservoir: The Geysers, California. *Geophys. Res. Lett.*, 23(6): 685-688.
- Spencer, C. and Gubbins, D., 1980. Travel-time inversion for simultaneous earthquake location and velocity structure determination in laterally varying media. *Geophys. J. R. Astr. Soc.*, 63: 95-116.