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GEOTHERMAL VERTICAL EFFECTS IN THERMAL RESPONSE TESTS

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ABSTRACT

We present analytical solutions for the temperature around the borehole heat exchanger which account for: (I) finite length line-source; (II) natural vertical gradient of the ground temperature; (III) arbitrary changes of the ground surface temperature. We find that the time-dependent part of the solution decays as a power of the time approaching the steady state limit. The long-time limit and asymptotic approach to this limit differ from that predicted by the infinite line-source model for very large times. We present series for the temperature in time scaled by the longitudinal and the transverse borehole lengths that also enables to determine the thermal conductivity of the ground, that is a critical parameter in designing geothermal borehole, and clarifies the estimation of the duration of the thermal response test.

SECTION 1 – INTRODUCTION

The Kelvin's infinite line source model is the most widely used one for the determination of the ground thermal conductivity (Mogenson 1983, Claesson et al 1987, Hellström 1991). For large values of time, finite size effects are to be taken into account; otherwise the ground temperature changes all the time. That is not the case for the finite line-source: the solution has been obtained and expressed as one-dimensional integral (Claesson et al 1987, Carslowand et al 1971) given zero temperature at the boundary of semi-infinite medium. However, analytical asymptotic form of the solution is highly desirable to improve the estimation of the effective conductivity of the ground and the thermal resistivity of the borehole from data of the thermal response test (TRT). The results of such probes can be compared to the predictions based on the analytical formulas for the temperature as a function of time.

Recently special attention is devoted to the estimation of temperature around the borehole by averaging over its longitude (Zeng et al 2002, Lamarche et al 2007). Typically, to this end, one applies the so-called "g" - function introduced by Claesson and Elkinson (1987). The “g"- function presents the thermal response factor of the borehole by the dimensionless temperature of the wall (Hellström 1991) and is used for modeling 3-D temperature distribution about the borehole. This "g"-function methodology is implemented in the simulation package TRNSYS, and in the software tools EED, GLHEPRO (Spitler 2000), which are commonly used for design of geothermal heat pump systems. In addition to the widely used numerical methods, analytical approach allows to check physical processes behind a model and analyze how the solution varies with the parameters of the designed system. Further refinement is desirable, because of the fact that numerical calculations show that the evaluation at the middle of the borehole overestimates its steady state temperature (Hellström 1991, Zeng et al 2002, Lamarche et al 2007). Although modeling borehole as a thin ellipsoid (Hellström 1991) and empirical formula for the steady state temperature as function of r_b/H (based on the numerical results of the line-source theory) (Zeng et al 2002) mitigate this problem, still the mean integral temperature remains the best solution for engineering purposes (Zeng et al 2002, Lamarche et al 2007). Following this suggestion (Zeng et al 2002) a new expression for the "g"-function was introduced in order to increase accuracy and to decrease computational cost. The averaged exact solution for temperature was modified from the double to one-dimensional integral form (Lamarche et al 2007). Such "g" - function, though available and not computationally expensive, is still clumsy to use.

In this paper we pursue the following purposes: (I) Elaboration of the analytical formulas to determine *in situ* thermal conductivity and thermal resistance of a borehole by multi-variable parameter fitting; (II) Selecting an efficient model for the evaluation of the TRT data; justifying the usage of the mean temperature for self-consistent interpretation of the TRT data; (III) Obtaining approximations for both mean and middle temperature of the borehole; estimating the time of attainment of a steady state.

This paper presents two main contributions. First, we introduce a more efficient version of the estimation of the thermal conductivity and the thermal resistance. This strategy solves the long-standing problem of that the steady-state temperature at the middle of the borehole is overestimated in the traditional approach. Second, we conduct an exhaustive comparison of the above methods of assessment the borehole wall temperature. For this purpose, we also report our results on the series in time for the temperature about the middle point of a borehole.

The rest of the paper is organized as follows. Section 2 introduces the standard formulations for the finite line-source model, and presents the solutions obeying general boundary and initial conditions, highlights the limitation of infinite line-source model causing from infiniteness assumption. Section 3 first reviews the results of the classical infinite line-source theory, and then proposes a method to evaluate TRT data based on the integral mean of the borehole temperature for consistent account of the axial heat transfer effects. Section 4 summarizes our present results, compares them with the major features that emerge from evaluating borehole temperature at its middle. The findings are then analyzed theoretically and through illustrative examples. Finally, Section 5 concludes and gives some directions for further investigation.

SECTION 2 - LINE SOURCE THEORY

The notation used throughout this paper is summarized in Table 1. A comparative study between numerical results and results of the infinite line-source (ILS) model showed that it is valid under some conditions (Signorelli et al 2007), which we specify below. Within this framework, commonly applied for the estimates of thermal response test data, the infinite medium is considered at given undisturbed ground temperature, T_0. In this paper, we consider natural flow along the vertical z axis with a constant gradient $\nabla_z T_{geo}$ in semi-infinite region, ground surface temperature of which, ψ (t), varies with time. We assume medium to be homogeneous and denote its volumetric heat capacity by C and its coefficient of thermal conductivity by λ, $\alpha = \lambda / C$ represents the thermal diffusivity. The heat is released at a constant rate along the borehole heat exchanger (BHE), and is transferred by mechanism of thermal conductivity. Within the finite line-source (FLS) model the equation of heat diffusion is invariant under spatial rotation about z axis of the vertical BHE. The temperature of the ground, T, is defined by the heat conduction equation.

Notation	Description
H	Height of the borehole heat exchanger, [m]
r_b	Radius of the borehole heat exchanger, [m]
M_f	Mass flow rate of heat carrier fluid, [kg/s]
R_b	Thermal resistance between fluid and borehole wall
T	Temperature of ground, [^{0}C]
T_0	Reference temperature (of undisturbed ground), [^{0}C]
T_{in}	Inlet temperature of BHE [^{0}C]
T_{out}	Outlet temperature of BHE [^{0}C]
$\nabla_z T_{geo}$	Geothermal gradient, [Km^{-1}]
Q_z	Heat flux per unit length, [Wm^{-1}]
C	Volumetric heat capacity, [Jm^{-3}K^{-1}]
λ	Thermal conductivity of ground, [WK^{-1}m^{-1}]
$\alpha = \lambda / C$	Thermal diffusivity of ground, [m^{2}s^{-1}]
b(f)	Subscript associated with the borehole wall (fluid)
S	Subscript associated with the steady-state

Table 1. Notation in Latin and Greek letters used in this paper.

$$C\frac{\partial(\vec{r}_\perp, z, t)}{\partial t} = \lambda \Delta T(\vec{r}_\perp, z, t) + Q_z \delta(\vec{r}_\perp)(\theta(z) - \theta(z - H))$$

at $t \geq 0, z \geq 0$ (1)

where coordinate vector $\vec{r}_\perp$ is orthogonal to z axis, Q_z is the heat flux density per length unit of the BHE of radius r_b and $\theta(z)$ is the step function. The initial condition, $T(z, t=0) = T_0(z) = T_0 + \nabla_z T_{geo} z$, reflects natural flow; the constant $\nabla_z T_{geo}$ is known as the *geothermal gradient*. The boundary condition on the surface, $T(z=0, t) = \psi(t)$, captures the rather pronounced effect of the variation of the ambient air temperature with time (Sanner et al 2007) on the upper part of the BHE.

We write temperature as a sum of the solution of the inhomogeneous Eq. (1), v_d, and solutions of the homogeneous Eq. (1), v_0, v_s, which have to be determined; these satisfy the conditions indicated in Table 2. The solutions for the mixed problem with the prescribed temperature, $\psi(t)$, on the surface of the semi-infinite medium with natural flow are presented in Table 3.

Conditions at ground surface	$v_d(r, z=0, t) = 0$ $v_s(r, z=0, t) = \psi(t)$ $v_0(r, z=0, t) = 0$
Initial conditions	$v_d(r, z, t=0) = 0$ $v_s(r, z, t=0) = 0$ $v_0(r, z, t=0) = T_0 + \nabla_z T_{geo} z$ z > 0, z $\neq$ 0

Table 2. General boundary and initial conditions for the temperature of the ground $T = v_d + v_s + v_0$.

In fact, the solution provided in Table 3(1) is identical to the integral in Claesson et al (1987) but written in a different form. It is seen that the time has several characteristic scales, namely, H^2/α, r_b^2/α, and $1/\omega$, where ω is a frequency of the surface temperature change. The depth of thermal penetration from the Earth's surface is given by $d_p = \sqrt{2\alpha/\omega}$, which naturally appears in the frame of finite line-source model. It is noteworthy that v_0, v_S solutions do not vary with radial coordinate, see blocks: 3(1), 3(2) in Table 3. Therefore, the TRT data can be filtered from these axial effects, described by v_0, v_S, and can be compared with the solution v_d that varies also with *r* due to the heat release and to which we mainly refer below. We find that the steady state limit (SSL) appears in the long time asymptotic of the semi-infinite model; that is semi-infinite line-source (SILS) in the semi-infinite region, see Table 3(5). This result is different from the unlimited increase of the temperature predicted by the ILS model, but the asymptotic behavior is very close also to the one predicted by the FLS model for the medium values of the time.

The steady state temperature field of one borehole was already obtained in the framework of the FLS model (Zeng et al 2002), see time-independent terms in Table 3(4). Our new contribution is that we find how the system reaches its steady state. Namely, the ground temperature approaches its steady state limit from below and with the inverse 3/2 power of the time. That is, in contrast to the inverse 1/2 power of the time predicted within the SILS model, see Table 3(5). So far, the BHE attains its steady state faster, $t^{-3/2}$ vs. $t^{-1/2}$, than one might expect making use of the SILS model. Furthermore the onset of this asymptotic behavior is delayed to a later time the lower the thermal diffusivity of the system that is of importance for the TRT interpretation.

Figure 1(a) shows comparison between the time-dependence of the ground temperature calculated within the finite and semi-infinite line-source model for an intermediate long-time scale, for the deep BHE; notice that geothermal flow causes the increase up to 2.62^o of the temperature at the depth of H=50 m. Figure 1(b) shows that the asymptotic behavior of the ground temperature for the short borehole differs significantly from that predicted by the SILS model as time approaches an upper bound of the medium time interval: $r^2/\alpha << t << H^2/\alpha$, i.e. when $t \to H^2/\alpha$. On the other hand, *T(r,z,t)* obtained within the FLS model for medium time values coincides with that obtained from the ILS model, though in a very narrow time window, when $H/r_b < 1$, see Table 3(3). In contrast to the case of deep BHE, the short borehole is under the strong influence of the surface temperature change due to the fact that the depth of heat penetration, d_p, is about 0.27(4) m if one cycle per day (year). This effect is essential for the upper part of the vertical BHE and a horizontal borehole.

In the following section we proceed to determine the method of evaluating the TRT data: that are temperature of heat carrier fluid versus time.

SECTION 3 - SELF-CONSISTENT APPROACH TO EVALUATING TRT DATA

On one hand, data of the mean temperature of the heat carrier fluid, which are available from the TRT, vary with time only. On the other hand, the previous series in time for the temperature of the ground surrounding BHE depend also on z-coordinate. Then the question arises what is a proper z point when comparing with the experimental data. We propose a consistent way of estimation of the thermal conductivity of the ground and the thermal resistance of the BHE based on averaging the temperature of the ground over the z range for the heat source. Thus, one can apply all the obtained series with the average temperature in place of temperature at a given point. The temperature of the ground appears to be z-constant for the infinite line-source model. Indeed in the surroundings of the borehole the results of the infinite line-source model for sufficiently large time values read as (Mogenson

1. Exact solution	$v_d = \frac{Q_z}{4\pi\lambda} \int\limits_{r/2\sqrt{\alpha t}}^{\infty} \left\{ 2erf(\frac{z}{r}u) - erf(\frac{H+z}{r}u) + erf(\frac{H-z}{r}u) \right\} \frac{e^{-u^2}}{u} du$
2.1. Surface temperature: $\psi(t)$	$v_s + v_0 = \frac{2}{\sqrt{\pi}} \int\limits_{z/\sqrt{4\alpha t}}^{\infty} \psi(t - \frac{z^2}{4\alpha u^2}) e^{-u^2} du + T_0 erf(\frac{z}{2\sqrt{\alpha t}}) + \nabla_z T_{geo} z$
2.2. $\psi(t) = T_0 + T_s \sin(\omega t)$	$v_s + v_0 = T_0 + T_s e^{-z/d_p} \sin(\omega t - z/d_p) + \nabla_z T_{geo} z$
3. Series of $v_d(r,z,t)$ in time about a midpoint depth. Medium time values	$v_d = \frac{Q_z}{4\pi\lambda} \{ -\gamma + \ln \frac{4\alpha t}{r^2} + O(\frac{(\alpha t)^{3/2}}{L^{5/2}}) + O(\frac{r^2}{\alpha t}) - $ $- \frac{1}{\sqrt{\pi}} (\phi(z) - \phi(H+z) + \phi(H-z)) \}$ when $r << \sqrt{\alpha t} << L, \; L = min(z, H-z)$
4. Approach v_d to the steady state limit (SSL). Large time values	$v_d = \frac{Q_z}{4\pi\lambda} \{ 2 \sinh^{-1} \frac{z}{r} - \sinh^{-1} \frac{H+z}{r} + \sinh^{-1} \frac{H-z}{r} - $ $- \frac{zH^2}{2\sqrt{\pi}(\alpha t)^{3/2}} (\frac{1}{6} - \frac{(H^2 + 2(z^2+r^2))}{80\alpha t}) \} + O(\frac{\mathbf{H}}{2\sqrt{\alpha t}})^7$ when $H < 2\sqrt{\alpha t}$
5. Semi-Infinite ($H \to \infty$) line-source	$v_d = \frac{Q_z}{2\pi\lambda} \{ \sinh^{-1} \frac{z}{r} - \frac{2z}{(\pi\alpha t)^{1/2}} (1 - \frac{z^2+3r^2}{36\alpha t}) \} + O(\frac{\mathbf{z}}{\sqrt{\alpha t}})^5$ when $z < \sqrt{\alpha t}, \; z \in [0, \infty)$

Table 3: List of the finite line-source solutions $T(r,z,t) = v_d + v_s + v_0$*, see above, and their time-series for the temperature of the ground at a middle of BHE. Dimensionless function* $\phi(z) := \tilde{\phi}(z,r,t) = \frac{(4\alpha t)^{3/2}}{z^3} exp\left(-\frac{z^2}{4\alpha t}\right)\left(1 - \frac{r^2}{2\alpha t} - \frac{r^2}{z^2}\right)$ *denotes the finite-length corrections.*

1983, Claesson et al 1987),

$$T(r,t) = \frac{Q_z}{4\pi\lambda} Ei\left(\frac{4\alpha t}{r^2}\right) \approx \frac{Q_z}{4\pi\lambda}\left\{\ln\frac{4\alpha t}{r^2} - \gamma + O\left(\frac{r^2}{4\alpha t}\right)\right\} + $$

$$+ T_0, \quad if \; \frac{4\alpha t}{r^2} >> 1 \qquad (2)$$

The function *Ei(u)* denotes the exponential integral (Carslowand et al 1971) and γ is Euler's constant. The reference temperature T_0 in Eq. (2) is often chosen to be equal to the natural undisturbed ground temperature, whereas its identification with the local average ground temperature is another possibility (Hellström 1991). In the frame of our approach accounting axial effects, as one can see from the Table 3 (blocks: 3(1), 3(2)) the choice $T_0 = v_0 + v_s$ is more appropriate to solve the external problem of heat conduction in the ground. In the infinite line model the borehole wall temperature T_b does not depend on the z coordinate and is compared with the arithmetic mean of the inlet and outlet temperatures, $\overline{T}_f = (T_{in} + T_{out})/2$.

For the inner problem of heat transfer inside the BHE, the average temperature of fluid can be written through the temperature at the borehole surface T_b as (Claesson et al 1987, Zeng et al 2004)

$$\overline{T}_f(t) = T_b(t) + Q_z R_b, \quad T_b(t) = T(r_b, z, t) \qquad (3)$$

Besides finding the thermal conductivity λ, the evaluation of the effective thermal resistance of the borehole, R_b, is another objective of the thermal response test. In spite of the fact that R_b also can be calculated by solving the internal problem of heat exchange between the moving fluid and the borehole wall through the grout (Hellström 1991, Signorelli et al 2007), it is more reliable when evaluated from the TRT data. By substituting solution for the heat conduction problem in the ground we arrive at

$$\overline{T}_f = T_0 + \frac{Q_z}{2\pi\lambda} g(t) + Q_z R_b$$

where, in our notation, $g(t) = \frac{2\pi\lambda}{Q_z} v_d(r,z,t)$ is the extended thermal response function (Claesson et al 1987) or the dimensionless temperature of the borehole perimeter at z coordinate. So far, $\overline{T}_f$ (t) is parameterized by three main parameters: λ, R_b and T_0. In the frame of the finite line-source model, T_b and T_0 appear dependent on the *z*-coordinate, while the definition of thermal resistance R_b in Eq. (3) assumes the comparison of $\overline{T}_f$ (t) with both z-independent T_b (t) and undisturbed temperature T_0. To find the steady-state thermal resistance, R_b, taking into account geothermal gradient, Claesson and Elkinson (1987) proposed to evaluate the steady-state temperature of the ground at the midpoint of the deep borehole as $g(t \to \infty) = ln H/\sqrt{3}r_b$, and T_0 as

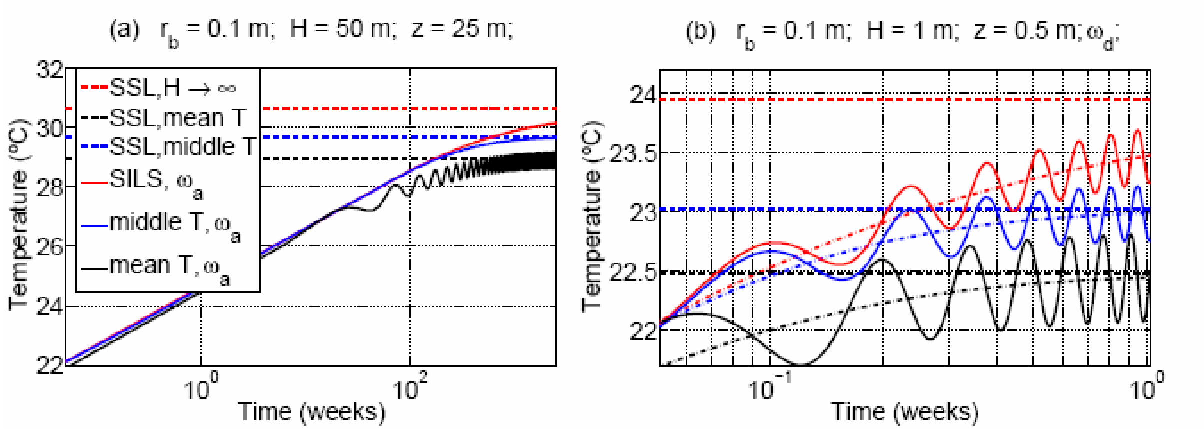

Figure 1: Temperature [ºC] for the deep BHE (left) and the short BHE (right) versus time in the logarithmic scale for different models. Models: temperature at z=H/2 (red line) and its steady state limit (SSL) (red dashed line) within semi-infinite line-source (SILS), middle temperature (blue line) and its SSL (blue dashed line) within finite line-source, mean temperature (black line) and SSL (black dashed line) within finite line-source. Temperature calculated in the frame of the exact FLS model on heat injection is shown within the range (Claesson et al 1987): $5r_b^2/\alpha < t < H^2/\alpha$ with $\alpha = 1.62\ 10^{-6} m^2/s$, $\lambda = 1$ W/m/s, $Q_z = 46.32$ W/m, $T_0 = 20°C$; the surface temperature is modeled as $T_0 + T_s \sin(t\omega_{a,d})$, $\nabla_z T_{geo} = 0$ with T_s taken 10ºC and 5ºC for annual (ω_a) cycle and diary cycle (ω_d), respectively; point dashed lines stay for T_s = 0.

$T_0 + \nabla_z T_{geo} H/2$, but finally selected the approximation $ln\, H/2r_b$ that better accounts for the surface condition (Claesson et al 1987). In order to make assessment of the borehole wall temperature T_b more accurate it was proposed to use the integral mean temperature (Zeng et al 2002). Following the same purpose of improving accuracy, the average temperature was used for T_b by the authors of reference (Lamarche et al 2007) who modified integral of the exact solution (Claesson et al 1987) for the external heat exchange problem.

We propose to use the averaging along the heat exchangers length,

$$<T> = \int_0^H T(r,z,t)\,dz\,/\,H,$$

in order to achieve three basics aims: firstly, to solve both internal and external heat transfer problem consistently; secondly, to evaluate λ, R_b and T_0 by multi-parameter fitting to the TRT data; and finally, to avoid ambiguity in the selection of an optimal point for evaluating borehole wall temperature as well as the undisturbed ground temperature. Indeed, choice of a point for estimation depends on a kind of exchange mode: heating or cooling because of the geothermal gradient (Signorelli et al 2007) and influence of the surface condition.

Self-consistent analysis of the heat transfer inside and outside the borehole is of considerable importance for the accurate determination of both effective thermal conductivity of the ground and the thermal resistance of one borehole and, so far, for dimensioning of a borehole array (Zeng et al 2003). In the following we first apply this proposal to a single borehole, and present results of the exhaustive study of both local middle and mean temperatures below in Table 4. Then we go on to an interacting BHE set in the long-time limit.

SECTION 4 - RESULTS AND DISCUSSION

By implicit integrating the exact solution (Claesson et al, 1987) of the form exposed in the Table 3 we arrive at the results presented in Table 4. This summarizes our analytical results and shows comparison between approximate expressions, for g-function, at the middle point (middle temperature) and that averaged over the heat exchange length (mean temperature) in the intermediate and the long time scales for one borehole. The series are available for straightforward use. In addition, if simulations are needed, the use of our one-integral form of solution for the mean temperature results in drastic decreasing of the CPU time, when compared whit the double integral of (Lamarche et al 2007).

Our main result clarifies the role that play finite-depth corrections in the heat exchange with the surrounding ground for each time scale. We propose new approximate expressions for the dimensionless ground temperature (around a deep borehole) within the medium and long time intervals for the external

	T at the middle point of BHE	Mean T of BHE
1. Exact solution for the g-function.	$g = \frac{1}{4}\int\limits_{r_b/2\sqrt{\alpha t}}^{\infty}\{3erf(\frac{h}{2}u) - erf(\frac{3h}{2}u)\}\frac{e^{-u^2}}{u}du,\ h = \frac{H}{r_b}$ $g = \frac{2\pi\lambda}{Q_z}v_d(r = r_b, z = H/2, t)$	$< g >= \frac{1}{2}\int\limits_{r_b/2\sqrt{\alpha t}}^{\infty}\{4erf(hu) - 2erf(2hu) - (3 + e^{-4h^2u^2} - 4e^{-h^2u^2})\frac{1}{\sqrt{\pi}hu}\}\frac{e^{-u^2}}{u}du,\ h = \frac{H}{r_b}$ $< g >= \frac{2\pi\lambda}{Q_z}\frac{1}{H}\int\limits_0^H v_d(r = r_b, z, t)dz$
2. Series of $T(r, z, t)$ in $\alpha t/H^2$ Medium times	$\frac{Q_z}{4\pi\lambda}\{-\gamma - 2\ln\frac{r}{H} + \ln\frac{4\alpha t}{H^2} + \frac{r^2}{4\alpha t}\}+$ $\nabla_z T_{geo}\frac{H}{2} + T_0 + T_s e^{-\frac{H}{2d_p}}\sin(\omega t - \frac{H}{2d_p})$ when $r << \sqrt{\alpha t} << H$	$\frac{Q_z}{4\pi\lambda}\{-\gamma - 2\ln\frac{r}{H} + \ln\frac{4\alpha t}{H^2} + \frac{\mathbf{3r}}{\mathbf{H}} - \frac{\mathbf{3}}{\sqrt{\boldsymbol{\pi}}}\frac{\sqrt{\mathbf{4\alpha t}}}{\mathbf{H}} - \frac{\mathbf{3}}{\sqrt{\boldsymbol{\pi}}}\frac{\mathbf{r^2}}{\mathbf{H}\sqrt{\mathbf{4\alpha t}}} + \frac{r^2}{4\alpha t}\}+$ $\nabla_z T_{geo}\frac{H}{2} + T_0 + < T_s e^{-z/d_p}\sin(\omega t - \frac{z}{d_p}) >$ when $r << \sqrt{\alpha t} << H$
3. Approach v_d to the SSL. Large times	$v_d = \frac{Q_z}{4\pi\lambda}\{3\sinh^{-1}\frac{H}{2r} - \sinh^{-1}\frac{3H}{2r} - \frac{H^3}{4\sqrt{\pi}(\alpha t)^{3/2}}(\frac{1}{6} - \frac{3H^2/2+2r^2}{80\alpha t})\} + O(\frac{\mathbf{H}}{2\sqrt{\alpha t}})^7,$ when $H < 2\sqrt{\alpha t}$	$v_d = \frac{Q_z}{4\pi\lambda}\{4\sinh^{-1}\frac{H}{r} - 2\sinh^{-1}\frac{2H}{r} + 3\frac{r}{H} - 4\sqrt{1 + \frac{r^2}{H^2}} + \sqrt{4 + \frac{r^2}{H^2}} - \frac{2H^3}{3\sqrt{\pi}(4\alpha t)^{3/2}}(1 - \frac{3(H^2+r^2)}{20\alpha t})\} + O(\frac{\mathbf{H}}{2\sqrt{\alpha t}})^7,$ when $H < 2\sqrt{\alpha t}$
4. SSL	$\frac{Q_z}{4\pi\lambda}2\ln(\frac{H}{\sqrt{3}r}),\ H > \sqrt{3}r_b$	$\frac{Q_z}{4\pi\lambda}(-2 - 2\ln\frac{H}{r} + 3\frac{r}{H}),\ H > \frac{3}{2}r_b$

Table 4: Comparison between the exact solutions and their time-series for the temperature of the ground at the middle point of the borehole with that averaged over the heat source depth, both obtained in the framework of the FLS model. Here <> denotes averaging.

heat conduction problem, $r \geq r_b$, in the following form:

$$< g\left(\frac{r}{H}, \frac{H}{\sqrt{4\alpha t}}\right) >= \frac{1}{2}\{-\gamma + \ln\left(\frac{4\alpha t}{r^2}\right) + \frac{3r}{H} - \frac{3}{\sqrt{\pi}}\frac{\sqrt{4\alpha t}}{H} - \frac{3}{\sqrt{\pi}}\frac{r}{H}\frac{r}{\sqrt{4\alpha t}}\} \qquad (4)$$

when $\frac{r^2}{\alpha} \leq t \leq \frac{H^2}{\alpha}$, and

$$< g\left(\frac{r}{H}, \frac{H}{\sqrt{4\alpha t}}\right) >= \frac{1}{2}\{-2 - 2\ln\left(\frac{H}{r}\right) + 3\frac{r}{H}\} \qquad (5)$$

when $t \geq \frac{H^2}{\alpha}$.

Note that, in the limiting case of $r_b / H \to \infty$, the proposed expression (4) recovers the well-known thermal response function in the ILS model, Eq. (2). In general, the approximate formulas (4), (5) for the averaged g-function differ from the original one (Claesson et al 1987) by the extra terms which are proportional to $1/H$. While the approximation expressions (Claesson et al 1987, Zeng et al 2003) fail to solve completely the overestimation problem, the exact expression for the mean temperature clearly shows the decreasing of the SSL value with respect to the middle temperature, see for comparison Table 4 (blocks 4(3), 4(4)). In the steady state case, the relative error of the overestimation of the middle temperature (Claesson et al 1987) or of the numerical approximation (Zeng et al 2002) attains 20% and 6.5% with respect to that given by the Eq.(5) for the borehole of $\frac{r_b}{H} = 0.002$.

The extended <g> function Eq. (4,5) can be easily applied to estimation of the steady-state temperature field for an arbitrary borehole configuration using the superposition principle (Carslowand et al, 1971). Indeed, to have the precise formula for evaluating the stationary temperature of one BHE becomes an important factor when interference between boreholes starts, as far as in this case the overestimation error accumulates. We hope that simple and precise expressions for the mean temperature obtained here may be convenient to use when designing a complex borehole configuration.

Figure 1 (a, b) illustrate the way in which mean temperature of one BHE calculated by our method differs from the other two estimations of temperature calculated at z=H/2 in the FLS and SILS models if heat injection flow is constant. Figure 1(b) shows up also that the middle temperature delays from the mean temperature due to the disappearence of

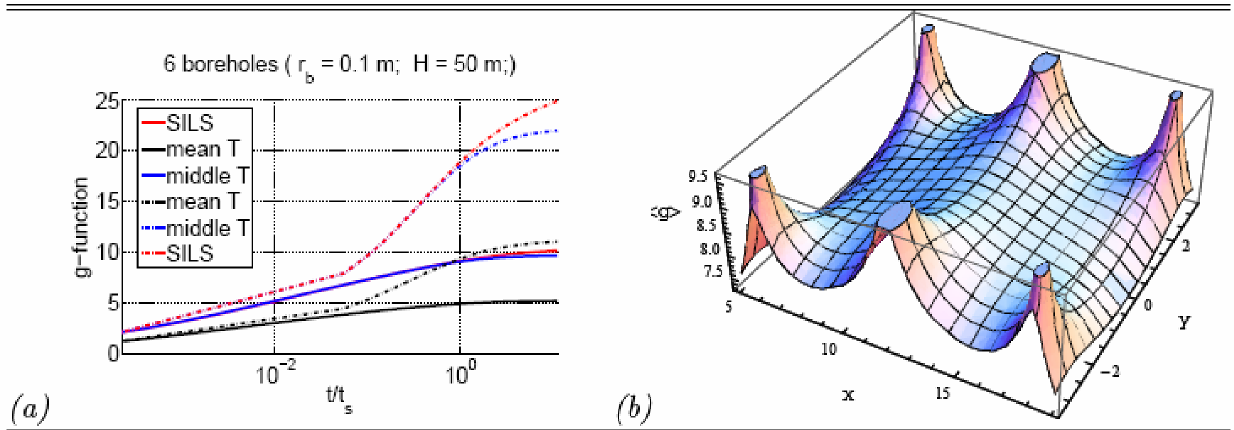

Figure 2: (a) Thermal response, g-function versus time scaled by $t_s = 9H^2/\alpha$. Temperature at z = H/2 for single borehole (red line) calculated within SILS model, middle temperature (blue line) and mean temperature (black line) calculated within FLS approach. Dashed lines stay for the central borehole in the rectangular 2×3 array with an inter-borehole distance of 3 meters.

(b) Mean steady-state temperature field (<g>) for six boreholes in a rectangular pattern with the relative distance: B/H=0.12 at a constant rate of heat injection. The <g>-function takes approximate values of 11.1 and 10.3 for the central borehole and a borehole situated at the vertex of the rectangle, respectively.

the surface temperature changes with the depth. Indeed, up to the exponentially small terms, when H<< d_p, we obtain

$$< e^{-z/d_p} \sin(\omega \cdot t - \frac{z}{d_p}) > \approx \sin(\omega \cdot t - \frac{\pi}{4}),$$

i.e. the 'time-lag' occurs in the form of the phase shift of $\pi/4$. Figure 2 (a) shows that the slope of the mean temperature with respect to the logarithm of time is smaller than the slope for the other two, hence, causing larger value for the effective thermal conductivity according to Eq.(2). This conclusion is essential for evaluating TRT data, while decrease of the local SSL is of importance for the long-term design of a borehole configuration. Figure 2(a) shows that the rate of accumulating overestimation error in time (slope) changes noticeably when the heat from the neighbor boreholes reaches a given BHE. In fact, the time scale of this heat diffusion is about B^2/α, this value falls within medium values of the time. Therefore, making use of the Eq.(5) we estimate the overall thermal response function for the central borehole as

$$g(\frac{r_b}{H},t) + 3g(\frac{B}{H},t)\theta(t - B^2/\alpha) + 2g(\frac{\sqrt{2}B}{H},t)\theta(t - 2B^2/\alpha).$$

Figure 2(b) illustrates the temperature field calculated as the superposition of the temperature of each borehole of the relative depth $r_b/H = 0.002$ for the 2×3 array by using approximating SSL, Table 4 (right block 4(3)).

SECTION 5 – CONCLUSIONS

In this paper we have presented the results of a systematic study for the ground temperature for the BHE modeled as finite depth line source of constant heat flow into the semi-infinite medium with time-dependent surface temperature, given initial distribution of the temperature with geothermal gradient. We have ordered the proposed analytical expressions for asymptotic behavior of the ground temperature in the increasing long-time scale. That is of importance for definition of the time interval when estimating model parameters from experimental data, and may give insight on the on-going discussion on duration of the TRT.

The proposed method consists in averaging the borehole temperature, rather than using its value at the middle point of the borehole. We have provided approximate expressions, which are immediately available in a wide range of the lengths and the time values. The obtained series of mean temperature differ from that predicted by the finite line-source model at the middle point by the terms of order 1/H, which we present explicitly. The classical result of the traditional infinite line-source model is applicable with a good accuracy for the medium values of the time, while finite-depth corrections are significant for the long time scale. The analytical form of our proposals for the mean ground temperature enables flexibility in the evaluation of the *in-situ* thermal response test data, and is quite suitable for the design of a ground heat exchanger.

Effect of anisotropy of the heat flow from a BHE in a non-homogeneous multi-layer ground is a subject of on-going research. Thermal response data are under comparative study with the proposed analytical formulas to show suitability of our proposal and interpretation.

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REFERENCES

Carslowand H.S., Jaeger J.C., *Conduction of Heat in Solids,* Marcel Dekker, Inc., New York, 1971.

Claesson, J., Eskilsson, P.; "*Conductive heat extraction by a deep borehole; analytical studies-Thermal Analysis of Heat Extraction Boreholes*", pp. 1-26, Dep. of Mathematical Physics, University of Lund, (Sweden), 1987.

Hellström G., *Thermal Analyses of Duct Storage System,* Dept. of Mathematical Physics, University of Lund, Sweden, 1991.

Lamarche L., Beauchamp B., *A new contribution to the finite line-source model for geothermal boreholes,* Energy and buildings 39, pp. 188-198, 2007.

Mogenson, P. (1983); "Fluid to duct wall heat transfer in duct system heat storage", Proceedings *of the International Conference on Surface Heat Storage in Theory and Practice,* pp. 652-657 *(Stockholm, Sweden), 1983.*

Sanner B., Hellström G., Spitler J. and Kolh T., *Numerical evaluation of thermal response tests,* Geothermics 36, pp. 141-166, 2007.

Signorelli S., Bassetti S., Pahud D., and Kolh T., *Numerical evaluation of thermal response tests,* Geothermics 36, pp. 141-166, 2007.

Spitler J., *GLHEPRO- a design tool for commercial building ground loop heat exchangers,* Proceedings of the Fourth International heat Pumps in Cold Climates Conference (Aylmer, Québec), 2000.

Zeng H., Diao N. and Fang Z., *A finite line-source model for boreholes in geothermal heat exchangers,* Heat Transfer Asian Research 31, pp. 558-567, 2002.

Zeng H., Diao N. and Fang Z., *Heat transfer analysis of boreholes in vertical ground heat exchangers,* International Journal of Heat and Mass Transfer 46, pp. 4467-4481, 2003.

Zeng H., Diao N. and Fang Z., Improvement in modeling of heat transfer in vertical ground heat exchanges, HVAC&Research, 10, pp. 459-470, 2004.