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GEOTHERMAL BRINE INVASION IN OIL RESERVOIRS: A 3D GENERALIZATION OF THE BUCKLEY-LEVERETT MODEL USING NON-LINEAR FINITE ELEMENTS

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ABSTRACT

In the southern coast of the Gulf of Mexico some deep geothermal aquifers are associated to hydrocarbon reservoirs. Some of their wells are invaded by geothermal brine, producing a variable mixture of hot water and oil. This water, at temperatures of 150°C and having a density of 1150 kg/m^3; flows vertically through a fault from the aquifer located 6000 m depth. The non isothermal conditions affect the effective saturations and the relative permeabilities of the immiscible phases. The relative permeability of oil is increased by the increase of temperature produced by the geothermal water. This effect reduces the residual saturation of heavy oils. At the same time the dynamic viscosities of water and oil are diminished, affecting the displacement of both fluids. Although the oil is extracted in wells finished upper the aquifer, the total volume of produced water, in some cases, equals or exceeds the oil production. The handling of this extra hot water becomes a practical serious problem. We introduce a numerical original model able to predict the critical oil rate for which the wells can be totally invaded by geothermal brine.

For the construction of the model we apply classic laws and equations. We use standard published formulas for both relative permeabilities and capillary pressure. We obtain a single non - linear partial differential equation (PDE) which depends only on water saturation, space and time. This PDE is a 3D generalization of the classical 1D Buckley-Leverett model. To solve the new PDE we use non linear finite elements. The numeric simulation could reproduce the effect of water invasion: After some time elapsed, the original oil volume diminishes abruptly, displacing the boundary of the water-oil contact and the transition zone in the vertical direction. Our objective is to estimate the optimum mass rate for producing wells in order to minimize the production of water or to achieve a mixture oil-water extraction where oil always prevails.

INTRODUCTION

The production of petroleum together with connate water is a common phenomenon in oil and gas reservoirs. This water is unusable, although its operation is, in general, quite expensive. The magazine Oilfield Review (Arnold, 2004) reported that only in the USA there are extracted 10 barrels (1.6 m^3) of water for each barrel of oil. In the whole world three barrels of water for each one of oil are produced. The cost of this water disposal is between 5 and 10 thousand million dollars in the USA and approximately of 40 thousand million dollars in the whole world. Even using the most advanced disposition techniques, water can represent 90% of the total volume of liquids at field's surface, impacting seriously the commercial feasibility of the field. Due to its null commercial utility, this water should be reinjected into the formation to maintain reservoir pressure. Another possible future use is its treatment to make it potable and usable in the hydraulic nets of cities close to the oil field.

GEOTHERMAL AQUIFERS AND OIL FIELDS

Geothermal areas related to hydrocarbons reservoirs also exist in different parts of the world. The presence of interstitial hot water in the pores alters several parameters of the reservoir. The non isothermal conditions affect the effective saturations and the relative permeabilities of both immiscible phases. The relative permeability of oil is increased by the increase of temperature originated by geothermal water. At the same time the dynamic viscosities of water and oil diminish, affecting the displacement of both fluids. The Bellota-Jujo hydrocarbon complex, located in the southern coast of the Gulf of Mexico (Fig. 7), is a remarkable example of this type of coupled processes. The Port Ceiba reservoir, which is part of this system, is associated to an aquifer located 6000 meters under the surface of the field. For this reason it contains brine and hydrocarbons. The water in this reservoir flows vertically toward the production wells, through conductive faults, which connect the oil zone with the deep aquifer.

The water of the aquifer is geothermal brine at 150°C, having a density of 1150 kg/m^3. Port Ceiba's wells are oil producers, but some of them are invaded by brine, producing a variable mixture of water and oil. Although the oil is extracted at the upper zone of the oil- water contact (COW), the total volume of produced water equals or exceeds the oil production. The effect of water invasion, together with oil extraction, produces a gradual decrease of the original volume of oil and a vertical displacement of the COW. In this way the well receives more and more water until it becomes completely invaded. The handling of this water in the formation is a serious practical problem costing millions of pesos to the company every year.

The main goal of this research is the understanding of the water invasion mechanism and the estimation of the critical volumetric rate in oil wells for which the invasion begins to happen. The model should allow predicting with precision this critical rate and, consequently, to be able to reduce the extraction rates in wells just on time, maximizing its productive life. In this work a numerical original model is developed, able to perform this task.

GENERAL DESCRIPTION OF THE PROBLEM

Hypothesis and Qualitative Information Available

The brine in the formation has different physical behavior compared to hydrocarbons. Water conducts as a substance having a molecular weight larger than 18. This behavior is due to the fact that intra-molecular forces of water are more intense than those of petroleum (Pedersen and Christensen, 2006). Due to superficial tensions, a great amount of oil is caught into the pores, in such a way that the mobility of the invasion water prevails. For heavier and more viscous oils, the mobility of water will dominate in the immiscible mixture of both fluids.

This phenomenon is described by the total mixture rate $q = q_w + q_o$ and by the quotient $q_w / q_o = \lambda_w / \lambda_o > 1$; $\lambda_j = \kappa_j / \mu_j$ is the phase mobility, κ_j its permeability and μ_j its dynamic viscosity (j = water, oil). If the volumetric rate of the well is very high, the produced fluid would be predominantly water. We call $B_w = \rho_{wS} / \rho_{wR}$ the volume factor of water in the formation (density of water at standard conditions divided by density of water at reservoir conditions). This factor represents the expansion of the volume of water between the formation and the surface of the field. Assuming this expansion small, we will take the value $B_w \approx 1$. The following information is available:

- Geothermal water invades the oil reservoir through a fault that penetrates an aquifer at 6000 m of depth and 150°C of temperature.
- The geothermal aquifer and the oil reservoir form a geologic unit system, delimited at their boundaries by impermeable rocks forming a profound closed and isothermal volume.
- Water flows from the deep aquifer to the reservoir because of pressure variations at the COW.
- Darcy's Law and Continuity equation are valid in both phases.
- Relative permeabilities and capillary pressure only depend on saturations.
- The following parameters are constants: Rock permeability, viscosities and densities of both phases.

FIELD DATA

Available numerical data are summarized in Table 1 (Suarez & Samaniego, 2006).

Average pressure	$p_a = 940$ *kg/cm*^{2}
Bottom flowing pressure	$p_{wf} = 700$ *kg/cm*^{2}
Volumetric rate	$q_0 = 11000\ B_{ce}/D$
Oil Density	$\rho_o = 770$ *kg/m*^{3}
Water Density	$\rho_w = 1145$ *kg/m*^{3}
Pressure difference Δp_w	$p_a - p_{wf} = 240$ *kg/cm*^{2}
Vertical distance between The well and the COW	$\Delta H = 375$ m.
Temperature of brine	150°C
Capillary pressure	$P_c(S_w) = p_o - p_w$
Saturations	$S_w + S_o = 1$

Table 1.- Numerical information from well PC-115 of the Puerto Ceiba Reservoir (PEMEX – PEP, 2004).

A fundamental formula relating capillary pressure and capillary height is:

$$P_c(S_w) = h_c \Delta\rho g\ ,\ \Delta\rho = \rho_w - \rho_o\ ,\ g = 9.8\ m/s^2 \qquad (1)$$

Where h_c is the height over the plane of capillary pressure $p_c = 0$. This surface is the boundary of the oil-water contact (COW) where $S_w \sim 1$, $S_o \sim 0$. The transition area is the place where both phases coexist. The residual saturation of water S_{wi} is reached at the point of the reservoir where $S_o \sim 1$.

Relative Permeabilities and Capillary pressure

For the capillary pressure the experimental values reported by Aziz (1999) were used, together with equation (1). The relative permeabilities for water and oil we used are the correlations proposed by Brooks and Corey in 1964 and verified experimentally in a recent publication (Cunha *et al.*, 1999). The analytic expression of these correlations are as follows:

$$k_{rw}(S_w) = 0.0525\left(\frac{S_w - 0.363}{0.326}\right)^{2.714}; \qquad (2)$$
$$k_{ro}(S_w) = 1.3180\left(\frac{0.689 - S_w}{0.326}\right)^{1.193}$$

The numbers inside the parentheses and the exponents were measured experimentally, while the values outside the parentheses were obtained by least squares fitting.

A GENERAL 3D MODEL

In this problem we considered the simultaneous flow of two immiscible fluids, oil and water, in a porous medium in three dimensions. For the deduction of the final differential equation we assumed that:

- There is no transfer of mass between fluids.
- The law of Darcy is applicable.
- The system is hot but isothermal.
- Rock permeability *K*, densities ρ_w, ρ_o and viscosities μ_w, μ_o, are constants.

Tridimensional Flow of Oil and Water

Using a traditional Cartesian reference system, the immiscible flow of oil and water takes place in a plane formed by an inclined fault, forming an angle θ, between 0 and $\pi/2$ radians, with the vertical direction. The fluids enter the fault from the formation and from the aquifer at initial constant velocity. For practical reasons and to simplify the writing of equations we define the following variables:

$$\begin{aligned} \vec{v}_w &= \vec{w} && \textit{velocity of water.} \\ \vec{v}_o &= \vec{v} && \textit{velocity of oil.} \\ \vec{u} &= \vec{w} + \vec{v} && \textit{velocity of both phases.} \end{aligned} \qquad (3)$$

$$\textit{that means: } \vec{u} = \begin{pmatrix} u_x \\ u_y \\ u_z \end{pmatrix} = \begin{pmatrix} w_x + v_x \\ w_y + v_y \\ w_z + v_z \end{pmatrix}$$

Using the continuity equations for both phases:

$$\rho_w \vec{\nabla}\cdot\vec{w} + \varphi\rho_w \frac{\partial S_w}{\partial t} = 0;\ \rho_o \vec{\nabla}\cdot\vec{v} + \varphi\rho_o \frac{\partial S_o}{\partial t} = 0$$
$$\Rightarrow \quad \vec{\nabla}\cdot\vec{w} + \varphi\frac{\partial S_w}{\partial t} = 0;\ \vec{\nabla}\cdot\vec{v} + \varphi\frac{\partial S_o}{\partial t} = 0 \qquad (4)$$
$$\Rightarrow \quad \vec{\nabla}\cdot(\vec{w}+\vec{v}) + \varphi\frac{\partial}{\partial t}(S_w + S_o) = 0$$
$$\Rightarrow \ \vec{\nabla}\cdot(\vec{w}+\vec{v}) = \vec{\nabla}\cdot\vec{u} = 0\ ;\ \ (\textit{because}: S_w + S_o = 1)$$
$$\Rightarrow \quad \vec{u}(t_0) = \vec{u}_0 = \vec{u} \ \Rightarrow\ \vec{u} = \text{constant}$$

The last equation results from both, the continuity and the fact that the initial total velocity is constant. We assume that the rock permeability tensor is the constant matrix:

$$\mathbf{K} = \begin{pmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{pmatrix} \qquad (5)$$

The Darcy's Law for each phase is:

$$\vec{w} = -\frac{k_{rw}}{\mu_w}\mathbf{K}\left(\vec{\nabla}p_w - \rho_w\vec{g}\right) = -\lambda_w \begin{pmatrix} k_x\,\partial_x p_w \\ k_y\,\partial_y p_w \\ k_z\dfrac{\partial p_w}{\partial z} - k_z\rho_w g\,Cos\,\theta \end{pmatrix} = \begin{pmatrix} w_x \\ w_y \\ w_z \end{pmatrix}$$

$$\vec{v} = -\frac{k_{ro}}{\mu_o}\mathbf{K}\left(\vec{\nabla}p_o - \rho_o\vec{g}\right) = -\lambda_o \begin{pmatrix} k_x\,\partial_x p_o \\ k_y\,\partial_y p_o \\ k_z\dfrac{\partial p_o}{\partial z} - k_z\rho_o g\,Cos\,\theta \end{pmatrix} = \begin{pmatrix} v_x \\ v_y \\ v_z \end{pmatrix}$$

$$\textit{the mobilities are: } \lambda_w = \frac{k_{rw}}{\mu_w};\ \lambda_o = \frac{k_{ro}}{\mu_o};\ \frac{1}{\lambda} = \frac{1}{\lambda_w} + \frac{1}{\lambda_o} \qquad (6)$$

From these vectorial equations pressure gradients are deduced:

$$\vec{\nabla}p_w = \begin{pmatrix} \dfrac{\partial p_w}{\partial x} \\ \dfrac{\partial p_w}{\partial y} \\ \dfrac{\partial p_w}{\partial z} \end{pmatrix} = \begin{pmatrix} -\dfrac{w_x}{\lambda_w k_x} \\ -\dfrac{w_y}{\lambda_w k_y} \\ \dfrac{-w_z}{\lambda_w k_z} + \rho_w g\,Cos\,\theta \end{pmatrix}; \qquad (7)$$

$$\vec{\nabla}p_o = \begin{pmatrix} \dfrac{\partial p_o}{\partial x} \\ \dfrac{\partial p_o}{\partial y} \\ \dfrac{\partial p_o}{\partial z} \end{pmatrix} = \begin{pmatrix} -\dfrac{v_x}{\lambda_o k_x} \\ -\dfrac{v_y}{\lambda_o k_y} \\ \dfrac{-v_z}{\lambda_o k_z} + \rho_o g\,Cos\,\theta \end{pmatrix}$$

Next, we define the fractional velocities as follows:

$$u_x = w_x + v_x \ \Rightarrow\ 1 = \frac{w_x}{u_x} + \frac{v_x}{u_x} = f_x + g_x$$
$$u_y = w_y + v_y \ \Rightarrow\ 1 = \frac{w_y}{u_y} + \frac{v_y}{u_y} = f_y + g_y \qquad (8)$$
$$u_z = w_z + v_z \ \Rightarrow\ 1 = \frac{w_z}{u_z} + \frac{v_z}{u_z} = f_z + g_z$$
$$or: \quad w_x = u_x f_x;\ w_y = u_y f_y;\ w_z = u_z f_z$$

The capillary pressure gradient is the matrix:

$$\vec{\nabla} p_C = \vec{\nabla} p_o - \vec{\nabla} p_w \qquad \Rightarrow$$

$$\vec{\nabla} p_C = \begin{pmatrix} \dfrac{\partial p_C}{\partial x} \\ \dfrac{\partial p_C}{\partial y} \\ \dfrac{\partial p_C}{\partial z} \end{pmatrix} = \begin{pmatrix} \dfrac{u_x f_x}{\lambda k_x} - \dfrac{u_x}{\lambda_o k_x} \\ \dfrac{u_y f_y}{\lambda k_y} - \dfrac{u_y}{\lambda_o k_y} \\ \dfrac{u_z f_z}{\lambda k_z} - \dfrac{u_z}{\lambda_o k_z} + (\rho_o - \rho_w) g \, Cos\,\theta \end{pmatrix} \qquad (9)$$

After doing some algebraic manipulations:

$$f_x = \frac{\lambda k_x}{u_x}\left[\frac{d\, p_c}{d\, S_w}\frac{\partial S_w}{\partial x} + \frac{u_x}{\lambda_o k_x}\right] = \underbrace{\frac{\lambda k_x}{u_x}\frac{d\, p_c}{d\, S_w}}_{\leftarrow F_x \rightarrow}\frac{\partial S_w}{\partial x} + \frac{\lambda}{\lambda_o}$$

$$f_y = \frac{\lambda k_y}{u_y}\left[\frac{d\, p_c}{d\, S_w}\frac{\partial S_w}{\partial y} + \frac{u_y}{\lambda_o k_y}\right] = \underbrace{\frac{\lambda k_y}{u_y}\frac{d\, p_c}{d\, S_w}}_{\leftarrow F_y \rightarrow}\frac{\partial S_w}{\partial y} + \frac{\lambda}{\lambda_o} \qquad (10)$$

$$f_z = \frac{\lambda k_z}{u_z}\left[\frac{d\, p_c}{d\, S_w}\frac{\partial S_w}{\partial z} + \frac{u_z}{\lambda_o k_z} + (\rho_w - \rho_o)\, g\, Cos\,\theta\right] =$$

$$= \underbrace{\frac{\lambda k_z}{u_z}\frac{d\, p_c}{d\, S_w}}_{\leftarrow F_z \rightarrow}\frac{\partial S_w}{\partial z} + \underbrace{\frac{\lambda k_z}{u_z}(\rho_w - \rho_o)\, g\, Cos\,\theta}_{\leftarrow \quad G_z \quad \rightarrow} + \frac{\lambda}{\lambda_o}$$

$$and: \; \frac{\lambda}{\lambda_o} = \frac{\lambda_w}{\lambda_w + \lambda_o} = \frac{1}{1 + \dfrac{\lambda_o}{\lambda_w}} = \left(1 + \frac{k_{ro}}{k_{rw}}\frac{\mu_w}{\mu_o}\right)^{-1}$$

We define the following auxiliary functions:

$$F_x = \frac{\lambda k_x}{u_x}\frac{d\, p_c}{d\, S_w}; \; F_y = \frac{\lambda k_y}{u_y}\frac{d\, p_c}{d\, S_w}; \qquad (11)$$

$$F_z = \frac{\lambda k_z}{u_z}\frac{d\, p_c}{d\, S_w}; \; G_z = \frac{\lambda k_z}{u_z}(\rho_w - \rho_o)\, g\, Cos\,\theta$$

The fractional velocities can be expressed as:

$$f_x = F_x \frac{\partial S_w}{\partial x} + \frac{\lambda}{\lambda_o}; \; f_y = F_y \frac{\partial S_w}{\partial y} + \frac{\lambda}{\lambda_o}; \qquad (12)$$

$$f_z = F_z \frac{\partial S_w}{\partial z} + G_z + \frac{\lambda}{\lambda_o}$$

Using these components in the equation of continuity for water (4), we deduce a final partial differential equation for water saturation:

$$\varphi \frac{\partial S_w}{\partial t} + \vec{\nabla} \cdot (\mathbf{F} \cdot \vec{\nabla} S_w) + \vec{G} \cdot \vec{\nabla} S_w = 0 \qquad (13)$$

$$S_w = S_w(x, y, z, t)$$

The auxiliary coefficients are of tensorial nature and are expressed as follows:

$$\mathbf{F} = \begin{pmatrix} u_x F_x & 0 & 0 \\ 0 & u_y F_y & 0 \\ 0 & 0 & u_z F_z \end{pmatrix};$$

$$\vec{G} = \begin{pmatrix} u_x \dfrac{\partial}{\partial S_w}\left(\dfrac{\lambda}{\lambda_o}\right) \\ u_y \dfrac{\partial}{\partial S_w}\left(\dfrac{\lambda}{\lambda_o}\right) \\ u_z \dfrac{\partial}{\partial S_w}\left(\dfrac{\lambda}{\lambda_o}\right) + u_z \dfrac{\partial G_z}{\partial S_w} \end{pmatrix} \qquad (14)$$

$\mathbf{F}$ *is the tensor of mobility of both phases.*

$\vec{G}$ *is the vector of relative mobility of both phases.*

The 1D Buckley-Leverett model

Using the functions definided in (14) and the general equation (13) in one dimension, let's say the vertical direction OZ, we obtain by direct substitution:

$$\varphi \frac{\partial S_w}{\partial t} + K \frac{\partial}{\partial z}\left(\frac{k_{rw}\, k_{ro}}{k_{rw}\,\mu_o + k_{ro}\,\mu_w}\right)\frac{\partial p_c}{\partial z} + u_z \frac{\partial}{\partial z}\left(\frac{k_{rw}}{k_{rw} + k_{ro}\dfrac{\mu_w}{\mu_o}}\right) +$$

$$+ K \frac{\partial}{\partial z}\left(\frac{k_{rw}\, k_{ro}}{k_{rw}\,\mu_o + k_{ro}\,\mu_w}\right)(\rho_w - \rho_o)\, g \; Cos\,\theta = 0$$

$$S_w = S_w(z, t) \qquad (15)$$

This equation is a some heavier form of the classic Buckely – Leverett equation with non-stationary solutions for the displacement of oil by water, when the action of capillary forces is neglected (see for example Marle, 1981).

Formulation and Numerical Solution using the Finite Element Method

The model represented by equation (13) is a non linear one, but it only depends on water saturation. We can formulate this model using finite elements (Zienkiewicz and Taylor, 1991; Reddy and Gartling, 2001). Multiplying both sides of equation (13) by a test function w_j defined over $\Omega \subset \mathbb{R}^3$, the solution domain and integrating:

$$\int_\Omega w_j\, \varphi \frac{\partial S_w}{\partial t} d\Omega = -\int_\Omega w_j\, \vec{\nabla} \cdot (\mathbf{F} \cdot \vec{\nabla} S_w)\, d\Omega - \int_\Omega w_j\, \vec{G} \cdot \vec{\nabla} S_w\, d\Omega \qquad (16)$$

Application of the general Green Theorem to eq. (16) and the definition of an interpolation function of the form: $S_w(x,y,z,t) \approx \sum_{i=1}^{N} s_i(t)\, w_i(x,y,z)$ allows the construction of the approximate ordinary differential system:

$$\begin{aligned} &\mathbf{M}\cdot\frac{d\vec{S}}{dt}+\mathbf{K}\cdot\vec{S} = \vec{Q} \\ &where: \\ &M_{ij}=\varphi\left\langle w_i, w_j\right\rangle,\ Q_j=\oint_{\Gamma}\mathbf{F}\cdot\vec{\nabla}S_w\cdot\vec{n}\,d\Gamma+\left\langle q_\alpha, w_j\right\rangle \\ &K_{ij}=\left\langle \vec{G}\cdot\vec{\nabla}w_i, w_j\right\rangle+\left\langle \mathbf{F}\cdot\vec{\nabla}w_i, \vec{\nabla}w_j\right\rangle \end{aligned} \tag{17}$$

After solving this system by classical techniques (Reddy and Gartling, 2001), we obtain an approximate numerical solution of model (13).

2D NUMERICAL SIMULATION OF BRINE INVASION

We proceed to make numeric simulations in two dimensions of an extraction oil zone presenting brine invasion. We used data from well PC-115 (table 1) located in Port Ceiba reservoir (Fig. 7). We assume the fault as a porous medium of very high permeability (~10^3 Darcy), located at the center of the production region. The lateral boundaries are at constant pressure and zero flow. The changes only occur in the vertical direction. To represent in a simple form the geometry of the area of interest, of dimensions (6000 m) x (800 m), a non-structured mesh (Fig. 1) of 552 triangular elements and 2640 grades of freedom was constructed. The interpolants are quadratic Lagrange polynomials.

CONCLUSIONS

The zone is subjected to several production rates in the interval (3000, 8000) B_{ce}/Day until the brine invasion is observed. Finally we could obtain a critical constant volumetric rate equal to 7057 B_{ce}/D for which the brine invasion is massive. Figure 2 shows the evolution of oil saturation in some most illustrative points. The coordinates of the point are indicated in the figure. Figures 3, 4, 5 and 6 are two-dimensional surfaces of the simulated area, illustrating the evolution of both distributions S_o and S_w in the whole region. For the critical oil rate of 7057 B_{ce}/D, an abrupt oil depression of 80% is obtained in a lapse of 6 days (Figure 2), after 52 days of continuous production.

The presence of water in hydrocarbon reservoirs is a practical serious problem. The estimate of the current cost of brine handling oscillates between 5 and 50 cents of dollar per barrel of water and it ascends to $4 US per barrel of petroleum in wells producing oil with 80% of water content. The worldwide cost of managing this water is about $40000 million dollars. We developed a 3D mathematical model to simulate the displacement of both phases. The model allows representing the brine invasion in wells producing oil. We presented applications with data of a well in the Gulf of Mexico

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FIGURES

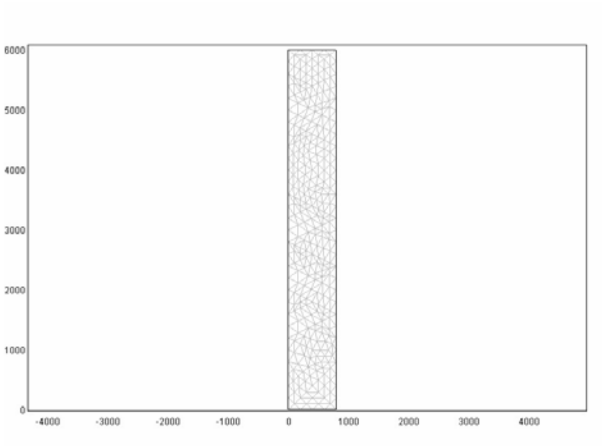

Figure. 1. Non structured mesh of the simulated porous medium, with 552 finite elements.

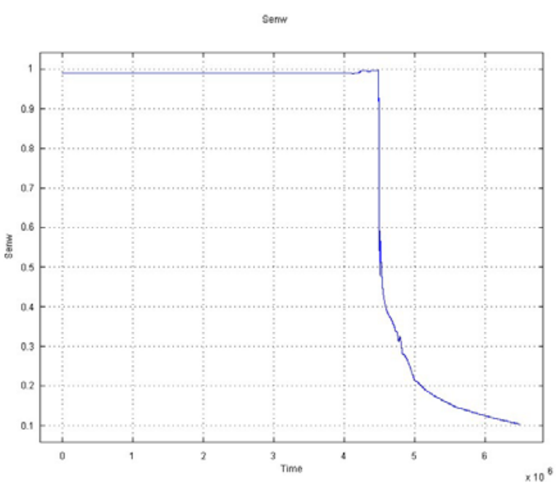

Figure. 2. Oil saturation affected by brine invasion at the point (150, 5500) m, after 52 days of production.

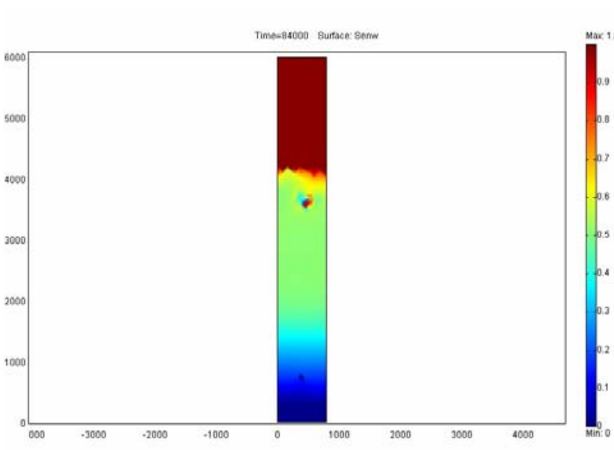

Figure. 3. Oil and brine saturation 23.3 hours after a transition zone (green) was established.

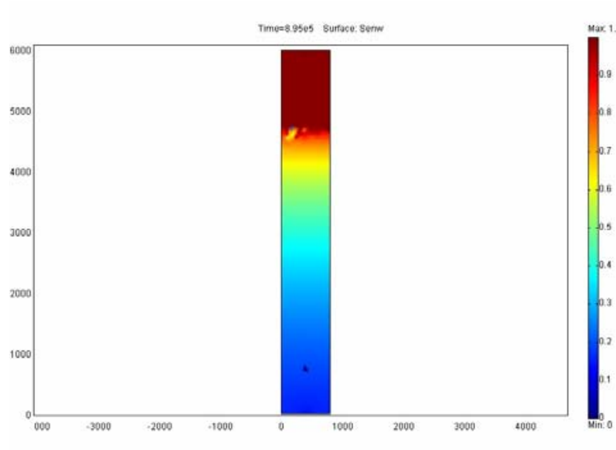

Figure. 4. Oil and brine saturation 10.4 days after the transitional zone was established.

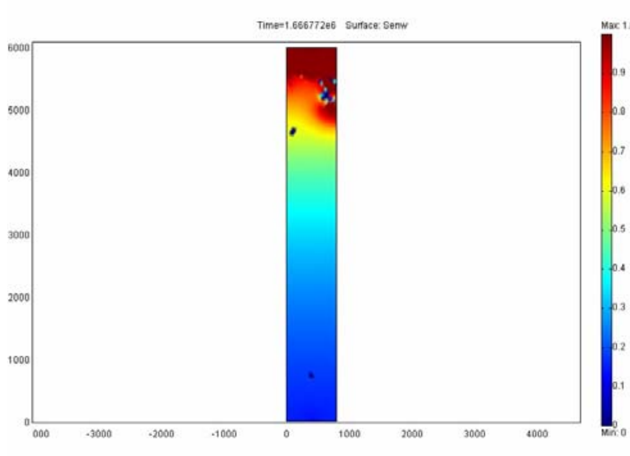

Figure. 5. Oil and brine saturation 20 days after the transitional zone was established.

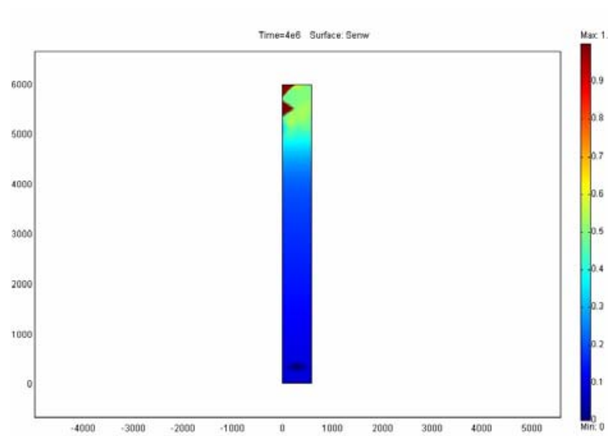

Figure. 6. Oil and brine saturation 46.3 days after the transitional zone was established.

Figure. 7. Geographical location of the Bellota – Jujo system.