

SORBING TRACERS - A POTENTIAL TOOL FOR DETERMINING EFFECTIVE HEAT TRANSFER AREA IN HOT FRACTURED ROCK SYSTEMS

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ABSTRACT

A key parameter controlling the performance and lifetime of a Hot Fractured Rock (HFR) reservoir is the effective heat transfer area between the fracture network and the matrix rock. In a vapor-dominated system, this area can be estimated by conducting a tracer test of non-adsorptive chemical, based on the unique signature of a long tail in a typical tracer breakthrough curve (BTC) due to diffusion of tracers between fractures and matrix rocks. The tailing strength increases systematically with the fracture-matrix interface area. In a water-dominated system, however, aqueous phase diffusivities are too small to generate a meaningful tail in the BTC of an inert solute tracer. Recent numerical studies have shown that reversible tracer sorption on matrix rocks will enhance tailing in BTCs for aqueous solutes in fracture-matrix systems. In this paper we develop an analytical solution to theoretically explore such a useful phenomenon. In deriving the solution, we used a boundary condition of a finite-length tracer slug, and neglected the diffusion along fractures. The solution shows that in a water saturated fractured rock system, increase of the retardation factor (that is practically manageable) should have the same effect on BTCs as increase of the diffusion coefficient (that is practically restricted). The strong enhancement in the BTC tails of sorbing tracers provides adequate sensitivity for determining the heat transfer area. The solution is useful for understanding transport mechanisms, verifying numerical codes, and for identifying chemicals with appropriate sorption properties as tracers for the characterization of a fractured reservoir.

INTRODUCTION

Fluid flow and chemical transport in fractured porous media has been a research topic for decades. The topic is important because many geologic formations are fractured to some extent. The topic is also difficult because fracture networks can have quite different geometry (such as the direction, aperture, and density of fracture sets, and the number of

fracture sets), which has significant impact on flow and transport processes. Thus, it is crucial to obtain a clear geometric picture of a fractured reservoir before developing it for energy extraction. For example, in the design and operation of hot fractured rock (HFR) reservoirs, it is very important to estimate the heat transfer area between the fracture network and the matrix rock. An effective way to obtain this geometric information is to conduct an appropriate tracer test. Extensive studies on tracer transport in fractured porous media have been conducted in the context of nuclear and chemical waste disposal (e.g., Moreno, et al., 1996; Polak, et al., 2003). Based on these studies, mathematical models have been developed for analyzing tracer test data. Since a naturally fractured reservoir is usually very complex, an appropriate numerical code is usually needed.

However, numerical codes must be verified against analytical solutions before application to practical problems. Analytical solutions for contaminant transport in fractured porous media were available as early as the early 1980s (Tang et al., 1981; Sudicky and Frind, 1982). The former paper is for a single fracture where the matrix is assumed to extend to infinity away from the fracture. The later paper is for the case of a set of parallel fractures with uniform fracture spacing and identical concentration at the entrance of the fractures. In both solutions the authors assumed a constant concentration at the fracture entrance. In a tracer test, however, this assumption is usually invalid. Instead, we may assume a slug-like flux boundary condition at the fracture entrance. More recently, Moridis (2002) developed semianalytical solutions for radioactive or reactive solute transport in variably fractured layered media. Here we develop a solution for a slug tracer test in a fractured rock under single-phase flow conditions, using some simplifying assumptions.

THEORY

To simplify the problem, we assume that the system has a single set of plane, parallel fractures with a uniform fracture spacing, $2B$ [L], and a uniform

fracture aperture, $2b$ [L]. We also assume that the tracer injection is uniformly distributed at each fracture entrance. Tracer concentration applied at the fracture entrance is denoted by C_0 [M/L³]. Taking advantage of the symmetry, we can restrict the solution to an elementary part of the system (one-half of a fracture and one-half of its adjacent matrix block) as shown in Figure 1. The z -axis is in the direction of the fracture, while the x -axis is perpendicular to the interface, pointing away from the fracture (Figure 1). The fracture is thus in the domain $-b \leq x \leq 0$ and $0 \leq z < \infty$; and the matrix is in the domain $0 \leq x \leq B$ and $0 \leq z < \infty$. The solid lines in Figure 1 represent zero-mass-flux boundaries.

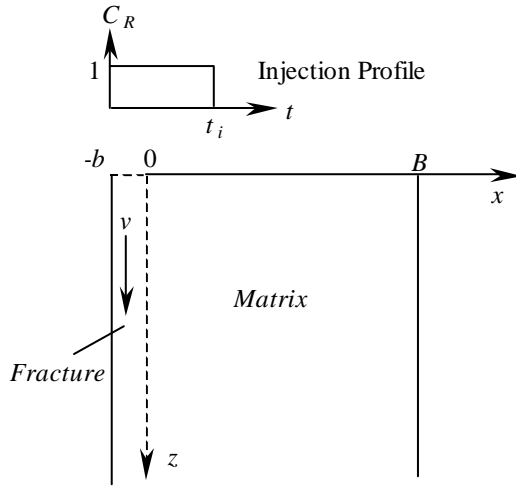


Figure 1. Schematic section of an elementary part of the parallel-fracture matrix system.

The aperture is assumed much smaller than the length of the fracture, and transport in the fracture is assumed one-dimensional along the fracture. The diffusive mass flux across the fracture-matrix interface is treated as a sink term in the mass conservation equation for the fracture. We neglect any advective flow across the interface. By this assumption, there is no advection in the matrix; and mass transport in the matrix is only through diffusion perpendicular to the fracture-matrix interface. We ignore diffusion and adsorption inside the fracture, as well as any tracer decay. Reversible sorption in the matrix is accounted for by a retardation factor. Based on these assumptions, we use the following governing equations simplified from Sudicky and Frind (1982):

$$\frac{\partial C_R}{\partial t} + v \frac{\partial C_R}{\partial z} + \frac{q}{b\phi C_0} = 0 \quad (z \geq 0) \quad (1a)$$

$$\frac{\partial C_R'}{\partial t} - \frac{D'}{R'} \cdot \frac{\partial^2 C_R'}{\partial x^2} = 0 \quad (0 \leq x \leq B) \quad (1b)$$

where C_R and C_R' (both dimensionless) are the relative solute concentrations in the fracture and the matrix, respectively, which are defined by:

$$C_R = C / C_0; \quad C_R' = C' / C_0 \quad (1c)$$

Here C [M/L³] and C' [M/L³] represent the chemical concentrations in the fracture and matrix, respectively.

In (1a) and (1b), t [T] is time, v [L/T] the groundwater velocity in fracture, q [M/L²/T] the diffusive mass flux across the fracture-matrix interface, ϕ the intrinsic porosity of the fracture, D' [L²/T] and R' the diffusion coefficient and retardation factor of the matrix, respectively. Both C_R and q are functions of z and t , i.e. $C_R(z, t)$ and $q(z, t)$. The relative concentration in the matrix, C_R' , depends on the migration distance in the fracture, z , as a parameter through the interface boundary conditions; in (1b) it is an explicit function of x and t only.

The following formulae are provided for estimating some parameters:

$$D' = \tau D^* \quad (2a)$$

$$R' = 1 + K_m \rho_b / \phi' \quad (2b)$$

where τ is the tortuosity of the matrix; D^* [L²/T] is the chemical molecular diffusion coefficient in water, typically of order 10⁻⁹ m²/s; and K_m [L³/M] is the matrix distribution coefficient (Freeze and Cherry, 1979). In (2b) ρ_b [M/L³] and ϕ' are the bulk mass density and the porosity of the matrix, respectively.

We assume an initially solute-free condition, i.e., the chemical concentrations in the fracture and matrix are both zero at the beginning:

$$C_R(z, 0) = C_R'(x, 0) = 0 \quad (3)$$

Setting $t = 0$ at the start time of tracer injection, and assuming an injection period t_i [T], the boundary conditions for the fracture are:

$$C_R(0, t) = 1 \quad (0 < t < t_i); \quad C_R(0, t) = 0 \quad (t > t_i) \quad (4a)$$

$$C_R(\infty, t) = 0 \quad (4b)$$

One boundary condition for the matrix is:

$$\left(\frac{\partial C_R'}{\partial x} \right)_{x=B} = 0 \quad (5)$$

At the fracture-matrix interface, we must have:

$$C_R(z, t) = C_R'(0, t) \quad (6a)$$

$$q = -\phi' D' C_0 \left(\frac{\partial C_R'}{\partial x} \right)_{x=0} \quad (6b)$$

Sudicky and Frind (1982) gave a general solution for the above problem under a constant boundary concentration that is different from (4a). We obtained an analytical solution for tracer concentration in the fracture for the case of a finite slug injection as (Shan and Pruess, to be published):

$$C_R = \begin{cases} 0 & (t \leq t_T) \\ \frac{2}{\pi} \int_0^\infty \frac{e^{\varepsilon_R} \varepsilon_A}{\varepsilon} d\varepsilon & (t > t_T) \end{cases} \quad (7)$$

where ε [$T^{-1/2}$] is the integration variable, ε_R and ε_A are given by:

$$\varepsilon_R = -\frac{t_T \varepsilon}{2\sqrt{t_F}} g_2(\sqrt{t_M} \varepsilon) \quad (8a)$$

$$\varepsilon_A = \sin \left[\frac{\varepsilon^2}{2} (t_s - t + t_T) + \frac{t_T \varepsilon}{2\sqrt{t_F}} \cdot g_1(\sqrt{t_M} \varepsilon) \right] + \sin \left[\frac{\varepsilon^2}{2} (t - t_T) - \frac{t_T \varepsilon}{2\sqrt{t_F}} \cdot g_1(\sqrt{t_M} \varepsilon) \right] \quad (8b)$$

The four time parameters, t_T , t_F , t_M , and t_s are defined by:

$$t_T = z/v \quad (9a)$$

$$t_F = \frac{\phi^2 b^2}{\phi'^2 D' R'} \quad (9b)$$

$$t_M = B^2 R' / D' \quad (9c)$$

$$t_s = \begin{cases} t - t_T & (t < t_i + t_T) \\ t_i & (t \geq t_i + t_T) \end{cases} \quad (9d)$$

The two functions, g_1 and g_2 , are defined by:

$$g_1(u) = \frac{\sinh(u) + \sin(u)}{\cosh(u) + \cos(u)} \quad (9e)$$

$$g_2(u) = \frac{\sinh(u) - \sin(u)}{\cosh(u) + \cos(u)} \quad (9f)$$

RESULTS

There are four characteristic times that determine the relative concentration, C_R . Among them, the injection period, t_i is a known parameter; t_T , t_F , and t_M are three parameters representing the properties of the fractured formation. According to (9a), t_T is the tracer transit (or travel) time to the observation point. Therefore, (7) indicates that the tracer concentration

at a specified calculation point will remain at its initial value (zero) before the transit time has elapsed. This is because, by assumption, tracer transport in the fracture is through advection only. At a given point, the shape of the BTC is thus affected by two other characteristic times, t_F and t_M . According to (9b) and (9c), t_F and t_M can be thought of as two characteristic times for crossing the interface and the matrix block, respectively.

The Role of t_M

By definition (9c), t_M is the product of B squared and the ratio R'/D' , both factors originating from (1b), the governing equation for the matrix. Thus t_M affects the tracer concentration distribution in the matrix, but does not affect the tracer concentrations in the fracture in most practical applications. Physically, t_M represents a time to reach the interior no-flow boundary of the matrix through diffusion; the longer the time, the later the boundary effect will come into play. Mathematically, t_M affects the concentration in the fracture only through the two functions, $g_1(u)$ and $g_2(u)$ that are essentially a constant of unity for $u > 6$ (see Figure 2).

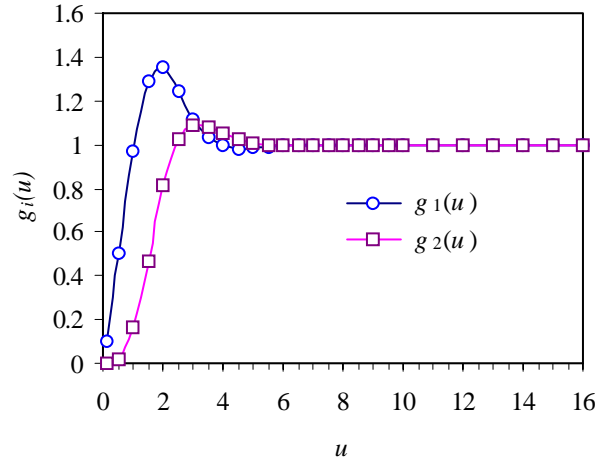


Figure 2. The functions, $g_1(u)$ and $g_2(u)$.

In other words, t_M affects the solution only in the interval $0 < \varepsilon < \varepsilon_0$ where

$$\varepsilon_0 = 6/\sqrt{t_M} = \frac{6}{B} \sqrt{\frac{D'}{R'}} \quad (10)$$

For a matrix block size of meters or larger, since $R' \geq 1$ and D' is usually less than 10^{-9} m²/s, the resulting value of ε_0 is usually on the order of or less than 10^{-4} s^{-1/2}. The integral in (7) usually has negligible contribution in the interval 0 to 10^{-4} . We used the input parameters in Table 1 to calculate the BTCs at two different locations (two different values of t_T).

Table 1. Base parameters used in Figures 3a, 3b and 4.

b (m)	t_i (s)	v (m/s)	D' (m^2/s)	ϕ	ϕ'	R'
0.005	28800	0.001	10^{-10}	1.0	0.05	1.0

At each location, we calculate the BTCs for three different values of fracture spacing ($2B$), keeping all other parameters fixed. Figures 3a and 3b show the comparisons of the calculated BTCs at a near location ($t_T = 0.5 t_i$) and a far location ($t_T = 10 t_i$), respectively. Both figures indicate that the effect of B (or t_M) is insignificant. At the near location ($t_T = 0.5 t_i$, Figure 3a), a smaller B does cause a slight increase in concentration due to the boundary effect. However, the transit time for a practical observation point is usually much larger than the injection period, t_i , which will not show such a difference.

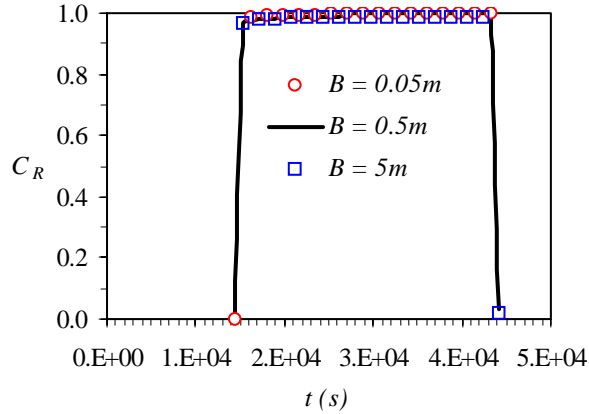


Figure 3a. Breakthrough curves for three different B values at $t_T = 0.5 t_i$.

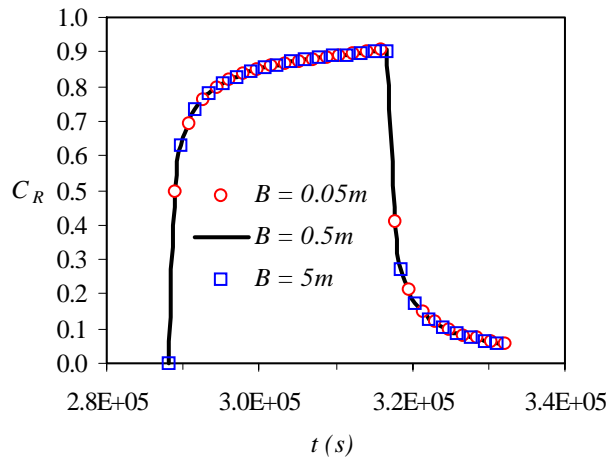


Figure 3b. Breakthrough curves for three different B values at $t_T = 10 t_i$.

In Figures 3a and 3b, the smallest B value is 5 cm. Although further decrease of B can cause larger differences in the BTC, very small B values are not of interest in practical applications. We should also remember the assumption of $B \gg b$ in deriving the solution.

The Role of t_F

In practice, the only parameter, apart from the tracer transit time t_T , which affects a fracture BTC is t_F . According to (9b), t_F depends on the matrix diffusion coefficient (D') and the retardation factor (R') only through the product $D'R'$. Therefore the impact of matrix retardation on tracer concentrations in the fracture is the same as that of an increased matrix diffusion coefficient. This equivalence of matrix retardation factor to diffusion coefficient is demonstrated in Figure 4, where the BTCs at $t_T = 50 t_i$ are calculated using three different pairs of D' and R' with a constant product ($10^{-10} m^2/s$).

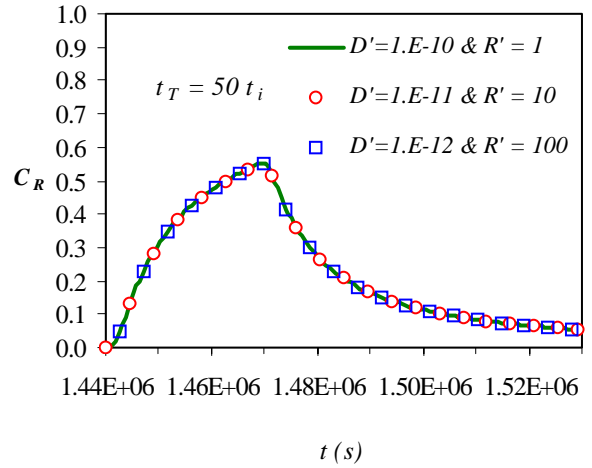


Figure 4. Breakthrough curves for three different combinations of D' (m^2/s) and R' .

The three calculated BTC curves in Figure 4 are identical because we maintained t_F a constant (10^8 s). Any reduction of t_F value implies a faster passage for tracer to enter the matrix, and thus causes a decrease of tracer concentration in the fracture during tracer injection but an increase of tracer concentration in the fracture after tracer injection.

In Figure 5, we used constant values for t_i (10^4 s), t_T (10^6 s), and t_M (10^9 s) but varied t_F (10^9 s, 10^8 s, and 10^7 s). The decrease of peak and the increase of tail of the BTC curve are both significant. For relatively large t_F (e.g., 10^9 s, or even 10^8 s), the BTC peak appears approximately at the time $t = t_T + t_i$. However, as t_F is further decreased to 10^7 s, the BTC peak appears at a time that is significantly larger than $t_T + t_i$ (Figure 5).

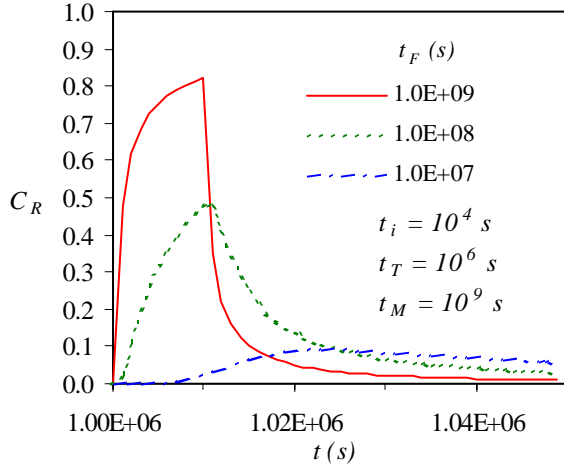


Figure 5. The effect of t_F on breakthrough curves.

Verification

TOUGH2 (Pruess, et al., 1999) is a numerical code for multiphase, multi-component flow, transport, and heat transfer problems. This code has been verified against many analytical solutions. Here we demonstrate the verification of TOUGH2 against the analytical solution (7). By using the same set of input parameters, we calculated the BTC curve at the same point in the fracture using TOUGH2 and (7). We compare the results in Figure 6.

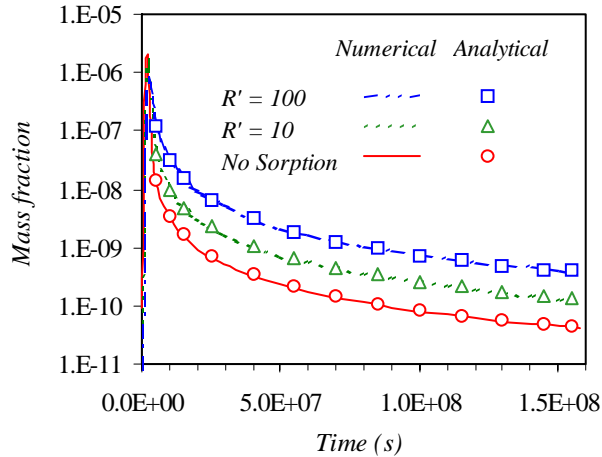


Figure 6. Comparison of the analytical solution with the numerical (TOUGH2) solution.

For three different retardation factors, the corresponding numerical and analytical solutions agree very well. Note that TOUGH2 uses “mass fraction” instead of relative concentration. The results of the analytical solution were converted to mass fraction for the comparison.

Application

Although analytical solutions are usually too idealized for actual field cases, here we offer some guidelines for the potential application to a simplified field condition, such as a one-dimensional tracer test in a horizontal fractured formation. The main purpose of application is to inversely estimate the average fracture spacing, $2B$, and from that the fracture-matrix interfacial (heat transfer) area per unit volume.

The injection period, t_i , is known; and the breakthrough time (the tracer transit time), t_T , is observed in the tracer test. A BTC can be calculated by applying the known t_i and t_T , and an assumed t_F and t_M (e.g., 10^9 s) to (7). Varying t_F , we obtain a set of BTCs. We then estimate t_F by fitting the observation data to the calculated BTCs. After t_F is estimated, we can calculate B by rewriting the definition formula, (9b) into:

$$t_F = \frac{\phi_f^2 B^2}{\phi'^2 D' R'} \quad (11)$$

where ϕ_f is the continuum (average) fracture porosity defined by:

$$\phi_f = \frac{\phi b}{B + b} \approx \frac{\phi b}{B} \quad (12)$$

For a given fracture porosity, variation of fracture spacing results in different BTC tails. Figure 7 shows the effect of fracture spacing by using $\phi'/\phi_f = 10$, $D' R' = 10^{-9}$ m²/s, and fixed values for t_i , t_T , and t_M . We see that smaller fracture spacing causes larger tracer concentration in the BTC tail.

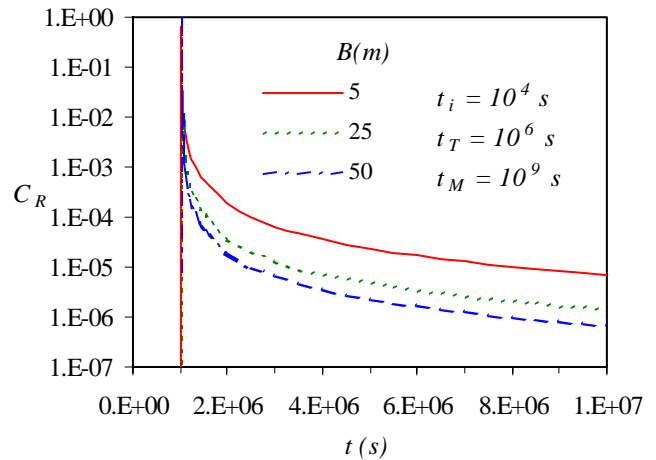


Figure 7. The effect of fracture spacing on BTC.

The fracture porosity ϕ_f can be estimated by means of the injection flux rate (q_i) and the pore velocity, v , by:

$$\phi_f = q_i / v = q_i \cdot t_T / z \quad (13)$$

The matrix porosity, ϕ' , can be estimated by laboratory tests on rock samples. The matrix tortuosity τ can be determined by laboratory tests or estimated using the Millington (1959) model:

$$\tau = (\phi')^{1/3} \quad (14)$$

Substituting this tortuosity and the known tracer molecular diffusion coefficient D^* into (2a) we obtain D' , the diffusion coefficient in the matrix. The bulk mass density, ρ_b , can be estimated by:

$$\rho_b = \rho_s (1 - \phi') \quad (15)$$

where ρ_s is the rock grain density (approximately 2650 kg/m³). The distribution coefficient of tracer in the matrix (K_m) can be determined by laboratory tests. Therefore, we can calculate the retardation factor, R' , using (2b). Finally we use (11) and obtain:

$$B = \frac{\phi'}{\phi_f} \sqrt{t_F D' R'} \quad (16)$$

The fracture-matrix interface area available for heat transfer per unit volume of the flow system is then $A/V = 1/(B + b) \cong 1/B$.

CONCLUSION

A simplified analytical solution for solute tracer migration in fractured porous media reveals some important facts. In most practical cases, the tracer concentration in the fracture is dependent on three characteristic times: the injection period t_i , the tracer transit time t_T , and the crossing-interface time t_F . The independence of the solution on another characteristic time t_M and the definition of t_F theoretically validate a new finding, namely, the retardation factor and the diffusion coefficient of the matrix have the same effect on tracer concentrations in the fracture. This equivalence provides the basis for using reversibly sorbing chemicals as tracers to test a fractured formation. The verification of a numerical code, TOUGH2, against the analytical solution demonstrates excellent agreement. The analytical solution can also be useful in selecting solutes with appropriate sorption properties, and analyzing field data under simplified conditions. Such analysis can inversely estimate the two important parameters: the average fracture porosity and fracture spacing, from which effective heat transfer area per unit reservoir volume may be obtained.

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