AN AUTOMATED WELL TEST ANALYSIS SYSTEM (AWTAS)

M.J. O’Sullivan¹, A.E. Croucher¹, J.A. Newson¹, E.B. Anderson², A. Hochwimmer³, S. Yamazawa³ and S. Lin⁴.

1. Department of Engineering Science, University of Auckland, Auckland, New Zealand
2. PB Power GENZL Division, Auckland, New Zealand.
3. New Energy and Industrial Technology Development Organisation (NEDO), Tokyo, Japan
4. Geophysical Surveying Co., Ltd., Tokyo, Japan

e-mail: m.osullivan@auckland.ac.nz

ABSTRACT

Systems developed in the past for the computerised analysis of well tests are based on mathematical models of the reservoir that can be solved using analytical methods. Unfortunately, for many geothermal well tests the flows are non-isothermal and often involve phase changes. This means that the corresponding mathematical models cannot be solved analytically and are complicated by fluid properties being neither linear nor uniform. Standard linear models based on uniform fluid properties can give a reasonable approximation of the behaviour of geothermal reservoirs for simple constant-rate drawdown tests (Grant and Sorey 1979, O’Sullivan 1987). However, the most common kinds of well tests for geothermal systems are pressure build-up tests or injection fall-off tests. Both of these tests involve considerable non-linearity and non-uniformity, because of the variation of fluid properties such as density, viscosity, and temperature with saturation.

During a geothermal well test the hot water in the reservoir may boil and then later re-condense. Thus, the reservoir behaviour is non-linear and non-uniform. Standard linear models based on uniform fluid properties can give a reasonable approximation of the behaviour of geothermal reservoirs for simple constant-rate drawdown tests (Grant and Sorey 1979, O’Sullivan 1987). However, the most common kinds of well tests for geothermal systems are pressure build-up tests or injection fall-off tests. Both of these tests involve considerable non-linearity and non-uniformity, because of the variation of fluid properties such as density and viscosity with temperature and saturation.

PURPOSE AND DESIGN OF SOFTWARE

Traditional methods of well-test analysis, developed for the ground water and oil industries, involved graphical methods such as Horner plots or type-curve methods (see Earlougher 1977, for example).

With the development of computers, non-linear regression methods became feasible. One of the most important early papers on the topic was by Earlougher and Kersch (1972). Recently a number of automated well-test analysis systems have been developed. They include non-linear regression techniques for fitting the reservoir/well-bore model to the field data, as well as a pre-processing stage to help with the choice of model or models (i.e. model recognition). An excellent survey paper by Horne (1994) describes these techniques.

With regard to the analysis of geothermal well-tests, there are two major problems with the existing automated well-test analysis systems. Firstly, the de-convolution techniques used for model recognition depend on the response of the reservoir being linear and the reservoir fluid properties being uniform.

Large-scale reservoir behaviour has also been used in system identification or inverse modelling studies. In the geothermal context, several modellers (Finsterle et al. 1997, O’Sullivan et al. 1998, White 1995, White et al. 1998) have used the iTOUGH2 code (Finsterle 1997), which employs the numerical simulator TOUGH2 (Pruess 1991) to generate the model response (instead of deriving the model response from an analytic formula, as in well-test analysis). In these inverse modelling studies, several selected reservoir parameters are allowed to vary until the model results fit the field data as well as possible. The basic “fitting” or optimisation techniques used are similar to those used by automated well-test analysis systems. However, the use of a numerical simulator means that the non-linear behaviour of geothermal reservoirs can be more closely reproduced.

The aim of the research described in the present paper is to construct an automatic well-test analysis system that can be applied to geothermal well tests, by combining the approaches of codes such as AUTOMATE for well-test analysis (Horne 1995) and numerical inverse reservoir modelling codes such as iTOUGH2 (Finsterle 1997). To produce an
automatic well-test analysis system based on numerically generated model responses, the following major ingredients are essential:

- Very fast and accurate numerical generation of the model response
- Very fast model fitting
- An easy-to-use graphical interface

The first of these requires the development of fast numerical solvers (see ‘Fast Numerical Solvers’) and the second requires a robust and fast fitting technique (see ‘Model Fitting’). The development of the user interface and related database management is described in the ‘Graphical Interface’ section.

**FAST NUMERICAL SOLVERS**

For a single layer uniform porous medium model, the flow near a well is radially symmetric and a 1D radial grid can be used. For such a grid the discrete mass and energy balance equations can be written as

\[ V_i \left[ M_{i+1}^{n+1} - M_i^n \right] + \Delta t \frac{n+1}{2} \left[ F_{i+1/2}^{n+1} A_{i+1/2} - F_{i-1/2}^{n+1} A_{i-1/2} \right] = 0 \]  

(1)

and

\[ V_i \left[ E_i^{n+1} - E_i^n \right] + \Delta t \frac{n+1}{2} \left[ F_{i+1/2}^{n+1} A_{i+1/2} - F_{i-1/2}^{n+1} A_{i-1/2} \right] = 0 \]  

(2)

It is an important feature of finite difference formulae (1) and (2) that all the flux terms are evaluated at time \( t^{n+1} \). Such schemes are called “fully implicit”. The implicit methods are required in geothermal simulators in order to handle highly nonlinear phase changes.

The quantities in (1) and (2) are defined as follows:

- \( i \) subscript labelling quantities in block \( i \)
- \( n, n+1 \) superscripts labelling quantities evaluated at times \( t^n \) and \( t^{n+1} \) respectively
- \( V_i \) volume of block \( i \)
- \( A_{i+1/2} \) area of interface between block \( i \) and block \( i+1 \)
- \( M_i^n \) mass content / unit volume
- \( E_i^n \) energy content / unit volume
- \( F_{i+1/2}^{n+1} \) mass flux
- \( F_{i+1/2}^{n+1} \) energy flux
- \( \Delta t^{n+1} \) time step between times \( t^n \) and \( t^{n+1} \)

The mass and energy contents are given in terms of the other quantities by the formulae

\[ M_i^n = \phi_i \left[ \rho_i^n S_i^n - \rho_\phi^n S_\phi^n \right] \]  

(3)

and

\[ E_i^n = (1 - \phi_i) \rho_i^n c_i^n T_i^n + \phi_i \left[ \rho_i^n S_i^n u_i^n - \rho_\phi^n S_\phi^n \right] \]  

(4)

where

- \( \phi_i \) porosity
- \( \rho_i^n \) density of water
- \( \rho_\phi^n \) density of steam
- \( S_i^n \) water saturation (volume fraction)
- \( S_\phi^n \) steam saturation
- \( \rho_d \) rock density
- \( c_i^n \) rock specific heat
- \( T_i^n \) temperature
- \( u_i^n \) specific energy of water
- \( u_\phi^n \) specific energy of steam

These formulae apply for general two-phase conditions (boiling water and steam) and in this case \( \rho_i^n , \rho_\phi^n , u_i^n , u_\phi^n \) are all dependent on the pressure \( p_i^n \). Steam table formulae are available for their calculation (see UK Committee on the Properties of Steam, 1970).

The total flow is given as the sum for the individual phases:

\[ \text{FML}_{i+1/2}^{n+1} = \text{FML}_{i+1/2}^{n+1} + \text{FMV}_{i+1/2}^{n+1} \]  

(5)

Here \( \text{FML}_{i+1/2}^{n+1} \) is the water flux and \( \text{FMV}_{i+1/2}^{n+1} \) is the steam flux, which are calculated using two-phase version of Darcy’s Law:

\[ \text{FML}_{i+1/2}^{n+1} = \left[ k_{i+1/2} \right] v_i^n \frac{p_{i+1/2}^n - p_i^n}{\Delta t^{n+1/2}} \]  

(6)

\[ \text{FMV}_{i+1/2}^{n+1} = \left[ k_{i+1/2} \right] v_\phi^n \frac{p_{i+1/2}^n - p_i^n}{\Delta t^{n+1/2}} \]  

(7)

Here \( k_{i+1/2} \) is the rock permeability, \( k_{i+1/2}^W \) and \( k_{i+1/2}^V \) are the relative permeabilities (see below) and \( v_i^n , v_\phi^n \) are the kinematic viscosities of each phase.

Similarly, the total energy flux is given by

\[ \text{FE}_{i+1/2}^{n+1} = h_i^n \text{FML}_{i+1/2}^{n+1} + \frac{T_i^n - T_i^n}{\Delta t^{n+1/2}} \]  

(8)
Here \( h_{i+1/2}^n \) is the enthalpy of water and \( h_{i+1/2}^{n+1} \) is the enthalpy of steam.

The last term in (8) is the energy flux resulting from conduction. The quantity \( \Delta r_{i+1/2} \) is the distance between the centres of block \( i \) and block \( i+1 \), and \( K \) is the thermal conductivity.

An important feature of (6), (7) and (8) is the evaluation of the interface relative permeabilities, kinematic viscosities and enthalpies. They are calculated using upstream weighting (see Pruess 1991 or O’Sullivan 1985).

At the first time step the initial data provides values for \( p_i^0 \) and \( T_i^0 \) (or \( S_i^0 \)) for \( i = 1, 2, \ldots, M \). The boundary conditions must also be specified. It is usually assumed that the model is sufficiently large so that it is “infinite acting” and no flow occurs at the end of the model.

Thus
\[
F_{M+1/2}^{n+1} = 0
\]
\[
F_{M+1/2}^{n+1} = 0
\]

At the well base or sand face end of the model the total flow is specified: thus
\[
F_{i+1/2}^{n+1} A_{i+1/2} = QM_{i+1}^{n+1}
\]
Here \( QM_{i+1}^{n+1} \) is the specified wellbore flow. Note \( QM_{i+1}^{n+1} \) is positive for injection and negative for production. The energy flux is calculated using
\[
F_{i+1/2}^{n+1} A_{i+1/2} = h_{i+1/2}^{n+1} QM_{i+1}^{n+1}
\]
Here \( h_{i+1/2}^{n+1} \) is the flowing enthalpy, which is specified as part of the test conditions for an injection test, or is calculated from \( p_i^{n+1} \) and \( T_i^{n+1} \) (or \( S_i^{n+1} \)) for production.

The equations provided by (1) and (2) are non-linear. For all the blocks they can be written in compressed vector form as
\[
\mathbf{F}(\mathbf{X}) = 0
\]
Here \( \mathbf{F} \) is a vector of \( 2M \) equations with the \( 2i - 1 \)st equation given by (1) and the \( 2i \)th equation given by (2). Similarly \( \mathbf{X} \) is a vector of \( 2M \) unknowns with the \( 2i - 1 \)st element \( p_i^{n+1} \) and the \( 2i \)th element \( T_i^{n+1} \) (or \( S_i^{n+1} \)). The system (13) is solved iteratively using the Newton Raphson method.

The \( k+1 \)st estimate of the solution \( \mathbf{X}^{k+1} \) is calculated using
\[
J^k \left[ \mathbf{X}^{k+1} - \mathbf{X}^k \right] = -\mathbf{F}^k
\]
Here \( J^k \) is the Jacobian matrix defined by
\[
J^k_{i,j} = \frac{\partial F_i(\mathbf{X}^k)}{\partial X_j}
\]
and \( \mathbf{F}^k = \mathbf{F}(\mathbf{X}^k) \).

The process is initiated by using the solutions at time \( t^k \) as the first estimate of the solution at time \( t^{k+1} \). If the scheme does not converge the time step is decreased (see Pruess, 1991).

In TOUGH2 and in the simplified simulator developed here the Jacobian derivative entries are calculated numerically using difference approximations.

Equation (14) represents a large system of sparse linear equations. In TOUGH2 there are several methods available for solving them. Some of the latest methods available are the conjugate gradient methods such as GMRES and BICGSTAB (Bullivant et al., 1991, Moridis and Pruess, 1997).

These methods apply for general models and general sparse structures for \( \mathbf{J} \). However careful examination of the Jacobian arising from (1) and (2) shows that it has a very special structure, namely it has at most six non-zero entries in each row. With some simple additions and subtractions the system of equations can be reduced to a pentadiagonal banded form. A special fast solver is available for such systems of linear equations and it has been implemented in the AWTAS code.

Some researchers (Black et al., 1986 and Noy et al., 1988) have suggested that a fractured reservoir can be represented by a fractal or fractional dimension model. For the numerical implementation of this model the same fast solver as for the single layer porous medium model can be used. All that has to be changed is the specification of the block volumes and interface areas. Table 1 and Table 2 give the formulae for calculating these quantities.
Dimension of model $V_i$

<table>
<thead>
<tr>
<th>Dimension</th>
<th>$V_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2b^2(r_i - r_{i+1})$</td>
</tr>
<tr>
<td>2</td>
<td>$\pi b^2(r_i^2 - r_{i+1}^2)$</td>
</tr>
<tr>
<td>3</td>
<td>$\frac{4\pi}{3}(r_i^3 - r_{i+1}^3)$</td>
</tr>
<tr>
<td>$n$</td>
<td>$\frac{\alpha_n b^{2(n-1)} r_{i+1}^{n-1}}{n}(r_i^n - r_{i+1}^n)$</td>
</tr>
</tbody>
</table>

Table 1: Block volumes

Dimension of model $A_{i+\frac{1}{2}}$

<table>
<thead>
<tr>
<th>Dimension</th>
<th>$A_{i+\frac{1}{2}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$2b^2$</td>
</tr>
<tr>
<td>2</td>
<td>$2\pi b r_{i+\frac{1}{2}}$</td>
</tr>
<tr>
<td>3</td>
<td>$4\pi r_{i+\frac{1}{2}}^2$</td>
</tr>
<tr>
<td>$n$</td>
<td>$\alpha_n b^{2(n-1)} r_{i+\frac{1}{2}}^{n-1}$</td>
</tr>
</tbody>
</table>

Table 2: Block interface areas

Here it is assumed that the $i^{th}$ block, with volume $V_i$, is located between radii $r_{i-1}$ and $r_i$. The $i^{th}$ interface with area $A_{i+\frac{1}{2}}$ is located between the $i^{th}$ and $(i+1)^{st}$ blocks at a radius of $r_{i+\frac{1}{2}}$. $b$ is the “layer” thickness and $\alpha_n$ is defined by

$$\alpha_n = \frac{2\pi r_{i+\frac{1}{2}}^{\frac{1}{n}}}{\Gamma(\frac{1}{n})}$$

A fractional dimension model has been implemented in AWTAS.

In a multi-layer fracture/matrix system the fluid flow occurs mainly in the radial direction along the fracture. There is also some flow perpendicular to the fracture into or out of the matrix. Flow in the radial direction in the matrix is very small. If it is ignored entirely (equivalent to assuming zero radial permeability in the matrix) then the structure of the Jacobian matrix in (14) has a simple form which enables the equations to be solved very quickly. In fact the system of equations has a “tree-like” structure with the equations for the fracture flow being the “trunk” of the tree and the equations for the matrix flow being the “branches” of the tree. The idea of the fast solver for this model is to work along each branch eliminating the matrix unknowns until modified equations containing only fracture unknowns remain. These fracture equations have the same structure as for the single layer case and the same single layer fast solver can be used.

The MINC system (Pruess and Narasimhan 1982) is a numerical generalisation of the double porosity idea introduced by Warren and Root (1963). The idea with MINC is to represent a fractured rock reservoir as a dual system of a fracture medium and a matrix medium. All the actual fractures are represented in an average sense by the fracture medium and all the multitude of “lumps” of matrix between the fractures is represented by the matrix medium. The flow through the fractured rock is assumed to take place in the high permeability, low volume fracture medium. However flow between the fracture medium and the local matrix medium can also occur, but no large-scale flow through the matrix medium itself is permitted.

Thus a numerical model based on the MINC system consists of a number of interconnected fracture blocks with nested matrix blocks connected to each fracture block. For a radial flow model this gives rise to a block structure which is identical to a multi-layer fracture-matrix model, where no radial flow is permitted in the matrix. The only difference between the various MINC models (of which the multi-layer fracture-matrix model is really a special case) is in the way the volumes and interface areas for the nested matrix blocks are calculated. The methods for calculating these for 1D, 2D and 3D fracture systems are given by Pruess (1983).

It is also possible to implement a fractional dimension version of the MINC method by suitable calculation of fracture and matrix block volumes and interface areas. This will be included as an option in AWTAS at a later stage.

Non-uniformities in reservoir properties caused by skin effects can also be easily modelled when a numerical simulator is used. This topic is currently being investigated.

The fast solvers described above are for models where the flow is radial ($r$) or cylindrical ($r-z$). More complex geometric configurations and boundary conditions could be included, but the practicability of their use in AWTAS depends on how fast the corresponding solvers can be made to run. Further research on this topic is required.

**MODEL FITTING**

The process of fitting a model to data falls into three main steps:

- Selection of model
- ‘Tuning’ model parameters to optimise agreement between data and model response
- Evaluating degree of agreement and reliability of parameter estimates

The first of these steps may be carried out manually, using knowledge of the reservoir and modelling experience, or (at least in simpler cases) some degree
of automation may be introduced by applying ‘model recognition’ techniques (Horne 1994).

Tuning of the model parameters may be cast as a non-linear optimisation problem, in which the discrepancy between the data and model response is viewed as a function of several variables (the unknown model parameters) to be minimised. Commonly, this discrepancy is measured using a ‘weighted least-squares error’ \( \varepsilon \) defined by:

\[
\varepsilon(p) = \sum_{i=1}^{nd} \left( \frac{d_i - m_i(p)}{\sigma_i} \right)^2
\]

where \( p \) is the vector of model parameters, \( nd \) is the number of data points, \( d_i \) is the \( i \)th data point, \( m_i \) is the modelled approximation to \( d_i \) and \( \sigma_i \) is the estimated measurement error in \( d_i \).

This approach has the advantage of flexibility, in that it is applicable to any data set (e.g. mixtures of pressure and temperature measurements). More importantly, any type of model can be used- be it analytical or numerical. As numerical models are a necessary ingredient in the AWTAS software, this is the parameter tuning approach adopted.

An efficient and robust nonlinear optimisation algorithm is required to minimise the function \( \varepsilon(p) \), since this function can become time-consuming to evaluate and very complex when numerical models are used. AWTAS employs the so-called ‘Full Newton’ method of Dennis et al. (1981) (better known as the ‘NL2SOL’ algorithm) which is a slightly more advanced version of the well-known Levenberg-Marquardt method. Both of these methods are specifically designed for least-squares minimisation problems and offer advantages over other methods in both efficiency and robustness. The algorithm requires the Hessian matrix of the function \( \varepsilon(p) \) to be calculated, which is carried out using finite difference approximations.

When more complex numerical models are used, it is possible that the function \( \varepsilon(p) \) could develop more than one local minimum, in which case it may become necessary to find the absolute minimum using a global searching method (such as the ‘simulated annealing’ algorithm- see Kirkpatrick et al. (1983)). Unfortunately such methods generally require far more function evaluations than do conventional local optimisation methods (Goffe et al. 1994), so their use is to be avoided if at all possible.

**EXAMPLE**

Perhaps the most demanding of the various tests so far conducted using AWTAS is a drawdown-buildup test in a hot water reservoir, starting from an undisturbed pressure close to the boiling point. Such a test involves very non-uniform processes near the wellbore. During the drawdown phase of the test, a boiling zone develops that expands outwards as time increases. After the well is shut in, the liquid saturation increases in the boiling zone as the pressure near the well rebounds. A ring of condensed water forms around the wellbore and slowly spreads outwards. Eventually the whole of the boiling zone, which was established during drawdown, re-condenses and liquid conditions are again present throughout the reservoir.

The results of numerical simulations (using the solver described above in 'Fast Numerical Solvers') of two drawdown-buildup tests are shown in Figures 1-5. These results represent pressure and temperature results at the wellbore. The parameter values used are given in Table 3.

Test I is for a reservoir with an undisturbed pressure high enough to prevent any boiling from occurring during drawdown. The Horner plot of pressure during buildup gives the expected straight line (Figure 2). However test II has an undisturbed pressure low enough to cause boiling (and consequently the processes described above) to occur. The two distinct sections of the Horner plot (see Figure 5) correspond to two-phase and liquid-only reservoir behaviour respectively.
Parameter | Value
--- | ---
Initial pressure | 50 bar (test I), 36 bar (test II)
Initial temperature | 240 °C
Pumping rate | 50 kg.s$^{-1}$
Layer thickness $b$ | 100 m
Permeability | $10^{-13}$ m$^2$
Porosity | 0.1
Fractional dimension | 2.0 (test I), 1.8 (test II)

Table 3: Model parameter values

In order to test AWTAS, an artificial data set was manufactured by adding random noise to the simulated results from test II. It was found that AWTAS was able to recover the reservoir parameters (permeability, porosity and fractional dimension) when started from some other initial parameter values. Table 4 shows the initial and fitted parameter values. The fitted parameter values are close to those used to generate the original data set (see Table 3), with the discrepancy being attributable to the added noise. The artificial test data and the model results for the fitted parameter values determined by AWTAS are shown in Figure 6 and Figure 7.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Initial value</th>
<th>Fitted value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Permeability</td>
<td>$8 \times 10^{-14}$ m$^2$</td>
<td>$9.83 \times 10^{-14}$ m$^2$</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.15</td>
<td>0.108</td>
</tr>
<tr>
<td>Fractional dimension</td>
<td>2.0</td>
<td>1.801</td>
</tr>
</tbody>
</table>

Table 4: Initial and fitted parameter values

Figure 5: Horner plot (test II)
A number of tests of AWTAS on field data from various geothermal fields have been carried out, with mixed success. The main difficulties that have arisen are related to the quality of the data and the types of tests. Most reasonable quality data sets are for multi-rate injection-fall off tests. Past analyses of these tests have used only the part of the record that mostly closely fits the traditional constant rate injection-fall off test. The analysts have used graphical methods such as a Horner plot.

With AWTAS the whole complex injection record and corresponding pressure response can be incorporated. However the non-isothermal nature of the test, often with an injection temperature very different from the reservoir temperature, makes analysis based on a simple reservoir model pointless. It is apparent even from a visual examination of the data that the temperature dependence of porosity and permeability is an important factor. A model for representing temperature dependence of rock properties suggested by Nakao and Ishido (1998) has been included in AWTAS and some success has been achieved with it in matching field data. Further work on the topic is proceeding.

ACKNOWLEDGEMENTS

NEDO (New Energy and Industrial Technology Development Organization of Japan) provided funding for this work as part of the New Sunshine Programme.

REFERENCES


LBL-15227, Lawrence Berkeley Laboratory, Berkeley, CA.


