CAPILLARY PRESSURE INFLUENCE ON WATER VAPORIZATION IN GEOTHERMAL RESERVOIRS

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ABSTRACT

The frontal mathematical model of water-steam phase transition with capillary forces within porous media is proposed. The complete system of boundary conditions at the boiling front is presented. Similarity analytical solution is derived for the one-dimensional problem of mass extraction from a contact boundary between a hydrothermal reservoir and impermeable rocks. It is shown that two different regimes of water vaporization exist: 1) with formation of a two-phase transition zone and 2) with formation of a sharp vaporization front within nonwetting porous media. The effects of capillary forces on the main characteristics of the vaporization process are investigated.

NOMENCLATURE

Latin symbols

a thermal diffusivity [m^2/s],

C heat capacity [J/(K kg)],

e intrinsic energy density [J/kg],

h enthalpy density [J/kg],

k permeability [m^2],

m porosity,

P pressure [Pa],

q specific heat of phase transition [J/kg],

R gas constant [J/(kg K)],

R_0 universal gas constant [J/(kmole K)],

r mean radius of the capillary meniscus [m],

T temperature [K],

u water or steam velocity [m/s],

V velocity of the vaporization front [m/s],

V_w molar volume of water [m^3 /kmole],

v filter velocity [m/s],

Greek symbols

$\boldsymbol{a}$ water compressibility coefficient [1/Pa],

$\boldsymbol{b}$ water thermal expansion coefficient [1/K],

$\boldsymbol{\varepsilon}$ dimensionless similarity coordinate of the vaporization front,

$\boldsymbol{\nu}$ dimensionless similarity variable

$\boldsymbol{q}$ contact angle,

$\boldsymbol{l}$ thermal conductivity [W / (m K)],

$\boldsymbol{m}$ viscosity [Pa s],

$\boldsymbol{r}$ density [kg / m^3],

$\boldsymbol{s}$ surface tension [J/m^2],

Subscripts

n normal,

0   initial value,

S porous medium skeleton,

v vapor,

w water,

$+$ quantities to the right of the front,

$-$ quantities to the left of the front,

$*$ values of the quantities at the front,

Superscript

0   boundary value.

INTRODUCTION

The frontal approach to phase transition problems is developed. This method has been applied to geothermal reservoir modeling by [Udell (1985)], Pruess, Calore, Celati and Wu (1987), Garg and Pritchett (1988), Woods and Fitzgerald (1993), Tsypkin (1994), etc.

Capillary pressure effects have been studied by Udell (1985) and Pruess and O'Sullivan (1992).

In Pruess and O'Sullivan (1992) numerical simulations were performed to evaluate the impact of capillarity and vapor adsorption on the depletion of vapor-dominated geothermal reservoirs. At the same time, treatment of simple physical situations for which solutions exist in a closed analytical form is

also useful for illustrating the essential features of the phenomena.

The main aim of the present study is to investigate the capillary pressure effects on the movement of a water-steam phase transition front in geothermal reservoirs. In this case the effects of capillary forces take place at the interface only. We assume that the rock may be modelled as an isotropic porous medium in which Darcy's law is valid and all components (skeleton of porous medium, water and vapor) coexist in local thermodynamic equilibrium. We describe the vaporization process as movement of a transition front that separates water-saturated and vapor-saturated permeable rock. The keystone of the work is the complete system of boundary conditions at the vaporization front, which takes into account capillary forces. Only a similarity solution of the one-dimensional depletion problem of mass extraction from a contact boundary between a hydrothermal reservoir and impermeable rocks is considered. The whole problem is reduced to a system of transcendental equations that is solved numerically for a wide range of physical parameters. The results of numerical calculations show that there is a domain of parameters in which frontal solutions exist.

We present distributions of the temperature and pressure functions in each zone and derive the analytical formula of the mass vaporizing function. We show that the mass vaporizing function depends nonmonotonically on porosity and decreases as a function of the contact angle.

GOVERNING EQUATIONS

Let us assume that the hydrothermal reservoir is an incompressible porous medium at rest, saturated with either water or steam. In order to describe the processes of heat and mass transfer we will use the following systems of mass and energy conservation equations, Darcy's law (O'Sullivan, 1985), the equations of state for water and steam, and the thermodynamic relations. In the water zone we have

$$m\frac{\partial}{\partial t}\rho_w + div\rho_w\vec{v}_w = 0$$

$$\frac{\partial}{\partial t}(\rho e)_1 + div(\rho_w h_w \vec{v}_w) = div(\lambda_1 grad T)$$

$$\vec{v}_w = -\frac{k}{\mu_w}grad P$$

$$r_w = r_{w0}(1 + a(P - P_0) - b(T - T_0))$$

$$e_w = h_w - \frac{P}{r_w}; \qquad de_S = C_S dT$$

$$dh_w = C_w dT + \frac{1 - bT}{r_w}dP$$

$$l_1 = ml_w + (1-m)l_S$$
$$(re)_1 = mr_w e_w + (1-m)r_S e_S$$

In the vapor zone the main system has the form

$$m\frac{\partial}{\partial t}\rho_v + div\rho_v\vec{v}_v = 0$$

$$\frac{\partial}{\partial t}(\rho e)_2 + div(\rho_v h_v \vec{v}_v) = div(\lambda_2 grad T)$$

$$\vec{v}_v = -\frac{k}{\mu_v}grad P$$

$$P = r_v RT, \quad e_v = h_v - \frac{P}{r_v}$$

$$dh_v = C_P dT, \quad de_S = C_S dT$$

$$l_2 = ml_v + (1-m)l_S$$
$$(re)_2 = mr_v e_v + (1-m)r_S e_S$$

After identical transformations, the systems of the basic equations reduce to the systems of two equations in T, P.

In the water zone, the system of basic equations has the form

$$\frac{\P P}{\P t} - \frac{b}{a}\frac{\P T}{\P t} - \frac{k}{mm_w}(grad P)^2 +$$

$$+\frac{kb}{mam_w}grad P\, grad T = \frac{k}{mam_w}\Delta P$$

$$(1)$$

$$(rC)_1\frac{\P T}{\P t} - mbT\frac{\P P}{\P t} + \frac{k}{m_w}bT(grad P)^2 -$$

$$-\frac{k}{m_w}r_w C_w\, grad P\, grad T = l_1\Delta T$$

$$(rC)_1 = mr_w C_w + (1-m)r_S C_S$$

In the vapor zone we have

$$\frac{\P P}{\P t} - \frac{P}{T}\frac{\P T}{\P t} - \frac{k}{mm_v}(grad P)^2 +$$

$$+\frac{k}{mm_v}\frac{P}{T}grad P\, grad T = \frac{k}{mm_v}P\Delta P$$

$$(\rho C)_2 \frac{\partial T}{\partial t} - m \frac{\partial P}{\partial t} -$$

$$- \frac{k}{\mu_v} \frac{P C_P}{RT} \, grad \, P \, grad \, T = \lambda_2 \Delta T$$

$$(\rho C)_2 = m \rho_v C_v + (1-m) \rho_S C_S$$

In the impermeable rock zone the usual heat equation is valid.

BOUNDARY CONDITIONS

The formulation of the water-steam phase transition problem admits the existence of a phase transition front. The conditions at this interface can be obtained from the mass conservation law for H_2O and the energy conservation law on discontinuities of the water saturation function

$$\left[\rho \, (V_n - u_n) \right]_{-}^{+} = 0$$

$$\left[\rho \, h(V_n - u_n) + \lambda \, (grad \, T)_n \right]_{-}^{+} = 0 \tag{3}$$

This system is not complete and must be supplemented by the thermodynamic relations.

The first group of relations is based on the assumption of local thermodynamic equilibrium between all phases at the phase transition front (since the velocity of the front is small)

$$T_+ = T_- = T_*, \tag{4}$$

$$P_+ = P_- = P_* \tag{5}$$

There is another relation between the saturated vapor pressure and transition temperature. Saturated vapor pressure above the flat surface of a bulk liquid can be written by the formula (Vukalovitch, 1955):

$$\lg \frac{P_*}{P_a} = A + \frac{B}{T_*} \tag{6}$$

$$A = 5.44, \quad B = -2005.1 \, K, \quad P_a = 10^5 \, Pa ;$$

Considering capillary forces inside the porous medium, condition (5) is given by

$$P_{w+} = P_{v-} + P_c \tag{7}$$

where capillary pressure P_c is defined by

$$P_c = -\frac{2\sigma \, \cos\theta}{r} \tag{8}$$

Here, σ is the surface tension, which depends on temperature by the formula:

$$\sigma = \sigma_0 - w(T - T_0) \qquad w = 10^{-4} \frac{J}{m^2 K}$$

$$\sigma_0 = 4 \cdot 10^{-2} \frac{J}{m^2} \quad (T_0 = 463 \, K)$$

We will assume that the characteristic pore dimension is equal to $\sqrt{k/m}$.

The reduction of saturated vapor pressure above the meniscus of water is given by Kelvin's equation (wetting case, $\sigma > 0$):

$$P_{v-} = P_* \exp\left(-\frac{2\sigma V_w}{r R_0 T} \right) \tag{9}$$

From (6) and (9) we have the generalized Vukalovitch's equation for the capillary phase transition front:

$$P_{v-} = P_a \exp\left(2.3A + 2.3 \frac{B}{T_*} - \frac{2\sigma V_w}{r R_0 T_*} \right) \tag{10}$$

The complete multidimensional nonlinear system of boundary conditions at the water-steam phase transition front can be written as

$$m\left(1 - \frac{P_{v-}}{\rho_w R T_*} \right) V_n = \frac{k P_{v-}}{\mu_v \rho_w R T_*} (grad \, P)_{n-} -$$

$$- \frac{k}{\mu_w} (grad \, P)_{n+}$$

$$mq\rho_w V_n = (\lambda \, grad \, T)_{n+} - (\lambda \, grad \, T)_{n-} -$$

$$- q\rho_w \frac{k}{\mu_w} (grad \, P)_{n+}$$

$$T_+ = T_- = T_*, \tag{11}$$

$$P_{w+} = P_{v-} + P_c$$

$$P_{v-} = P_a \exp\left(2.3A + \frac{2.3B}{T_*} - \frac{4\sigma V_w}{R_0 T_*} \sqrt{\frac{m}{k}} \cos\theta \right)$$

$$P_c = -2\sigma \sqrt{\frac{m}{k}} \, \cos\theta$$

LINEAR APPROACH

We will consider the solutions to the problem in linear approximation when the pressure and temperature variations in each zone are small. We represent these functions in the form

$$P = \tilde{P} + P', \quad T = \tilde{T} + T'$$

Here, $\tilde{f}$ is the constant undisturbed value and f' is the perturbation of the function. In both domains the basic systems of equations in linear approximation have the form

$$\frac{\partial P'}{\partial t} + d_j \frac{\partial T'}{\partial t} = k_j \Delta P'$$

$$\frac{\partial T'}{\partial t} + w_j \frac{\partial P'}{\partial t} = a_j \Delta T' \qquad j = 1,2 \tag{12}$$

$$d_1 = -\frac{b}{a}, \quad k_1 = \frac{k}{a\,m m_w}, \quad w_1 = -\frac{mb\tilde{T}}{(rC)_1},$$

$$a_1 = \frac{ml_w + (1-m)l_S}{m\tilde{r}_w C_w + (1-m)r_S C_S}$$

$$d_2 = -\frac{\tilde{P}}{\tilde{T}}, \quad k_2 = \frac{k\tilde{P}}{m m_v}, \quad w_2 = -\frac{m}{(rC)_2},$$

$$a_2 = \frac{ml_v + (1-m)l_S}{m\tilde{r}_v C_v + (1-m)r_S C_S}$$

The systems of equations (12) can be simplified in each domain if the temperature and pressure are smaller than the critical values. For the energy equation in the water zone the absolute value of the second-to-first ratio term is equal to dimensionless parameter

$$\frac{mb\tilde{P}}{rC} \approx \frac{10^{-1}\,10^{-2}\,10^7}{10^6} \ll 1$$

This estimate shows that we can neglect the work of the water pressure forces compared with heat conduction. The system of equations in the water domain can then be reduced to the following form

$$\frac{\partial P'}{\partial t} + d_1 \frac{\partial T'}{\partial t} = k_1 \Delta P'$$

$$\frac{\partial T'}{\partial t} = a_1 \Delta T' \tag{13}$$

Since $\dfrac{m\tilde{P}}{(rC)_2 \tilde{T}} \approx \dfrac{10^{-1}\,10^7}{10^6\,5\cdot 10^2} \ll 1$, the second term in the energy equation of the vapor domain can be neglected. Both terms on the left side of the motion equation have identical orders of magnitude, which coincide with the left side of energy equation. Hence we have a motion equation which in first order is given by

$$a_2 \Delta \frac{T'}{\tilde{T}} = k_2 \Delta \frac{P'}{\tilde{P}}$$

Since $\dfrac{a_2}{k_2} \approx \dfrac{l_2\,m m_v}{(rC)_2\,k\tilde{P}} \approx \dfrac{10^{-1}\,5\cdot 10^{-5}}{10^6\,10^{-17}\,10^7} \ll 1$

the system of the main equations in the vapor domain can be written as

$$\Delta P' = 0$$

$$\frac{\partial T'}{\partial t} = a_1 \Delta T' \tag{14}$$

In the domain of impermeable rock heat transfer equation has the form

$$\frac{\partial T'}{\partial t} = a_3 \Delta T' \tag{15}$$

where $\qquad a_3 = \dfrac{l_S}{r_S C_S}$

ONE-DIMENSIONAL BLOCK DEPLETION PROBLEM

Let us consider the simplest model problem of fluid extraction from a contact boundary between a hydrothermal reservoir and the surrounding rocks. This situation arises, for example, when the heat-transfer medium flows out into a fracture between a permeable block and impermeable rocks. The pressure drop in the process of extraction leads to vaporization and, consequently, to a decrease in the temperature of the reservoir, which, as estimates show, may be considerable. In this case there is a heat inflow from the surrounding rocks, resulting in intensification of the vaporization process and the formation of the steam-filled zone (of considerable size).

Let impermeable rocks occupy the half-space $x < 0$ while the water at a temperature T_0 and a pressure P_0 occupies the half-space $x > 0$. The initial pressure must satisfy the thermodynamic condition for the existence of water

$$P_{w+}(T_0) > P_{v-}(T_0) + P_c$$

We assume that on the stationary wall $x = 0$ (corresponding to the fracture between blocks) the pressure drops to a relatively small value P^0.
Then the boiling front $x = X(t)$ ($\dot{X}(t) = V_n$) propagates to the right from the surface $x = 0$.
The initial and boundary conditions have the form:

$$
\begin{aligned}
t &= 0: \quad X(0) = 0, \quad x < 0: \quad T = T_0 \\
x &> 0: \quad T = T_0, \quad P_w = P_0, \\
x &= 0: \quad P_v = P^0 \ (P^0 < P_0), \\
&\quad (\boldsymbol{l}\ grad\ T)_{n+} = (\boldsymbol{l}\ grad\ T)_{n-}
\end{aligned}
\tag{16}
$$

If T_0, P_0, P^0 are constant quantities, then the problem (13)-(16) possesses a self-similar solution that describes the propagation of a plane vaporization front through a water-saturated hot rock:

$$T = T(\boldsymbol{z}), \quad P = P(\boldsymbol{z}), \quad X(t) = 2\boldsymbol{g}\sqrt{a_1 t},$$

$$\boldsymbol{z} = \frac{x}{2\sqrt{a_1 t}}$$

For the water domain $x > X(t)$, solutions of equations (13) giving the pressure and temperature , respectively, are

$$(\tilde{T} \equiv T_0, \quad \tilde{P} \equiv P_0)$$

$$T(\boldsymbol{z}) = (T_* - T_0)\frac{erfc(\boldsymbol{z})}{erfc(\boldsymbol{g})} + T_0$$

$$(17)$$

$$P(\boldsymbol{z}) = P_0 + R_1 \frac{erfc(\boldsymbol{z})}{erfc(\boldsymbol{g})} + R_2 \frac{erfc(\boldsymbol{z}\sqrt{a_1/\boldsymbol{k}_1})}{erfc(\boldsymbol{g}\sqrt{a_1/\boldsymbol{k}_1})}$$

$$R_1 = \frac{c_1 a_1}{\boldsymbol{k}_1 - a_1}(T_* - T_0); \quad R_2 = P_{w*} - P_0 - R_1$$

Similarly, the pressure in vapor domain $0 < x < X(t)$ can be written as

$$P_v = \frac{\boldsymbol{z}}{\boldsymbol{g}}\left(P_{v*} - P^0\right) + P^0 \tag{18}$$

Because $a_2 \approx a_3$ the unique heat transfer equation can be used in vapor and impermeable domains. Hence, the distribution of temperature in zone $-\infty < x < X(t)$ has the form

$$T(\boldsymbol{z}) = (T_* - T_0)\frac{1 + erf(\boldsymbol{z}\sqrt{a_1/a_2})}{1 + erf(\boldsymbol{g}\sqrt{a_1/a_2})} + T_0 \quad (19)$$

where

$$erf(z) = \frac{2}{\sqrt{\boldsymbol{p}}}\int_0^z e^{-s^2}ds, \quad erfc(z) = 1 - erf(z)$$

By substituting these solutions into the boundary conditions at the moving fronts we obtain the systems of transcendental equations for T_*, P_{w*}, P_{v*}, $\boldsymbol{g}$

$$P_{v*} = P_a \exp\left\{2{,}3A + \frac{2{,}3B}{T_*} - \frac{4\boldsymbol{s}V_w}{R_0 T_*}\sqrt{\frac{m}{k}}\cos\boldsymbol{q}\right\}$$

$$P_{w*} = P_{v*} + P_c$$

$$\frac{\sqrt{\boldsymbol{p}}mqa_1\boldsymbol{r}_w}{T_0}\boldsymbol{g} + \boldsymbol{l}_1\left(\frac{T_*}{T_0} - 1\right)\frac{\exp(-\boldsymbol{g}^2)}{erfc(\boldsymbol{g})} + \boldsymbol{l}_2\sqrt{\frac{a_1}{a_2}}\cdot$$

$$\left(\frac{T_*}{T_0} - 1\right)\frac{\exp(-\boldsymbol{g}^2 a_1/a_2)}{1 + erf(\boldsymbol{g}\sqrt{a_1/a_2})} + \frac{kq\boldsymbol{r}_w\boldsymbol{b}}{m_w T_0\boldsymbol{a}}\frac{a_1/\boldsymbol{k}_1}{1 - a_1/\boldsymbol{k}_1}\cdot$$

$$\left(\frac{T_*}{T_0} - 1\right)\left[\frac{\exp(-\boldsymbol{g}^2)}{erf(\boldsymbol{g})} - \sqrt{\frac{a_1}{\boldsymbol{k}_1}}\frac{\exp(-\boldsymbol{g}^2 a_1/\boldsymbol{k}_1)}{erf(\boldsymbol{g}\sqrt{a_1/\boldsymbol{k}_1})}\right] -$$

$$-\frac{kq\boldsymbol{r}_w P_0}{m_w T_0}\sqrt{\frac{a_1}{\boldsymbol{k}_1}}\left(\frac{P_{w*}}{P_0} - 1\right)\frac{\exp(-\boldsymbol{g}^2 a_1/\boldsymbol{k}_1)}{erf(\boldsymbol{g}\sqrt{a_1/\boldsymbol{k}_1})} = 0$$

$$\frac{\sqrt{\boldsymbol{p}}a_1\boldsymbol{k}_1 mm_w}{kP_0}\left(1 - \frac{P_*}{\boldsymbol{r}_w RT_*}\right)\boldsymbol{g} - \frac{\sqrt{\boldsymbol{p}}}{2\boldsymbol{g}}\sqrt{\frac{\boldsymbol{k}_1}{a_1}}\frac{m_w}{m_v}\cdot$$

$$\frac{P_{v*}}{\boldsymbol{r}_w RT_*}\left(\frac{P_{v*}}{P_0} - \frac{P^0}{P_0}\right) + \frac{\boldsymbol{b}T_0}{\boldsymbol{a}}\frac{a_1}{P_0}\frac{a_1}{\boldsymbol{k}_1 - a_1}\left(\frac{T_*}{T_0} - 1\right)\cdot$$

$$\left[\sqrt{\frac{\boldsymbol{k}_1}{a_1}}\frac{\exp(-\boldsymbol{g}^2)}{erfc(\boldsymbol{g})} - \frac{\exp(-\boldsymbol{g}^2 a_1/\boldsymbol{k}_1)}{erfc(\boldsymbol{g})}\right] -$$

$$-\left(\frac{P_{w*}}{P_0} - 1\right)\frac{\exp(-\boldsymbol{g}^2 a_1/\boldsymbol{k}_1)}{erfc(\boldsymbol{g}\sqrt{a_1/\boldsymbol{k}_1})} = 0$$

The pressure and temperature distributions calculated in the solution process must be fitted with thermodynamical conditions of existence of steam and water. In the water-saturated domain the local phase transition temperature, calculated in accordance with the pressure obtained from formula (17), must be no lower than the local water

temperature, otherwise the water will be in the superheated state. Analogously, the steam phase transition temperature must be no higher than the steam temperature, otherwise the steam will be in supercooled state. If the conditions of thermodynamical existence of water and steam are not satisfied, then the physical phenomenon cannot be described using the model proposed.

The steam supercooling condition is

$$\left[\frac{dT_f}{dz}\right]_2 < \left[\frac{dT}{dz}\right]_2$$

where the phase transition temperature T_f is determined by equation (10) and the steam pressure function is defined by formula (18). As a result we have:

$$2\sqrt{\frac{a_1}{pa_2}}\frac{T_* - T_0}{1 + erf(g\sqrt{a_1/a_2})}\exp(-g^2 a_1/a_2) -$$

$$-\frac{dT_f}{dP_f}\frac{P_{v*} - P^0}{g} > 0 \qquad (20)$$

The water superheating condition can be similarly determined:

$$\left[\frac{dT_f}{dz}\right]_1 < \left[\frac{dT}{dz}\right]_1$$

Using solution (17) we obtain:

$$1 - \frac{dT_f}{dP_f}\left[\frac{P_{w*} - P_0}{T_* - T_0} + \frac{b}{a}\frac{a_1}{k_1 - a_1}\right]\sqrt{\frac{a_1}{k_1}}$$

$$\frac{\exp(-g^2 a_1/k_1)erfc(g)}{\exp(-g^2)erfc(g\sqrt{a_1/k_1})} + \qquad (21)$$

$$+\frac{dT_f}{dP_f}\frac{a_1}{k_1 - a_1}\frac{b}{a} > 0$$

The disappearance condition of the left side of these relations determines the critical surfaces in the parameter space.

Let us consider how the rate of vaporization depends upon the main parameters of the physical process. Mass vaporizing is given by the quantity

$$M = m\int_0^\infty r_{w0}\,dV - m\int_g^\infty r_w(V)\,dV =$$

$$= mr_{w0}\left[\int_0^\infty\left(1 - \frac{r_w}{r_{w0}}\right)dV + \int_0^g \frac{r_w}{r_{w0}}\,dV\right] \quad (22)$$

Substituting in (22) the state equation of water and using (17) after identical transformation we obtain the final formula

$$M = mr_{w0}\left\{g\left[a(P_{w*} - P_0) - b(T_* - T_0)\right] +\right.$$

$$+ \frac{\exp(-g^2)}{\sqrt{p}\,erfc(g)}\left[b(T_* - T_0) - aR_1\right] - \quad (23)$$

$$\left.\sqrt{\frac{k_1}{pa_1}}\frac{R_2\exp(-g^2 a_1/k_1)}{erfc(g\sqrt{a_1/k_1})}a\right\}$$

M not only depends on the parameters, initial and boundary conditions but also contains T_*, P_{w*}, g, which are found in the process of solving the problem.

NUMERICAL RESULTS

The system of transcendental equations was solved numerically for the following characteristic values of initial and boundary conditions and parameters (Grigoriev, 1997). Parameters of water and vapor are computed as a functions of the fixed initial temperature.

$$T_0 = 450\,K,\ P_0 = 1.5\cdot 10^6\,Pa,\ P^0 = 5\cdot 10^5\,Pa$$

$$m = 0.1,\ k = 10^{-16}\,m^2,\ l_S = 2\,W/(m\cdot K),$$

$$C_S = 10^3\,J/(kg\cdot K),\ r_S = 2\cdot 10^3\,kg/m^3,$$

$$a = 0.75\cdot 10^{-9}\,Pa^{-1},\ r_w = 0.888\cdot 10^3\,kg/m^3,$$

$$b = 1.13\cdot 10^{-3}\,K^{-1}, C_w = 4.39\cdot 10^3\,J/(kg\cdot K),$$

$$l_w = 0.678\,W/(m\cdot K),\ q = 2.04\cdot 10^6\,J/kg,$$

$$R = 461\,J/(kg\cdot K), m_w = 0.148\cdot 10^{-3}\,Pa\cdot s,$$

$$m_v = 0.159\cdot 10^{-4}\,Pa\cdot s,$$

Some calculation results for neutral ($q = 90^o$), wetting ($0 \le q < 90^o$) and nonwetting ($90^o \le q < 180^o$) cases are presented below. There are two typical regimes of vaporization. One of them is plotted in Figures 1 and 2. Figure 1 shows a typical distributions of temperature (curve 1) and phase transition temperature $T_f = T_f(P)$ (curve 2). The latter is determined from relations (7) (10). As may be seen from Figure 1, the fall in pressure in the water domain leads to a change in the local phase transition temperature, which lies below the temperature curve in all cases, i.e. the relation (21) is valid. Physically, this correponds to the superheating

of water ($T_w > T_f$) in the zone ahead the vaporization front. Hence, the conditions of thermodynamical equilibrium existence of water in this domain are not satisfied. Thus, the mathematical model has a thermodynamical contradiction and the vaporization process cannot be described within a framework of a front approach; hence formation of a two-phase domain should be taken into account. The degree of superheating determines the extension of the two-phase zone.
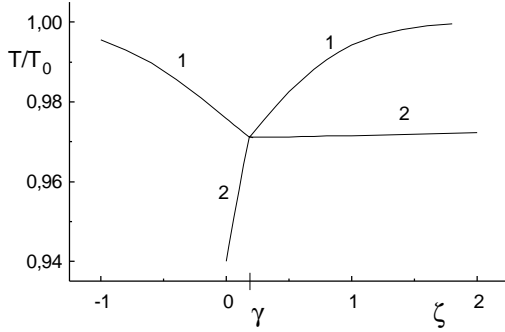

Fig.1 Distributions of the dimensionless temperature and phase transition temperature functions (curves 1 and 2, respectively).

Figure 2 presents the distributions of the pressure functions in the vicinity of the interface for neutral, wetting and nonwetting cases. In the neutral case (without capillary forces) the pressures of water and vapor at the phase transition front are equal. In the presence of capillary forces there is a jump in the pressure function at the front. In the wetting case (Fig.2b), the water pressure is less than that of vapor. In the nonwetting case (Fig.2c) the opposite is the case.
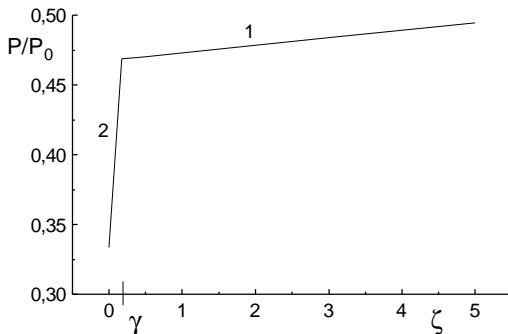

Fig.2a. Distribution of the dimensionless pressure function in water (curve 1) and vapor domains (curve 2). Neutral case ($q = 90^{\circ}$)

$g = 0.176;$ $P_{v*} = P_{w*} = 7.04 \cdot 10^5 \ Pa$
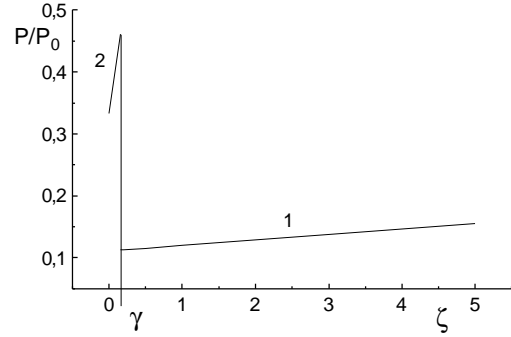$T_* = 436.68 \ K.$

Fig.2b. Distribution of the dimensionless pressure function in water (curve 1) and vapor domains (curve 2). Wetting case ($0 \le q < 90^{\circ}$)

$g = 0.157;$ $T_* = 436.18 \ K;$ $q = 78^{\circ};$
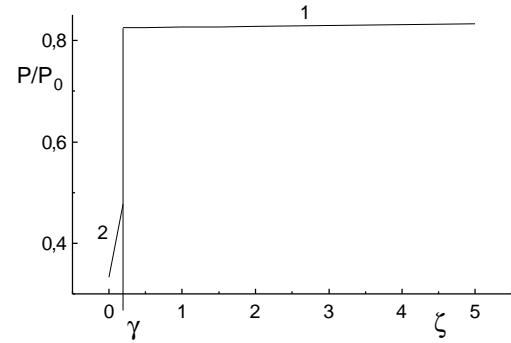$P_{v*} = 6.92 \cdot 10^5 \ Pa;$ $P_{w*} = 1.69 \cdot 10^5 \ Pa;$

Fig.2c. Distribution of the dimensionless pressure function in water (curve 1) and vapor domains (curve 2). Nonwetting case ($90^{\circ} < q \le 180^{\circ}$)

$g = 0.195;$ $T_* = 437.13 \ K;$ $q = 102^{\circ}$
$P_{v*} = 7.15 \cdot 10^5 \ Pa;$ $P_{w*} = 1.24 \cdot 10^5 \ Pa;$

There is a range of parameters with a second regime of vaporization, described by a thermodynamical non-contradictory solution, when the conditions of equilibrium existence of water and steam are satisfied, i.e. $T_w < T_f$ in the water domain and $T_v > T_f$ in the vapor domain. As an example, we present the results of numerical calculations for the following values of parameters, initial and boundary conditions:

$T_0 = 400\,K, \ P_0 = 3 \cdot 10^6 \ Pa, \ P^0 = 1.5 \cdot 10^5 \ Pa$

$m = 0.1, \ k = 10^{-17} \ m^2$

$\boldsymbol{a} = 0.54 \cdot 10^{-9} \ Pa^{-1}, \ \boldsymbol{r}_w = 0.926 \cdot 10^3 \ kg/m^3,$

$\boldsymbol{b} = 0.93 \cdot 10^{-3} \ K^{-1}, C_w = 4.28 \cdot 10^3 \ J/(kg \cdot K),$

$\boldsymbol{l}_w = 0.685 \ W/(m \cdot K), q = 2.18 \cdot 10^6 \ J/kg,$

$\boldsymbol{m}_w = 0.213 \cdot 10^{-3} \ Pa \cdot s, \quad \boldsymbol{q} = 102^o$

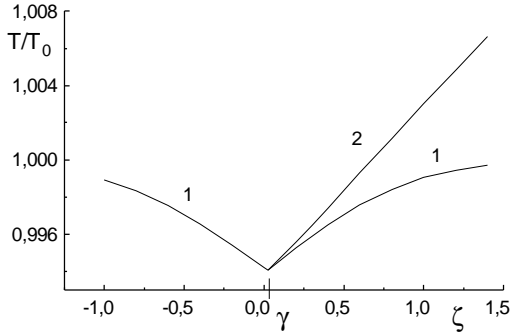$\boldsymbol{m}_v = 0.145 \cdot 10^{-4} \ Pa \cdot s,$

Fig.3. Distributions of the dimensionless temperature and phase transition temperature functions (curves 1 and 2, respectively) in nonwetting case.

$\boldsymbol{g} = 0.024; \ T_* = 397.63 \ K;$

$P_{v*} = 2.55 \cdot 10^5 \ Pa; \ P_{w*} = 2.11 \cdot 10^6 \ Pa;$

Figure 3 shows that the temperature in the water zone is lower than the phase transition temperature ($T_w < T_f$), which indicates the consistency of the frontal model of the vaporization process in nonwetting porous media. Hence, capillary forces play a stabilizing role for the vaporization front in the nonwetting case. Distributions of pressure in this case are as in Figure 2c.

Figure 4 presents, for different values of initial pressure, the domains of existence of frontal solutions in the plane (k, T_0), which lie under the critical curves. Above the curves are frontal solutions that are characterized by thermodynamical contradiction ahead of the vaporization front.
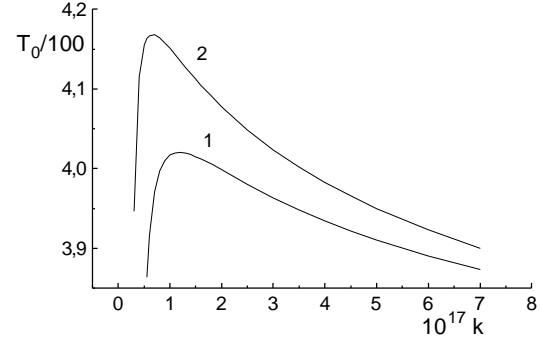

Fig.4. Critical curves of existence of a frontal solution for the initial pressures $P_0 = 3 \cdot 10^6 \ Pa$ (curve 1) and $P_0 = 4 \cdot 10^6 \ Pa$ (curve 2)

$\boldsymbol{q} = 102^o$

The results of our numerical calculations show that an increase in initial pressure leads to the expansion of the domain of a noncontradictory solution.

Figure 5 shows how the total mass vaporizing depends upon the contact angle, permeability and porosity if other parameters are constants:

$T_0 = 400 \ K; \ \boldsymbol{q} = 102^o; \ m = 0.1; \ k = 10^{-17} \ m^2;$

$P_0 = 4 \cdot 10^6 \ Pa; \ P^0 = 1.5 \cdot 10^5 \ Pa;$

As the contact angle decreases, the mass vaporized increases linearly.
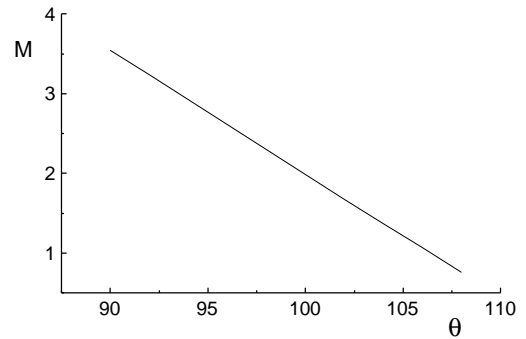

Fig.5a. Mass vaporizing as a function of contact angle.

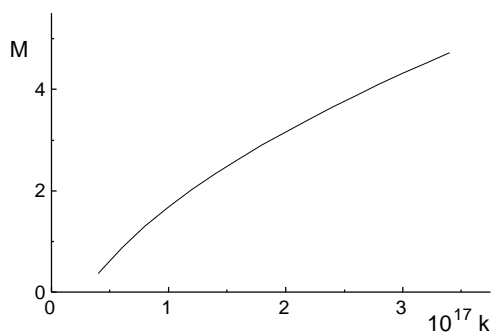Figure 5b shows that the mass vaporized increases with permeability.

Fig.5b. Mass vaporizing as a function of permeability.

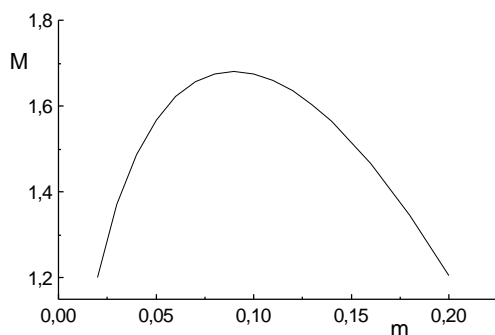The mass vaporizing is a nonmonotonic function of porosity.

Fig.5c. Mass vaporizing as a function of porosity.

CONCLUSIONS

A new model of frontal water vaporization in geothermal reservoirs, which takes into account capillary forces, is presented. A similarity solution is derived for the one-dimensional problem of fluid extraction from a contact boundary between a hydrothermal reservoir and impermeable rocks. The model shows that for nonwetting porous media there exists a range of parameters where capillary forces play a stabilizing role for the vaporization front, hence a sharp front develops. A parametric study of the vaporizing mass fraction shows that the latter has a maximum as a function of porosity.

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