MODELING SILICA DEPOSITION IN INJECTION WELLS
OF THE OTAKE GEOTHERMAL FIELD, JAPAN

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ABSTRACT
A fully implicit numerical model, which describes the permeability decrease caused by the silica deposition in porous medium, has been formulated and applied to simulate the filed data. The rate constants of silica deposition used in calculations were based on the Steefel and Lasaga, Tester et al., and Carroll et al. correlations. A reasonable match was obtained between the measured and calculated dimensionless permeability and water levels.

INTRODUCTION
In a water dominated geothermal system, reinjection of the wastewater is mostly employed instead of surface discharge. However, it is a serious problem that injection wells for wastewater decrease rapidly in their injectivity (Itoi et al., 1987, 1989). The quartz formed in reservoir near wellbore probably causes the injectivity decrease.


Lai et al. (1985) solved the mass, energy and silica mass balance equations by a combination of explicit monotonised upwind central difference method and the operator splitting technique. The numerical model was implemented for one-dimensional problems and two-dimensional problems were solved using by a conventional finite difference method. They have used the rate for silica-water reaction equation of Rimstidt and Barnes (1980) for the reaction term. Their numerical model was used for simulation studies of silica deposition in a single fracture. They also applied their model to study the temperature, pressure, and silica decline in the Ellidaar geothermal field in Iceland.

Itoi et al. (1985, 1986, 1986a, 1987) studied the isothermal one dimension and radial transport of silica in porous medium. They solved the silica conservation equation using finite difference methods. They represented the deposition rate by several alternative kinetic models based on their experimental results.

Wells and Ghiorso (1991) have calculated the rate of decrease in porosity and permeability in a porous medium as fluid flowed at constant rate against a uniform geothermal gradient. They solved one-dimension rate equation using a finite difference method, and applied to problems of silica mass transfer in mid ocean ridge hydrothermal systems. To represent the deposition of quartz, they used a general kinetics rate equation of Lasaga (1984).

Malate and O’Sullivan (1992) presented a mathematical model to describe silica transport and deposition in porous medium. In particular, they derived analytic solutions for the idealized problem of isothermal constant rate and variable rate injection into packed column or a one-dimensional channel. They used several forms of kinetic models of silica deposition and solved the problem using the method of characteristics. The changes in porosity and permeability resulting from deposition were included in their models.

Malate and O’Sullivan (1992a) performed a study on transport and deposition of silica in non-isothermal flows, either in porous medium or a single fracture. Same as the previous work, they obtained analytic solutions for both the one-dimensional problem of constant rate injection into channel or packed column and radially symmetric problem of the flow away from a reinjection well. They represented silica deposition by a rate equation of Rimstidt and Barnes (1980). The model was applied to some field data from the Otake geothermal field in Japan.
Malate and O’Sullivan (1993) extended work to analyze silica deposition effects into a uniform layer from a well that produces radially symmetric flow. The mathematical models developed also use a standard chemical transport-reaction term representing the deposition of silica. The first order rate equation of Rimstidt and Barnes (1980) was used to present silica deposition. The model derived was applied to simulate the changes in injectivity of some reinjection well in the Tongonan geothermal field in Philippines.

Lowell et al. (1993) performed calculations similar to Wells and Ghiorso (1991) but considered flow in discrete fractures and accounted for heat transfer between fluid and adjacent rock as well as the effect of pressure on silica solubility. They showed that decrease in permeability resulting from silica precipitation occurred about an order of magnitude more slowly than for thermal expansion.

Steefel and Lasaga (1994) developed a numerical model for computing coupled multi-component chemical reactions, multi-species chemical transport, hydrodynamic flow, and heat transfer. The model was solved using a finite difference method and simultaneously for multi-component reaction and solute transport in one and two dimension. They questioned the validity of maintaining equilibrium between dissolved silica and quartz on the fracture walls as assumed by Lowell et al. (1993). Steefel and Lasaga showed, in fact, that the flow rate is an important parameter in controlling the amount of super-saturation of dissolved silica, and argued that silica reaction kinetics need to be considered in order to correctly model permeability reduction by silica precipitation. Steefel and Lasaga (1994) also considered thermal convection in an initially homogeneous, porous box heated from below in which permeability was affected by kinetically controlled silica precipitation/dissolution. They showed that the reduction in permeability by precipitation caused the flow to be more diffuse, whereas mineral dissolution caused the flow to be more focused.

White (1995) presented an algorithm for the transport of reacting chemical species in multi-phase fluid systems such as those found in geothermal reservoirs. This algorithm has been incorporated into the geothermal simulator TOUGH2 (Pruess, 1987, 1991). He studied several example problems of geothermal reservoirs including similar problems that presented by Steefel and Lasaga (1994).

In recent work on effect of thermoelasticity, Martin and Lowell (1997) developed a numerical model for the evolution of fracture permeability resulting from combined effects of thermoelastic stresses and precipitation of silica as high-temperature, reactive fluid traverses temperature and pressure gradients. They validated the model by comparing the results with those from Moore et al. (1983), on cylindrical granite cores. They confirmed that the model results show a rapid initial decrease in permeability resulting from thermoelastic stresses, followed by a further decrease resulting from silica precipitation. They suggested that disagreement between the model and laboratory data caused by complication such as reaction kinetics, precipitation of other minerals, and nonhomogenous crack distributions.

In this paper we describe the application of a silica transport model to the long-term experiments of wastewater injection in a geothermal field (Itoi et al. 1987, 1989). The model includes a kinetics reaction to model the precipitation of quartz. A simple one-dimensional reservoir model was used for simulation study.

**MATHEMATICAL MODEL AND FORMULATION**

The following presents a summary of the governing transport equations. The mass continuity equation of water can be expressed as

\[
\nabla \cdot \left( \frac{k_w}{\mu_w} (\nabla P - \rho_w g \nabla Z) \right) = \frac{\partial}{\partial t} \phi \rho_w + q_w
\]

where \( \phi \) is the porosity, \( \rho_w \) is the water density, \( q_w \) is the rate of water per unit volume. The left-hand side is the flux term, where \( k \) is the absolute permeability, \( \mu_w \) is the water dynamic viscosity, \( P \) is the pressure, and \( g \) is the gravitational acceleration.

The equation for conservation of silica in porous media can be express as

\[
\nabla \cdot \left( \frac{k_w}{\mu_w} C (\nabla P - \rho_w g \nabla Z) \right) - \nabla \cdot (\rho_w D \nabla C) = \frac{\partial}{\partial t} (\phi \rho_w) + q_c - R_c \phi
\]

where \( C \) is the silica concentration, \( D \) is the diffusion coefficient, \( q_c \) is the rate of silica per unit volume, and \( R_c \) is the reaction rate of silica.

The equation for conservation of energy:

\[
\nabla \cdot \left( \frac{k_w}{\mu_w} h_w (\nabla P - \rho_w g \nabla Z) \right) - \nabla \cdot (K_T \nabla T) = \frac{\partial}{\partial t} (\phi \rho_w U_w + (1 - \phi) \rho_r U_r) + q_h
\]
where \( h_w \) is the water specific enthalpy, \( K_T \) is the thermal conductivity, \( U_w \) is the specific internal energy, \( \rho \) is the rock density, \( c_r \) is the heat capacity of the rock, \( T \) is the temperature, and \( q_r \) is the rate of generation heat per unit volume.

### Reaction Rate of Silica Deposition/Precipitation

The following discussion covers the kinetics model of silica deposition, which will be used in this paper in modeling silica transport in geothermal reservoir. The dissolution/precipitation of quartz is generally described by the reaction as

\[
\text{SiO}_2(s) + 2\text{H}_2\text{O} \rightleftharpoons \text{Si(OH)}_4(aq)
\]

Ramstidt and Barnes (1980) have shown that the growth of quartz is surface-controlled. In a general form (Steefel and Lasaga, 1994; Lasaga, 1981), the rate of growth or dissolution of a mineral in water solution can be expressed as

\[
R = \frac{A}{V} \left( \frac{Q}{K_s} - 1 \right)
\]

and

\[
k = k_{25} \exp \left[ \frac{-E_a}{R} \left( \frac{1}{T_K} - \frac{1}{298.15} \right) \right]
\]

where, \( k_{25} \) is the reaction rate constant at 25 °C (4.30 x 10^{-14} moles m^{-2} s^{-1}), \( E_a \) is the activation energy (75.0 kJ/mol), \( A/V \) is the area over which the reaction occurs per unit volume of fluid (m^2/m^3), \( T_K \) is the temperature in Kelvin, \( R \) is the gas constant (8.31456 J/mol K), \( Q \) is the activity of aqueous SiO_2, and \( K_s \) is the equilibrium constant for dissolution of quartz reaction.

Tester et al. (1994) performed their experiment for quartz dissolution kinetics in pure water at temperature 25 to 625 °C by five different experimental apparatuses. Their rate can be expressed as

\[
R = k_f \frac{A}{M_w} \left( 1 - \frac{Q}{K_s} \right)
\]

or

\[
R = k_r \frac{A}{M_w} (K_s - Q)
\]

where

\[
k_r = k_f / K_s
\]

and \( k_f \) is the forward (dissolution) rate constant, mol/m^2-s, \( k_r \) is the reverse (precipitation) rate constant, mol/m^2-s, and \( A/M_w \) is ratio of the active surface area to the mass of water, m^2/kg

Based on the results of rate data from the five reactor systems plus other ten previous experimental studies, an empirical equation for \( k_f \) based on the least-squares lines, was derived as

\[
k_{f,\text{geom}} = (276 \pm 193) \exp \left( -\frac{90.1 \pm 2.5}{RT} \right)
\]

for geometric surface area basis, and

\[
k_{f,\text{BET}} = (24 \pm 34) \exp \left( -\frac{87.7 \pm 4.7}{RT} \right)
\]

for BET (Brunauer-Emmett-Teller) derived surface area basis.

Carroll et al., (1998) investigated amorphous silica precipitation behavior in simple laboratory experiments and more complex field experiments in the Wairaki geothermal field, New Zealand. They found that in simple laboratory solution supersaturated with the absence of chemical impurities, precipitation rates have a first-order form

\[
\text{Rate}_{\text{ppt}} = k_{\text{ppt}} \exp \left( -\frac{E_a}{RT} \left( 1 - \frac{Q}{K_{eq}} \right) \right)
\]

where \( k_{\text{ppt}} \) is a rate constant of precipitation, 10^{1.9} molal m^{-2} s^{-1} and \( E_a \) is the activation energy, 61±1 kJ mol^{-1}.

### Porosity-Permeability Changes

The porosity of rock that decreases as silica deposits is expressed as

\[
\phi^{t+\Delta t} = \phi^t \times \frac{R^{t+\Delta t} \times \Delta t}{\rho_s}
\]

where \( \rho_s \) is the molar density of quartz.

The change in permeability as a result of deposition/precipitation is calculated using the model derived by Weir and White (1996). The permeability is given by

\[
k^{t+\Delta t} = k_0 \times \left( 1 - \left( \frac{\phi^{t+\Delta t} - \phi_c}{\phi_o - \phi_c} \right)^{1.58} \right)^{0.460}
\]
where $k_0$ is the initial permeability, $\phi_0$ and $\phi_c$ are the initial porosity and a critical value of porosity at which the permeability reduces to zero, respectively.

The updated porosity is then used to calculate the corresponding total surface area $A$ Linthner (1992) and Takeno et al. (1998a, 1998b) give a straightforward approach relationship between reaction surface area and porosity as

$$A^{t+\Delta t} = A_0 \times \left(\frac{\phi^{t+\Delta t}}{\phi_0}\right)^n$$  \hspace{1cm} (12)

where $A_0$ is the initial total surface area. Takeno et al. (1998a, 1998b) assumed a value of $n = 1/2$ and Linthner (1992) proposed $n = 2/3$.

The average permeability of the radial model system is simply given by spatial average of $k_{avg}$:

$$k_{avg} = \frac{\ln \left(\frac{R_{m+1}}{R_1}\right)}{\sum_{i=1}^{m} \ln \left(\frac{R_{m+1}}{R_i}\right)}$$  \hspace{1cm} (13)

where $R$ is the radial distance and $m$ is the number of grid block.

Numerical Formulation

A finite difference method is used to discretize the mass and energy balance equation. The diffusion term in Eq. 3 is neglected. Time is discretized as fully implicit, to ensure the numerical stability. The model employs upstream weighting to calculate these interface fluid enthalpy and concentration, for intrinsic permeability, which is based on harmonic weighting, and the mass density and viscosity are calculated from an arithmetic average.

SIMULATION RESULTS

Itoi et al. (1989) conducted long-term of wastewater injection tests in the Otake geothermal field (Fig. 1). Water was injected into two wells (Well R-1 and R-2) for up to 656 days. Injection conditions for two wells are shown in Table 1. The re-injection water was injected into Well R-1 in three stages, with different injection temperature at each stage. Similarly the wastewater was injected into Well R-2 at four stages each with a different injection temperature.

Table 1. Injecting conditions for well R-1 and R-2

<table>
<thead>
<tr>
<th>Well</th>
<th>Time, days</th>
<th>$Q_{inj}$, m$^3$/h</th>
<th>$T_{inj}$, $^\circ$C</th>
<th>pH</th>
<th>$C_t$, mg/l</th>
<th>$C_{mon}$, ppm</th>
</tr>
</thead>
<tbody>
<tr>
<td>R-1</td>
<td>0–227</td>
<td>30</td>
<td>50</td>
<td>8.2</td>
<td>587</td>
<td>495</td>
</tr>
<tr>
<td></td>
<td>227–503</td>
<td>30</td>
<td>60</td>
<td>8.1</td>
<td>564</td>
<td>552</td>
</tr>
</tbody>
</table>

Table 2 Parameters used for calculation

<table>
<thead>
<tr>
<th>Parameters</th>
<th>R-1</th>
<th>R-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Well radius ($r_w$), m</td>
<td>0.0451</td>
<td>0.0689</td>
</tr>
<tr>
<td>Outer boundary, m</td>
<td>45.1</td>
<td>68.9</td>
</tr>
<tr>
<td>Reservoir thickness (h), m</td>
<td>32.7</td>
<td>38.92</td>
</tr>
<tr>
<td>Initial Pressure ($P_o$), Pa</td>
<td>1.34E6</td>
<td>1.67E6</td>
</tr>
<tr>
<td>Initial porosity ($\phi_0$)</td>
<td>0.1</td>
<td>0.1</td>
</tr>
<tr>
<td>Density of rock ($\rho_r$), kg/m$^3$</td>
<td>2720</td>
<td>2720</td>
</tr>
</tbody>
</table>
Density of silica scale ($\rho_s$), kg/m$^3$

<table>
<thead>
<tr>
<th></th>
<th>2040</th>
<th>2040</th>
</tr>
</thead>
</table>

During the injection test, both the fall off test and the maximum injectivity test were repeated to examine the decrease of injectivity. They observed that a rapid decline of $k_h/k_{oh}$ and the maximum injectivity occurred at early stages of injection in both wells. After 615 days the water level of Well R-1 reached up to well head. Tracer test, caliper log, and borehole televiewer survey were also carried out. They found that the wells connectivity between two wells in three of five set as R-1 and K-1, K-1 and R-2, and K-2 and R-2, but no connection was observed between R-1 and R-2 nor between R-1 and K-2. According to the result of the caliper log and borehole televiewer, they found that silica deposited much more heavily in deeper part, where the water level located in the first half of injection period.

A simple one-dimensional reservoir model configuration was used for numerical study, as illustrated in Fig. 2. The parameters used in the model are listed in Table 2. Porosity of 0.10 is assumed in the model same as Malate and O'Sullivan (1992). The reaction rate of Steefel and Lasaga (1994), Tester et al. (1994), and Carroll et al. (1998) are used in this calculation. We used the silica solubility correlation of Rimstidt (1997), that is applicable for temperature range 0 to 300 °C:

$$\log K = -\frac{1107}{T_K} - 0.0254$$  \hspace{0.3cm} (14)

Here $K$ is the molal solubility of quartz and $T_K$ is in Kelvin. The initial silica concentration was calculated using Eq. 14.

The initial value of permeability thickness from Itoi et al. (1989) did not give a good match of the model results to the water level of field data. The values of
initial permeability and reaction surface area were then varied to obtain a match to the observed water level and decline of permeability of the well. The value of reaction surface area was fixed while the permeability value was first varied at the value given by the formulae Eq. 4, 5, and 9.

In Fig. 5 and Fig. 6, the curves of decline of average dimensionless permeability with time and depth of water level versus time are shown for Carroll’s, Tester’s, and Steefel’s equation, respectively for the \( n \) values range from 0–2/3. The observed data make good match with the initial permeability of 3.338E-16 m\(^2\) and the \( n \) value was 0.5 and 2/3.

The average dimensionless permeability versus time for Well R-1 is presented in Fig. 3 for Carroll’s, Tester’s, and Steefel’s equations, respectively. The calculated curves for the \( n \) values of equation (12) of 1/2 and 2/3 show a reasonable match with measured data. The best fit to the measured data was obtained with the initial permeability of 6.850E-16 m\(^2\). In Fig. 4, the behavior of the depth of calculated and measured water levels are illustrated. It is clear that the curves for the \( n \) value of 1/2 and 2/3 show a reasonable match.

The distributions of pressure, porosity, and permeability with time around the well R-1 using Steefel and Lasaga model at the \( n \) value of 0.5 are illustrated in Fig. 7. The pressure drop distribution around well R-1 shows steeply within 5 m from the well at late stages of injection. This is because the permeability and porosity decreased in this area. Our results agree with those of Itoi et al. (1987). At places more than 7 m from the well remain in initial value. In this calculation we found that the maximum deposition is 0.01 of the porosity at a distance of 1 m from the well.
CONCLUSIONS

- The long-term experiments of wastewater injection of a geothermal field were numerically analyzed.
- The first order equation rate (Steefel and Lasaga, 1994; Tester et al., 1994; and Carroll et al., 1998) for silica deposition gave a good match to the permeability decline and depth of water level measured in Well R-1 and R-2.
- The silica deposited mainly within 7 m from the well bore and the maximum deposition at distance of 1 m from the injection well.
- The relationship of reaction surface area with porosity is important and should be included when modeling deposition.

REFERENCES


