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STUDY OF DESORPTION IN A VAPOR DOMINATED RESERVOIR WITH FRACTAL GEOMETRY

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ABSTRACT

This paper is an attempt to model well decline in a vapor dominated reservoir with fractal geometry. The fractal network of fractures is treated as a continuum with characteristic anomalous diffusion of pressure. A numerical solver is used to obtain the solution of the partial differential equation including adsorption in the fractal storage space. The decline of the reservoir is found to obey the empirical hyperbolic type relation when adsorption is not present. Desorption does not change the signature of the flow rate decline but shifts it on the time/flow rate axis. Only three out of six model parameters can be estimated from field data, due to the linear correlation between parameters. An application to real well data from The Geysers field is presented together with the estimated reservoir, fractal space and adsorption parameters. Desorption dominated flow is still a questionable approximation for flow in fractal objects.

INTRODUCTION

Classical modeling approaches available in the literature for describing fractured reservoirs are based either on discrete deterministic and/or statistical fracture geometry information or on a lumped parameter multiple (usually dual) continuum approach. In many cases, however, the discrete fracture network approach is bound to fail due to the prohibitively large number of fracture parameters required by the model. This is the case with The Geysers geothermal field for which studies of the fracturing patterns have concluded that it can only be characterized as "effectively random" (Beall and Box, 1989).

At the other extreme, the continuum approach, usually involving the presence of double continua with different storage parameters coexisting in a Euclidean space with dimension D=2 or 1, offers little flexibility for characterization at the reservoir scale. The implicit assumption embedded in these models is that there exists a typical scale at which the variation of reservoir parameters is bounded, in other words that there exists a Representative Elementary Volume (REV).

Alternatively, fractal geometry of fracture networks provides a promising framework for a realistic reservoir model unconstrained by the requirement of REVs and without ruling out the possibility of using the continuum approach. Fractal geometry is characterized by self-similarity over all scales: zooming in a fractal object results in deterministic and/or statistically identical copies of the whole. This repetitivity determines on one hand the degree to which a fractal object fills the embedding space, or in other words the mass fractal dimension D and on the other hand the connectivity of the object, expressed by its spectral dimension. The discontinuous character of a fractal object, as opposed to the perfectly connected embedding Euclidean space, renders functions defined on it nondifferentiable. Thus the very idea of a partial differential equation written for flow in fractals would not be justified unless the properties of the medium were replaced by analytical equivalents. Following the model developed by O'Shaughnessy and Procaccia (1985) for diffusion on fractals, Chang and Yortsos (1990) have established the differential equation which governs flow in a fractal network of fractures. The solution of the welltest problem presented by them illustrates the phenomenon of anomalous diffusion typical for fractal geometry.

Often natural fracture systems have been successfully described within the framework of fractal geometry. Sammis et al. (1991) studied graywacke outcrop fracture patterns at Geysers and found that the scaling relationship characteristic to fractal objects is applicable over a span of scale of two orders of magnitude. An interesting application of fractal analysis to pressure transients in the Geysers field was shown by Acuna et al. (1992) who found that the mass fractal dimension could range between 1.2 and more than 2.0. According to them this type of analysis is more likely to give a plausible explanation for pressure transients than an alternative single finite conductivity fracture model. To our knowledge, no attempt has been made to date to interpret reservoir decline data using the fractal network of fractures model. Studies of well decline at Geysers have usually used the semiempirical Fetkovich method.

It has been suggested that adsorption is a plausible storage mechanism in vapor dominated reservoirs (Nghiem and Ramey, 1991). Measurements of adsorption on cores from Geysers have shown that adsorbed water completely saturates micropores in the reservoir rock (Shang at al., 1993). However, the numerical model set up by Hornbrook (1994) to compute the pressure response to production in a one-dimensional reservoir where adsorption was modeled by Langmuir isotherms has shown very small adsorption effects at late times.

The objective of this paper is to analyze the adsorption effects on reservoir decline when fluid flow takes place in a fractal network of fractures. The partial differential equation established for pressure diffusion and incorporating an adsorption source term is solved using a numerical solver. Following the theoretical analysis of the results, the application of the model to specific well decline data from The Geysers gives an estimate of the model parameters and allows for an evaluation of the present approach.

MATHEMATICAL MODEL

The physical model considered consists of a fracture network embedded in the Euclidean rock matrix, considered to be impermeable. Traditionally the storage and flow properties of the porous medium included in the mathematical model are porosity and permeability, respectively. In a fractal object neither porosity nor permeability have a constant value across the model. Both porosity and permeability are considered to decrease with the Euclidean distance, r, from the well in a power law fashion (Chang and Yortsos, 1988):

$$porosity: \quad \phi = \frac{aV_s}{B} r^{D-d} \tag{1}$$

$$permeability: \quad k = \frac{aV_s}{B} m r^{D-d-\theta} \tag{2}$$

where a is a constant characteristic of the fractal object expressing the number of sites per fractal mass (dimension L^{-D}), V_s the volume of each site assumed constant across the fractal object (dimension L^3), B a geometric constant (of dimension L^{3-D}) which describes the appropriate symmetry ($B = A$, $2\pi h$, 4π for rectilinear, cylindrical and spherical symmetry, respectively, where A and h represent cross-sectional area and reservoir thickness respectively), m is a parameter akin to permeability which expresses connectivity and flow conductance, D is the mass fractal dimension, d is the dimension of the embedding Euclidean space and θ is a parameter related to the spectral dimension of the fractal network (O'Shaughnessy and Procaccia, 1985). Note that for a Euclidean fracture network: $D = d$ and $\theta = 0$. In this case the two relations above reduce to:

$$B\phi = aV_s \tag{3}$$

$$m = \frac{k}{\phi} \tag{4}$$

The partial differential equation describing diffusion on a fractal network of fractures has been derived by Chang and Yortsos (1988). The nondimensional form of this equation and the associated dimensionless group is:

$$(r_D)^\theta \frac{\partial p_D}{\partial t_D} = \frac{\beta}{r_D} \frac{\partial p_D}{\partial r_D} + \frac{\partial^2 p_D}{\partial r_D{}^2} \tag{5}$$

$$r_D = \frac{r}{r_D}; \; t_D = \frac{t}{t\star}; \; p_D = \frac{p_i{}^2 - p^2}{p_i{}^2} \tag{6}$$

$$with \quad t\star = \frac{\overline{c_f \mu} r_w{}^{\theta+2}}{m} \tag{7}$$

where r_w is the well radius and p_i the initial reservoir pressure. The choice for this nondimensionalization of the pressure was determined by the fact that for the purposes of a well decline analysis the boundary condition at the well is usually prescribed (constant/variable) pressure.

Two important observations should be made with respect to this equation: 1) nowhere in Eqn. (5) does the embedding Euclidean space dimension appear and 2) the anomalous diffusion phenomenon is captured by the power law relation between the diffusivity and radial distance.

The addition of a new term representing the desorption of water in the fracture network as a source at each site in the fractal object will bring the partial differential equation to the desired form for studying the adsorption effects on the fluid flow. In order to derive the adsorption source term we will consider the Langmuir isotherm model for the mass X of water adsorbed in a unit volume of rock:

$$X\left(\frac{p}{p_s}\right) = d\frac{c\left(\frac{p}{p_s}\right)}{1+(c-1)\left(\frac{p}{p_s}\right)} \tag{8}$$

where d is the magnitude factor which determines the maximum amount adsorbed at $p = p_s$, c is a shape factor which determines the rate at which desorption occurs and p_s represents the saturation pressure at initial reservoir temperature, hereafter considered equal to the initial pressure in the reservoir. Let V be the volume of rock available for adsorption in a cylinder of radius r. If the space available for adsorption has a mass fractal dimension, Z then:

$$V = V_a r^Z \tag{9}$$

where V_a is a geometric factor of dimension L^{3-Z}. Total adsorbed mass in the cylinder of radius r is, therefore:

$$A_t = dcV_a\frac{\left(\frac{p}{p_s}\right)}{1+(c-1)\left(\frac{p}{p_s}\right)}r^Z\rho_R \tag{10}$$

where ρ_R represents the rock density. The dimensional form of the adsorption source term can be obtained by differentiating A_t with respect to r and t:

$$Q_A = (Z+1)\,r^{Z-1}dcV_a\frac{p_s}{2p\,(p_s+(c-1)\,p)^2}\frac{\partial\left(p^2\right)}{\partial t}\rho_R \tag{11}$$

Expressing p in terms of the nondimensional p_D:

$$p = p_i\sqrt{1-p_D};\quad \frac{\partial\left(p^2\right)}{\partial t_D} = -p_i{}^2\frac{\partial p_D}{\partial t_D} \tag{12}$$

and using the assumption that the reservoir was initially at saturation conditions, $p_i = p_s$, after normalization we obtain:

$$Q_A = Ar_D{}^{\alpha}F\left(p_D\right)\frac{\partial p_D}{\partial t_D} \tag{13}$$

with constant A given by:

$$A = \frac{(Z+1)\,dcV_a m\rho_R}{2\overline{\mu c_f}\rho_s r_w{}^{D+\theta-Z+2}\left(aV_s\right)} \tag{14}$$

where ρ_s is the density of saturated vapour, exponent α is $Z-D$ and the pressure function F is given by:

$$F\left(p_D\right) = \frac{1}{\sqrt{1-p_D}\left(1+(c-1)\sqrt{1-p_D}\right)^2} \tag{15}$$

Note that $F(p_D) > 0$ for any value of the shape factor $c > 0$ and any p_D in $(0,1)$. Also note that large values for this term can be expected when the dimensionless pressure is small: $p_D \rightarrow 0$ (which occurs at the beginning of the drawdown) and the shape factor has a small value: $c \rightarrow 0$ (favorable for desorption). Reasonable estimates of parameter A are of the order of 10^3.

The partial differential equation including desorption effects becomes:

$$\frac{\partial p_D}{\partial t_D}\left[r_D{}^{\theta} + Ar_D{}^{\alpha}F\left(p_D\right)\right] = \frac{\beta}{r_D}\frac{\partial p_D}{\partial t_D} + \frac{\partial^2 p_D}{\partial r_D{}^2} \tag{16}$$

It can be assumed that the dimension of the storage space coincides with the dimension of the adsorption space: $D = Z$.

A numerical solver based on the method of lines was used to solve Eqn. (16) for prescribed wellbore pressure and infinite acting outer boundary conditions. The solution, represented with cubic Hermite polynomials, is presented in the next section.

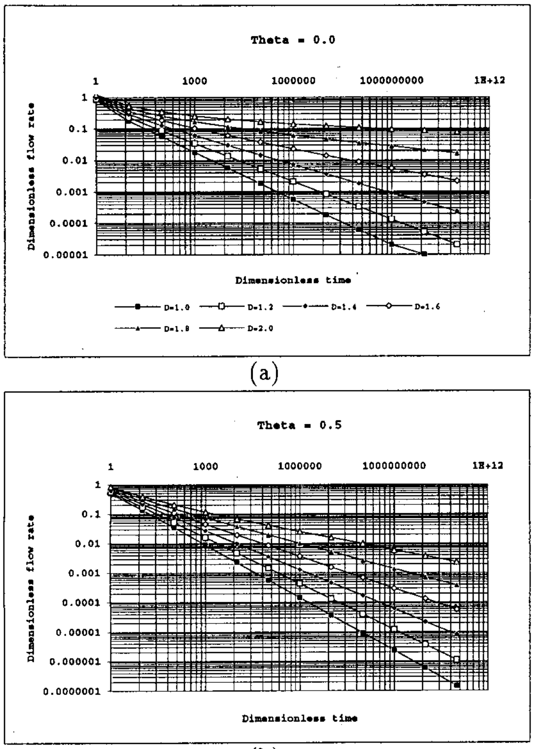
RESULTS AND DISCUSSION

The solution of the equation of flow in a fractal network of fractures was computed first for the case where the adsorption term was neglected ($A = 0$). The results are presented in terms of dimensionless flow rate vs. dimensionless time in Fig. 1. As expected, they reduce to the well-known solutions for one-, two- and three-dimensional cases, when the flow space has a Euclidean geometry. The distinct feature of the solution for $D < 2$ is a practically constant log-log slope at late time:

$$log\ q = a\ log\ t + b \tag{17}$$

$$or\quad q = B\ t^a \tag{18}$$

where a and $B = 10^b$ are constants related to the model parameters: D and θ. Differentiating relation



(a)

(b)

Figure 1: Well decline with no adsorption, (a) $\theta = 0.0$ and (b) $\theta = 0.5$

(18) with respect to time and replacing time t with the power $1/a$ of the flow rate q divided by the constant B we obtain:

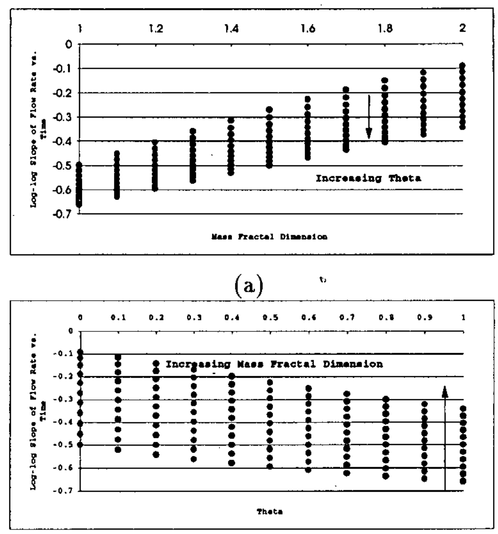
$$\frac{dq}{dt} = \frac{a}{B^{1-\frac{1}{a}}} q^{1-\frac{1}{a}} \tag{19}$$

The generic form of (19):

$$\frac{dq}{dt} = C\ q^{h} \tag{20}$$

is known as the general empirical hyperbolic decline relationship and has been used in specific forms to analyze the decline of wells at The Geysers.

A sensitivity analysis was conducted for the two model parameters involved: mass fractal dimension D and the spectral dimension dependent parameter θ. The results, presented as a graph of the log-log slope of flow rate - time relation versus D and θ respectively (Fig. 2) shows that the solution is more sensitive to the mass fractal dimension D than to the parameter θ, specially at lower values of D (closer to 1.0). Also, Fig.



(a)

(b)

Figure 2: Effects of fractal parameters on the log-log slope of the well decline (a) Mass fractal dimension effect (b) Effect of parameter θ

3 clearly shows that D and θ are linearly correlated.

The next step was to compute the solution for the case where adsorption was present. For this a fractal geometry with parameters $D = 1.9$, $\theta = 0.25$ was considered (in agreement with the results obtained by Acuna et al., 1992, in one of the wells at The Geysers), a dimensionless adsorption parameter between 1 and 100 and an adsorption curve shape coefficient ranging between 0.01 and 10. Fig. 4a) shows the effects of increasing the dimensionless adsorption coefficient. It can be seen that beyond a threshold value of A (10 for this set of parameters) the decline curves start with a plateau

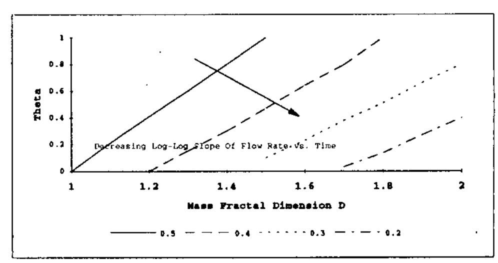


Figure 3: Linear correlation of fractal parameters

value corresponding to a maximum flow rate that can be sustained by the specific geometry of the reservoir. Although the ratio of the flow rates sustained by the reservoir for different dimensionless adsorption coefficients decreases with time, the curves remain distinct during the entire time span examined. The adsorption curve shape coefficient c has practically the same influence as A on the decline curve (curves in Fig. 4b) were obtained for $A = 1000$). The maximum sustainable flow rate was obtained for $c = 1$, which is exactly the value which divides the interval of possible values of c into two classes: the adsorption favorable class ($c > 1$) and the desorption favourable class ($c < 1$). For values of adsorption coefficients in excess of the threshold values the reservoir can be characterized as desorption dominated.

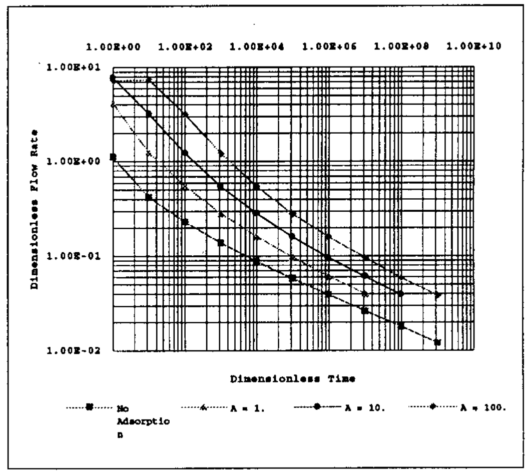
Examining the radius r_i at which interference can be noted (r for which $p_{wf} = 0.01$) it can be seen from Fig. 5a) and 5b) that a power law relation between r_i and the two adsorption coefficients A and θ is applicable. The value of r_i itself for desorption dominated flow is extremely low: $r_i < 300 r_w$ at $t_d = 10^9$, which lays a question mark on the validity of the assumption that pressure can be approximated by its smooth envelope at such small radii.

APPLICATION TO FIELD DATA ON WELL DECLINE

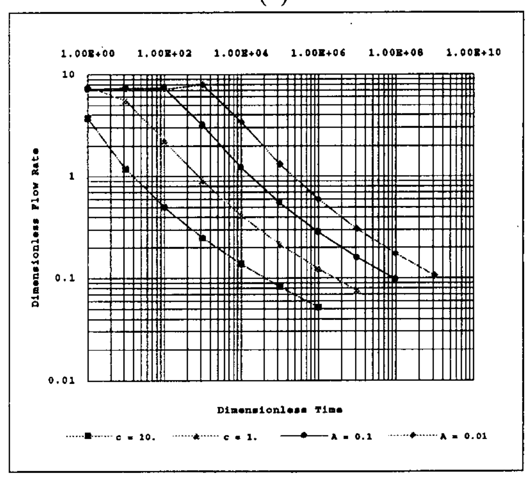
The parameters of the model which have to be estimated from field data are: $q\star$ and $t\star$, the flow rate and time normalizing constants, D and θ, the fractal object parameters and A and c, the adsorption coefficients. However, not all these parameters are independent of one another.

We have seen already from Fig. 3 that the fractal object parameters are not independent. Also, overall both adsorption parameters, A and c, have the effect of shifting the decline curve in time, therefore being correlated with the normalizing time constant. This is of consequence in analyzing field data and estimating reservoir parameters, causing the problem to be poorly constrained.

Another aspect important in estimating the reservoir parameters is that the boundary condition at the well may be very important for the 'signature' of the decline curve. An illustration of the effect of changing the level of the well pressure is given in Fig. 6. The case where p_{wf} is changed from 0.3 to 1.0 within a period of time spanning two orders of magnitude, although less likely to be encountered in practice, shows that the shape of the pressure decline curve can be dis-



(a)



(b)

Figure 4: Effects of adsorption parameters on well decline (a) c=0.01 and variable A and (b) A=1000 and variable c

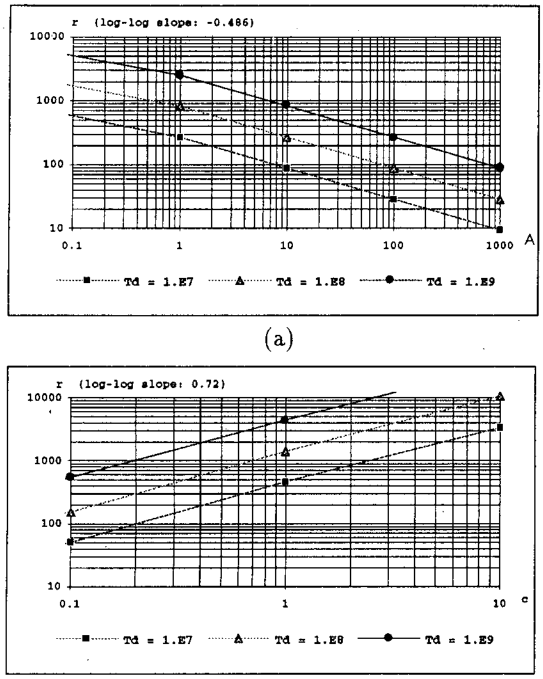


Figure 5: Interference radius (a) vs. adsorption parameter A and (b) vs. adsorption parameter c

torted sufficiently to make a simple type-curve match with type curves computed for constant wellbore pressure impractical. Therefore an automated model fitting using the appropriate boundary conditions is required.

Finally, we can set as a target for an automated type-curve matching program the estimation of 3 out of 6 parameters. As an illustration of the application of the model to real field data we have tried to estimate the reservoir parameters for well LF 425 of Unit 12, Geysers geothermal field. The parameters to be estimated were: $q*$, D, A. The bottomhole corrected pressure was computed using Goyal's method (1986) and used in defining the boundary condition at the well for the partial differential equation. For the short part of the decline data the parameters that were used as input to the program were: $\theta = 0.1$, $t* = 0.19$, $c = 0.1$, and the parameters resulting from the automated evaluation were: $D = 1.15$, $q* = 12.5$, $A = 2137$. This set of parameters is by no means unique. It belongs to an entire family of correlated parameter sets. For instance the data could have been modeled with $D = 1.45$ and $\theta = 0.55$.

In spite of the underdetermination of the model it can,

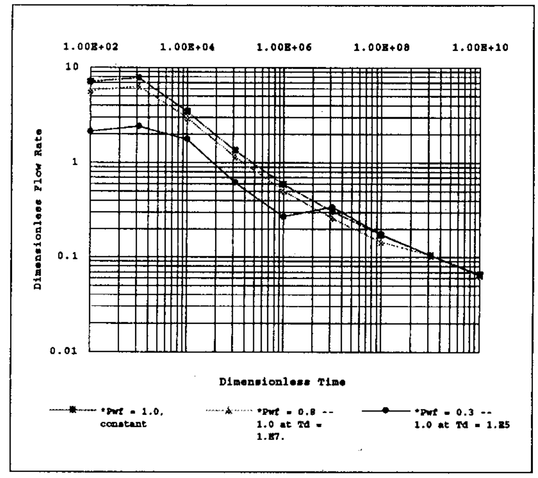


Figure 6: Effects of changing the bottomhole pressure

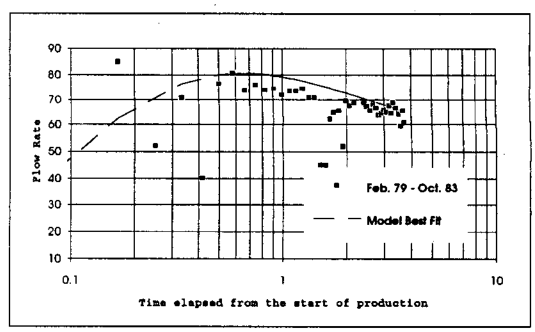


Figure 7: Model fitting to decline data from well LF State 425, The Geysers

nevertheless be used to predict future decline in wells.

CONCLUSIONS

The characteristics of flow denoted by 'anomalous diffusion' are determined by a power law relation of diffusivity and Euclidean radius. Models of flow in a fractal network of fractures require two additional parameters: D, the mass fractal dimension, associated with the storage properties of the reservoir and θ, a parameter representative for the spectral dimension of the fractal object, associated with the permeability field. Since adsorption is determined by the vapor pressure lowering phenomenon it can be assumed that the same mass fractal dimension of the storage space can be applied to the adsorption space.

Flow rate decline in a fractal reservoir without adsorption obeys the general relation for hyperbolic decline. The signature of the flow rate decline in the presence of adsorption does not change significantly at late times. However, at early times a constant maximum attainable flow rate develops when threshold values of the adsorption coefficients A and c are exceeded and desorption becomes dominant. Model results show that when desorption dominantes interference effects may be completely eliminated. A question mark remains as to whether the assumption of a good approximation of the actually nondifferentiable pressure function defined on a discontinuous fractal space is the smooth envelope anymore.

Parameters akin to storage properties: adsorption coefficients A and c and the invariant part of the diffusivity are strongly correlated and thus difficult to estimate from field data on flow rate decline. Reservoir response is less sensitive to the spectral dimension related parameter θ than to the mass fractal dimension D. Estimation of a range of (D,θ) values is possible using real well decline data. Due to the correlation of model parameters a set of maximum three independent parameters out of six can be obtained by automated type curve matching. The model was successfully tested on a well from The Geysers geothermal field.

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