A MODEL OF LOCAL DISTRIBUTION OF SATURATION IN A FRACTURED LAYER, AND ITS APPLICATION

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ABSTRACT
This paper describes a model of local distribution of liquid water (or steam) saturation in a fractured layer. The model, based on the Bernoulli trials as a probability density function of saturation, gives the following relation between the average value of the relative permeability for the water phase, \( k_{rw} \), and the arithmetical mean of saturation, \( S_w \):

\[ k_{rw} = S_w^m, \]

where \( m \) is an index representing the non-uniformity of saturation (\( 1 \leq m < 4 \)). When \( m = 4 \), the saturation is distributed uniformly. The proposed model also gives the average value for the relative permeability of the steam phase, \( k_{rga} \), as follows:

\[ k_{rga} = (k_{w1} + k_{w2})/2 = 0.5, \]

where \( k_{w1} \) and \( k_{w2} \) are the relative permeabilities of the path 1 and 2, respectively. The arithmetical mean of the path 1 and 2 is \( S_w = (S_{w1} + S_{w2})/2 = 0.5 \). Consequently, we have the two kinds of relative permeabilities, \( k_{rw} \) and \( k_{rga} \), which are different from each other. This discrepancy suggests that the relation between relative permeabilities and saturation depends not only on the average of saturation but also its distribution, apparently.

MODELING SATURATION DISTRIBUTION
In the first place, let us consider a simple problem on local distribution of the saturation. Now assume that the two values of water saturation, \( S_{w1} = 0 \) and \( S_{w2} = 1 \) in the flow path 1 and 2, respectively, in a spatial region. Then, the average relative permeability, \( k_{rw} \), of liquid water phase (here in after, referred as water phase) in the region is calculated as follows:

\[ k_{rw} = (k_{w1} + k_{w2})/2 = 0.5, \]

These relations are applied to analysis of some experimental data already reported by the authors. Also, this presentation shows the validity of the Bernoulli trials as a density probability function of saturation in comparison with other kinds of such functions: the normal distribution, the triangle distribution and the beta distribution.

INTRODUCTION
The relative permeability, \( k_r \), plays an important role in analyzing two-phase flow through a porous medium, because \( k_r \) determines the velocity of each phase, \( v \), according to the Darcy's law and the concept of relative permeability (also, saturation) are established.

This presentation indicates a model of local distribution of \( S_w \), using the Bernoulli trials as a probability density function of \( S_w \). The model gives correlation equations between the relative permeabilities and the arithmetical mean of \( S_w \) in the local distribution. Validity of the model is investigated through experimental data.
a unit (e.g., $\Delta x$) of discrete region, whose size is about 10m through 1km in the numerical analysis. $S_w$ is not always distributed uniformly.

Using such a probability density function, $F(S_w)$, Niibori & Chida (1989) defined the average, relative permeabilities, $k_{rwa}$ and $k_{rge}$, respectively, as follows:

$$k_{rwa} = \frac{\int_0^1 F(S_w)k_{rwa}dS_w}{\int_0^1 F(S_w)dS_w},$$

$$k_{rge} = \frac{\int_0^1 F(S_w)k_{rge}dS_w}{\int_0^1 F(S_w)dS_w}.$$  (5, 6)

(Here in after, $k_{rwa}$ and $k_{rge}$ are referred as the apparent relative permeability). Then the arithmetical mean of saturation is

$$S_{wa} = \frac{\int_0^1 F(S_w)S_{wa}dS_w}{\int_0^1 F(S_w)dS_w}.$$  (7)

As for $k_r$ and $k_g$ (not $k_{rwa}$ and $k_{rge}$), we use the empirical equations, Eq.(2) and the following equation, which have been widely used in geothermal reservoir analyses by, for example, Faust & Mercer (1979), Zyvoloski et al. (1980), Sorey et al. (1980), Pruess et al. (1983), O'Sullivan et al. (1985), Gudmundsson et al. (1986) and Niibori et al. (1987):

$$k_r = (1 - S_w)^2(1 - S_w^2),$$  (8)

where $S_w$ is the normalized saturation (Aziz & Settari, 1979, p.33) considering the residual saturations of water and gas phases in the strict sense.

Niibori & Chida (1992) investigated relationships between $k_r$ and $S_{wa}$, assuming some kinds of probability density function of saturation, the Beta distribution, the triangle distribution, the normal distribution and the Bernoulli trials, because it is impossible to determine the probability density function a priori. The results suggest that the average relative permeabilities depend not on kind of such function, but on the arithmetical mean, the standard deviation, $\sigma$, and the skewness $\mu_3/\sigma^3$ of saturation as follows:

$$\sigma^2 = \frac{\int_0^1 F(S_w)(S_w - S_{wa})^2dS_w}{\int_0^1 F(S_w)dS_w},$$

$$\mu_3 = \frac{\int_0^1 F(S_w)(S_w - S_{wa})^3dS_w}{\int_0^1 F(S_w)dS_w}.$$  (9, 10)

For example, the apparent relative permeabilities based on the Beta distribution agree quite well with those on the Bernoulli trials, when the values of $S_{wa}$, $\sigma$ and $\mu_3/\sigma^3$ each coincide with those of the Beta distribution. Such tendency is appeared, even if the probability function is described by any type function (the Beta distribution, the Normal distribution, the triangle distribution, and the Bernoulli trials).

Figure 2(a) displays the regions of $k_{rwa}$ and $k_{rge}$, given by the Beta distribution (with one peak), also comparing the experimental data already reported by Wyckoff & Botset (1936). In the same way, Figure 2(b) shows the regions given by the Bernoulli trials (representing a probability density function with two peaks). These figures suggest that some experimental data are accompanied with the saturation distribution with relatively small value of the standard deviation, because its maximum value for the Beta distribution is smaller than that for the Bernoulli trials under the condition $0 < S_w < 1$.

The Bernoulli trials are composed of $S_{wa}$, $S_{wa}$ ($0 < S_{wa} < S_{wa} < 1$) and $f$ (which is ratio of $S_{wa}$) thus the ratio of $S_{wa}$ is $1-f$, as shown in Figure 3. To derive correlation equations between $k_r$ and $S_w$ for the water- and the gas-phase, respectively, we assume such a distribution as Figure 4. This figure shows an illustration of the saturation distribution on the radial direction, considering the heat exchange surface locally in a fractured layer. In such a case, we assume the smaller saturation $S_{wa}$ to be zero, that is:

$$S_{wa} = 0,$$  (11)

because the region, close to the heat exchange surface, is occupied with the steam-phase. On the other hand, when $S_{wa}$ is described as follows:

$$S_{wa} = S_{wa}^{(m-1)/2},$$  (12)

the model of local distribution of saturation gives

$$k_{rwa} = S_{wa}^m.$$  (13)
Figure 2 Application of the apparent relative permeability, based on (a) the Beta distribution and on (b) the Bernoulli trials, to the experimental data.

Figure 3 Illustration of the Bernoulli trials.

Figure 4 Modeling saturation distribution.
where \( S_{wa} \) is the arithmetical mean of the water saturation. Then, the value of \( f \) is described by

\[
f = S_{wa}^{(4-m)/3}.
\]

(14)

\( m \) is an index representing the non-uniformity of saturation \( (1 \leq m \leq 4) \). When \( m = 4 \), the saturation is distributed uniformly. The proposed model also gives the average value for the relative permeability of the steam phase, \( k_{r_{wa}} \), as follows:

\[
k_{r_{wa}} = 1 - S_{wa}^m - 2S_{wa}^{m^2} - 2S_{wa}^{m+1/3}.
\]

(15)

Figure 5 shows the relation between \( S_{wa} \) and \( k_{r_{wa}} \) for each \( m \) (Fig. 5(a) is for the water-phase, and Fig. 5(b) is for the steam-phase). From Figs. 5(a) and (b), we can recognize that Eqs. (13) and (15), respectively, describe conveniently the regions of \( k_{r_{wa}} \) indicated by Fig. 2(b) for water- and steam-phase.

APPLICATION

Experiments

The experiments are carried out by using a packed glass beads bed submerged horizontally in a thermostat at a temperature of more than 373K (Niibori et al., 1992). Figure 6 shows a schematic diagram of an experimental apparatus. Hydrostatic heads at the inlet and the outlet are fixed by overflows, respectively. Water is injected continuously into the bed under a constant pressure gradient and is heated from the surrounding, and part of the water vaporizes in the bed. When the temperature and the flow rates of steam and hot water attain to the steady state, 0.5 cm³ of ammonia water solution (about five volume percent of \( \text{NH}_3 \)) is injected into the bed as a tracer. The tracer vaporizes in the bed, and ammonia gas flows out of the bed with steam. Then, the \( \text{NH}_3 \) content in the exit gas is measured by a gas chromatography every few minutes to evaluate the tracer response. Temperature in the bed is measured by thermocouples and an AD converter. Flow rates of water and steam at the outlet are also measured by a separator, a cooler and two electric balances. These data are stored to a magnetic disc through a personal computer every 15 seconds. Table 1 summarizes the experimental conditions.

<table>
<thead>
<tr>
<th>Table 1 Experimental conditions</th>
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<tbody>
<tr>
<td>length of the packed bed</td>
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<tr>
<td>Inner diameter of the bed</td>
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<tr>
<td>Average diameter of glass beads</td>
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<tr>
<td>Average porosity</td>
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<tr>
<td>Temperature of thermostat</td>
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<tr>
<td>Hydrostatic head difference</td>
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Figure 6 Schematic diagram of experimental apparatus. (C: cooler, B: packed bed, TW: water tank, TS: three-way tube, TH: thermostat, O: overflow, EB: electric balance, AD: A/D converter, D: disk unit, TC: thermocouples, PC: personal computer.)
The boundary conditions are described by

\begin{align}
0 = 1, \quad S_{wa} = 1, \quad & \text{at } X = 0 \text{ ,} \\
0 = P, \quad \frac{\partial S_{wa}}{\partial X} = 0, \quad & \text{at } X = 1 \text{ .}
\end{align}

In regard to the mass transfer, let us assume that:
(a) the tracer is injected into the water phase at the inlet;
(b) the tracer vaporizes into the steam phase by boiling;
(c) the mixing diffusion of the steam phase is ignored;
(d) the density and the viscosity of each phase, and the mixing diffusivity and the latent heat of the water phase are constant.

These assumptions give the following equations (Niibori et al., 1992):

\begin{align}
(1 - S_{wa}) \frac{\partial C_g}{\partial T} = - \frac{\partial (V_w C_w)}{\partial X} + D_a C_w \text{ ,} \\
S_{wa} \frac{\partial C_w}{\partial T} = - \frac{\partial (V_w C_w)}{\partial X} + \frac{1}{P_e} \frac{\partial}{\partial X} \frac{\partial C_w}{\partial X} - D_a C_w \text{ .}
\end{align}

Dimensionless variables used above are defined as follows:

\begin{align}
C_w &= c_w / c_w^*, \quad C_g = y \rho / c_w^*, \quad P_e = v_w^* / E, \\
T &= \tau / \tau^*, \quad \tau^* = (x_i - x_B) / v_w^*, \quad V_w = v_w / v_w^*, \\
V_g &= v_g / v_w^*, \quad D_a = K_{GT} \tau^* / \epsilon, \\
X &= (x - x_B) / (x_i - x_B), \quad X_{IN} = -x_B / (x_i - x_B) \text{ .}
\end{align}

where $c_w^*$ is the concentration of the tracer at the inlet of the bed, $y$ is the fraction of the tracer in the gas phase, and $K_{GT}$ is the over-all mass transfer coefficient (Bird et al., 1960, p. 654).

The boundary conditions are

\begin{align}
V_w C_w - V_w C_{w+} - \frac{1}{P_e} \frac{\partial C_w}{\partial X}, \quad & \text{at } X = X_{IN} \text{ ,} \\
\frac{\partial C_w}{\partial X} = 0, \quad \frac{\partial C_g}{\partial X} = 0, \quad & \text{at } X = 1 \text{ ,}
\end{align}

where $C_w^*$ is the tracer concentration at

\begin{align}
X = \lim_{\delta \to 0} (X_{IN} - \delta) = X_{IN-} \\
X = \lim_{\delta \to 0} (X_{IN} + \delta) = X_{IN+}
\end{align}

Eq.(22) is derived from the so-called, closed vessel
assumption, which is available in a system which has larger back-mixing magnitude than that of the surrounding (Levenspiel, 1972, p.276).

The initial condition and the tracer injection are described by

\[ T=0, \quad C_w=0, \quad C_g=0, \quad \text{in} \quad X_{IN} \leq X \leq 1, \quad (24) \]

\[ 0 \leq T < T_{in}, \quad C_w=1, \quad \text{at} \quad X=X_{IN}, \quad (25) \]

\[ T=0, \quad T_{in} \leq T, \quad C_w=0, \quad \text{at} \quad X=X_{IN}. \quad (26) \]

where \( T_{in} \) is the injection time of tracer.

Comparisons with the experimental data

\( m, x_g, G, D_a, \) and \( P_e \) are unknown parameters in the numerical model mentioned above. Of them, \( m \) and \( x_g \) are determined from the flow rate into the bed and the temperature data in the bed, respectively. The other parameters \( G, D_a, \) and \( P_e \) are estimated as follows:

(a) The value \( G \) is calculated from the steam flow rate \( Q \) in the steady state, because the relationship between \( G \) and \( Q_g \) is expressed by

\[ G=Q/Q_w^*, \quad (27) \]

(Niibori & Chida, 1992), where \( Q_w^* \) is the characteristic flow rate, defined as follows:

\[ Q_w^* = \pi r_{in}^2 \rho_w v_w^*, \quad (28) \]

where \( r_{in} \) is the inner diameter of the bed, and \( v_w^* \) is velocity of water phase, when the saturation is unity.

(b) The physical meaning of \( D_a \) is the rate constant to describe the mass transfer of tracer from the water phase to the steam phase. The transfer rate is fast enough as compared with the rate through water or steam phase. Thus, the value can be basically assumed to be infinity in this paper. However, in the numerical analysis of the mathematical model, we must set \( D_a \) large enough. Figure 8(a) shows the sensitivity of \( D_a \) on the tracer response, where \( W_{IN} \) is the dimensionless total amount of tracer, \( C_g \) is the dimensionless concentration (the gas content in the steam phase) at the outlet, and \( T \) is the dimensionless time. The sensitivity decreases, as the value of \( D_a \) larger. When the \( D_a \) is greater than 5, the response does not depend on \( D_a \). From these calculations, the value is assumed to be 10 in the numerical analysis. For reference, Figure 8(b) shows the time-change of \( C_w \) at the outlet in the same way as Fig.8(a). Fig.8(b) denotes that the injected tracer almost moves from the water-phase to the steam phase in the bed for case of \( D_a \) larger than 5.

(c) This paper determines the value of \( P_e \) form the tracer response in a single-phase flow system of the bed. Figure 9 denotes the tracer responses without boiling of water. The experiment (Run No.C1) is carried out in room temperature, by using the same bed. The tracer is \( 2cm^3 \) of KCl solution (about \( 7.5mol/m^3 \)), whose concentration at the outlet is estimated by the electronic conductivity. In Fig.9, the experiment data agree quite well with the calculation of \( P_e=60 \).
As mentioned above, all values of the parameters are evaluated in advance. Figure 10 indicates between \( G \) and \( m \), evaluated from the flow rate into the bed in the steady state. From this figure, we can recognize that the value of \( m \) decreases from 4 to about 3, as the value of \( G \) increases.

Figure 10 Relation between the dimensionless heat flow into the bed \( G \) and the value of \( m \) estimating the local distribution of saturation. (symbols ○:290mm, +:145mm in length of the packed bed.)

Figure 11 Comparisons of the calculated values with the experimental data conducted at \( m=3 \).

Figure 12 Comparisons of the calculated values with the experimental data conducted at \( m=4 \).

**CONCLUSION**

We discussed a model of local distribution of saturation in a fractured layer, whose detail structure is not detected. The model, based on the Bernoulli trials as a probability density function of saturation, gives correlation equations of the apparent relative permeabilities to the arithmetical mean of saturation. The equations agrees quit well with the experimental results of mass transfer in water-steam flow with boiling in a permeable medium.
NOMENCLATURE

\(a\): specific surface area \([1/m]\)

\(c\): concentration of tracer \([\text{mol/m}^3]\)

\(c_i\): mixing diffusivity \([\text{m}^2/\text{s}]\)

\(k\): permeability \([\text{m}^2]\)

\(K_{\text{overall}}\): over-all mass transfer coefficient \([\text{s}]\)

\(k_r\): relative permeability \([\text{--}]\)

\(k_{r,\text{apparent}}\): apparent relative permeability \([\text{--}]\)

\(L_v\): latent heat \([\text{J/kg}]\)

\(p\): pressure \([\text{Pa}]\)

\(Q\): flow rate \([\text{kg/s}]\)

\(r\): inner diameter of the bed \([\text{m}]\)

\(r_s\): saturation, ratio in volume \([\text{--}]\)

\(t\): time \([\text{s}]\)

\(t^*\): space time \([\text{s}]\)

\(U\): over-all heat transfer coefficient \([\text{W/m}^2 \text{K}]\)

\(v\): velocity \([\text{m/s}]\)

\(v_w\): velocity of water phase, when the saturation is unity \([\text{m/s}]\)

\(W_0\): total amount of tracer \([\text{--}]\)

\(x\): coordinate in flow direction \([\text{m}]\)

\(x_{\text{bf}}\): leading edge of boiling (position at which water phase starts to vaporize in the bed \([\text{m}]\)

\(y\): fraction of tracer gas \([\text{--}]\)

\(\theta\): temperature \([\text{K}]\)

\(\mu\): viscosity \([\text{Pa s}]\)

\(\rho\): density \([\text{kg/m}^3]\)

\(\sigma\): standard deviation \([\text{--}]\)

Subscript

a: arithmetical mean

f: fluid

g: steam

t: thermostat

w: water

0: inlet

1: outlet

REFERENCES


