A New Derivative Type-Curve for Pressure Buildup Analysis with Boundary Effects

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ABSTRACT

This study investigates pressure buildup behavior of wells with wellbore storage and skin in bounded circular reservoirs, when inner and outer boundary effects interact to fully or partially dominate the well pressure response. Using dimensionless pressure derivative as the dependent variable, we show that early time response is governed by $C_D e^{2s}$ and late time response by r_{eD}^2/C_D . Equations are provided to estimate the limits of the intermediate time period, which corresponds to infinite acting radial flow and a semi-log straight line on a pressure-time graph. We present a new buildup derivative type curve, incorporating inner boundary (early-time) and outer boundary (late-time) effects. Applications of this type curve in buildup test design and interpretation are discussed.

INTRODUCTION

For a well with wellbore storage and skin and producing from a bounded reservoir, it is possible for inner and outer boundary effects to interact and dominate the well pressure response. *Chen and Brigham* (1978) investigated conditions under which such interference might obscure the semi-log straight line, corresponding to infinite acting radial flow, on a Horner buildup graph (*Horner*, 1951). It was found that this could occur even for small values of wellbore storage coefficient in reasonably sized drainage areas. However, they observed that a semi-log straight line with 5-10% error in slope could be found in almost all cases.

In this study, we expand upon the work of *Chen and Brigham* (1978) to identify conditions under which the inner and outer boundary combine to dominate the well pressure response. A second objective is to develop a general buildup type curve incorporating inner and outer boundary effects.

THEORY

The dimensionless wellbore pressure response for a well with storage and damage may be expressed using the convolution theorem as (*van Everdingen and Hurst*, 1949)

$$p_{wD}(t_D) = \frac{2\pi kh}{q\mu} [p_i - p_{wf}] = L^{-1} \left[\frac{1}{C_D l^2 + \frac{l}{S + l\bar{p}_D}} \right] \quad (1)$$

Here, L^{-1} is the inverse Laplace transform operator, l the Laplace space variable, S the skin factor, t_D the dimensionless time defined as

$$t_D = \frac{kt}{\phi\mu c_f r_w^2} \quad (2)$$

and C_D the dimensionless wellbore storage coefficient defined by

$$C_D = \frac{C}{2\pi\phi c_f h r_w^2} \quad (3)$$

In eqn. (1), $\bar{p}_D$ is the appropriate reservoir model in Laplace space, and for the case of a well draining from the center of a closed circle, it is given as (*van Everdingen and Hurst*, 1949)

$$\bar{p}_D = \frac{1}{l^{3/2}} \left\{ \frac{K_0(\sqrt{l}) I_1(r_{eD}\sqrt{l}) + I_0(\sqrt{l}) K_1(r_{eD}\sqrt{l})}{K_1(\sqrt{l}) I_1(r_{eD}\sqrt{l}) - I_1(\sqrt{l}) K_1(r_{eD}\sqrt{l})} \right\} \quad (4)$$

where I_n and K_n are modified Bessel functions, of order n , of the first and second kind respectively, and r_{eD} is the dimensionless outer radius r_e/r_w .

Defining the dimensionless buildup pressure as

$$p_{Ds}(\Delta t_D) = \frac{2\pi kh}{q\mu} [p_{ws} - p_{wf}] \quad (5)$$

we obtain, from eqn. (1), by superposition

$$p_{Ds}(\Delta t_D) = p_{wD}(t_{pD}) + p_{wD}(\Delta t_D) - p_{wD}(t_{pD} + \Delta t_D) \quad (6)$$

where Δt_D is the dimensionless shut-in time and t_{pD} the dimensionless producing time. Eqn. (6) is used for computing buildup pressure response. Appropriate p_{wD} terms are calculated by substituting eqn. (4) in eqn. (1), and then numerically inverting the Laplace space solution with the algorithm of *Stehfest* (1970).

In the recent literature, use of the pressure derivative has been shown to enhance the pressure response signal (e.g. *Bourdet et al.*, 1984). Hence, pressure derivative, rather than pressure, is chosen as

the dependent variable in this work. The dimensionless buildup derivative group is defined as

$$p'_{Ds} = \frac{dp_{Ds}}{d\ln(\Delta t_D)} = \frac{2\pi kh}{q\mu} \frac{d(p_{ws} - p_{wf})}{d\ln(\Delta t_D)} \quad (7)$$

and is easily seen to be the slope of a dimensionless MDH buildup graph (Miller, Dyes and Hutchinson, 1950).

CORRELATIONS

For large producing times such that $t_{pD} > t_{Dps}$, Ramey and Cobb (1971) show that, for a given system, all dimensionless MDH buildup graphs are identical. This observation facilitates eliminating producing time effects from our analysis. Under these conditions, we examined dimensionless buildup behavior for several combinations of C_D , S and r_{eD} and found that two parameters are sufficient to correlate the buildup response when p'_{Ds} is graphed as a function of $\Delta t_D/C_D$. The early-time behavior, reflecting inner boundary (wellbore storage and skin) effects, is controlled by the group C_De^{2S} . Late-time data, influenced by outer boundary effects, are correlatable with r_{eD}^2/C_D . Fig. 1 demonstrates the appropriateness of these correlating parameters for a variety of conditions.

Such behavior suggests that the overall pressure response may be approximated by superposing the independent effects of (a) a well with storage and skin in an infinite system, (b) a well without storage and skin in a finite system. This logic is similar to that used by Bourdet and Gringarten (1980) in generating their double-porosity system type-curves, and forms the basis for developing the new type-curve.

It should be pointed out that the use of C_De^{2S} as a correlating parameter for wellbore storage dominated pressure data was first theoretically justified by Bourdet and Gringarten (1980). Moreover, since the onset of outer boundary effects can be correlated with t_D/r_{eD}^2 for drawdown data, the use of $\Delta t_D/C_D$ as the time group intuitively suggests r_{eD}^2/C_D as the late-time correlating parameter for buildup data. These choices have been validated for buildup derivative data in Fig. 1.

A NEW TYPE-CURVE

Fig. 2 shows the buildup derivative type-curve developed in this study. The dimensionless derivative group p'_{Ds} is graphed as a function of the time group $\Delta t_D/C_D$ on log-log coordinates. As mentioned earlier, the early and late time correlating parameters are C_De^{2S} and r_{eD}^2/C_D respectively. The intermediate time period reflects the degree of interference between inner and outer boundary effects, which determines whether the semi-log straight line will develop or not on a pressure-time graph. On the type-curve of fig. (2), the semi-log straight line is equivalent to a p_{Ds} value of 0.50.

Since the semi-log straight line is integral to conventional analysis of buildup pressure data, it is

Fig. 1. Validation of C_De^{2S} and r_{eD}^2/C_D as correlating parameters.

Fig. 2. New buildup derivative type-curve.

useful to specify its limits. By examining the behavior of p'_{Ds} as it approaches (and deviates from) the value of 0.50, we find that

$$\frac{\Delta t_D}{C_D} \Big|_{begin} \approx 30 \log(C_De^{2S}) \quad (8)$$

$$\frac{\Delta t_D}{C_D} \Big|_{end} \approx \frac{r_{eD}^2}{100 C_D} \quad (9)$$

Conditions under which boundary effects might dominate the well pressure response can also be quantitatively established. Defining interference between inner and outer boundary to be such that less than half a log-cycle of semi-log straight line is present, we have

$$\frac{\Delta t_D}{C_D} \Big|_{end} < 10^{0.5} \frac{\Delta t_D}{C_D} \Big|_{begin} \quad (10)$$

Substitution from eqn. (8) and (9) then yields

$$\frac{r_{eD}^2/C_D}{\log(C_D e^{2S})} > 10000 \quad (11)$$

as the limiting condition for a semi-log straight line to develop on an MDH buildup graph.

APPLICATIONS

The type-curve presented in fig. 2 can be used for buildup test design and interpretation. In test design, an often sought information is the duration of the semi-log straight line. Based on *a priori* estimates of wellbore/reservoir parameters, this can be calculated using eqn. (8) and (9).

For interpretation, field data is plotted on log-log paper with $d(p_{wf}-p_{wi})/d(\log \Delta t)$ on the y-axis and Δt on the x-axis. This is then overlain on fig. (2) and a match is obtained for the entire curve. $C_D e^{2S}$ and r_{eD}^2/C_D are read-off as the parameters of the early and late time match respectively. Permeability-thickness is obtained from the pressure match. C_D can be calculated from the early-time line of unit-slope on the log-log graph, and then both r_{eD} and S can be estimated.

This procedure can be useful when the semi-log straight line has been obscured by interference between inner and outer boundary effects, in which case a conventional semi-log analysis or a type-curve analysis based on existing storage and skin type-curves may not work.

SUMMARY

We have presented a new buildup derivative type curve for wells with storage and damage in bounded circular reservoirs. Equations for estimating the beginning and end of the proper semi-log straight line on an MDH buildup graph are provided. The type-curve should be useful in buildup test design and interpretation for situations when inner and outer boundary effects interact to fully or partially dominate the well pressure response.

NOMENCLATURE

c_t	total system compressibility
C	wellbore storage coefficient
h	formation thickness
k	formation permeability
p_{wf}	flowing sandface pressure
p_{ws}	shut-in sandface pressure
p_i	initial pressure

q	well flow rate
r_w	well bore radius
r_e	external radius
t	time
t_p	producing time
Δt	shut-in time
t_{Dps}	dimensionless time to pseudo-steady state
ϕ	porosity
μ	viscosity

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