PROCEEDINGS, Eleventh Workshop on Geothermal Reservoir Engineering
Stanford University, Stanford, California, January 21-23, 1986
SGP-TR-93

BREAKTHROUGH TIME FOR THE SOURCE-SINK WELL DOUBLET

Will Menninger and Abraham Sageev

Stanford University

Stanford CA 94305

ABSTRACT

A pressure transient analysis method is presented for interpreting breakthrough time between two constant rate wells. The wells are modeled as two line source wells in an infinite reservoir. The first well injects at a constant rate and the second well produces at a constant rate. We studied the effects of transient pressure conditions on breakthrough time. The first arrival of injected fluid at the production well may be significantly longer under transient condition than under steady state condition. A correlation of the deviation of the breakthrough time for transient pressure conditions from the steady state condition is presented.

INTRODUCTION

Pressure transient analysis methods are used to estimate reservoir properties so that exploitation schemes may be evaluated. The method presented in this paper permits the interpretation and design of a special case, which will be referred to as the rate-rate doublet. In this two-well system, one well produces at a constant rate and the other well injects at the same constant rate. The two time dependent parameters in the rate-pressure doublet model are the pressure responses of each well. These parameters and ensuing equations will be described further in the Theory section. We have assumed a unit mobility ratio and that the injected fluid completely displaces the inplace fluid (piston-like displacement).

A constant rate well is approximated as a line source, since the the well-to-well length scale is much larger than the finite radius of the well. The constant rate line source well has been used as a building block for calculating the responses of various reservoir systems. *Theis* (1935) presented the line source pressure solution in an infinite domain. *Carslaw and Jaeger* (1960) and *Van Everdingen and Hurst* (1949) presented the pressure solution for a finite radius well in an infinite system. *Mueller and Witherspoon* (1965) showed the geometrical and time conditions under which the line source and the finite radius solutions are practically identical. They concluded that for observation wells located at a distance twenty times the wellbore radius the line source approximation is applicable. Also, this approximation is applicable for any observation well after a dimensionless time of ten.

Since the diffusivity equation describing the flow in the system is linear, superposition in space of constant rate line sources may be used. *Stallman* (1952) presented the superposition of two constant rate line sources replicating the effects of constant pressure or impermeable linear boundaries. In the same way, superposition of arrays of rate sources (*Kruseman and De Ridder* 1970, and *Ramey et al* 1973) were used to generate the effects of combinations of rectangular boundaries around a well. Using superposition, the line source approximation, and fluid flow particle tracking, transient breakthrough times for a tracer flowing from an injecting well to a producing well are calculated. This paper presents the effects of the transient flow period on the breakthrough time for the rate-rate doublet, and compares the transient and the steady state breakthrough times.

The transient breakthrough time is computed using a fluid flow front tracking technique that is essentially the physical representation of the Euler method for solving differential equations. As initial conditions, we have a particle at the injecting well at a time t_i after both wells were activated. From these initial conditions, equations (6) and (9) give us the instantaneous velocity of the particle at that point. We allow the particle to move a small, finite distance, dx, in a given time, dt, at the calculated velocity. After this move, the velocity is recalculated at the new point in space and time, and the process repeats until the particle arrives at the producing well. As the length of the discrete time increments reduces, the transient numerical solution becomes accurate, and is the basis for a comparison to the steady state solution.

THEORY

Transient Breakthrough Time

The source-sink doublet model uses an imaging method to generate a constant pressure linear boundary (*Carslaw and Jaeger* 1960, *Kruseman and De Ridder* 1970, and *Ramey et al* 1973). Figure 1 presents a schematic diagram of the rate-rate doublet configuration. The dimensionless pressure solution for this model is:

$$P_D = -\frac{1}{2}\left[E_i(-X_1) - E_i(-X_2)\right] \tag{1}$$

where

$$X_i = \frac{r_{Di}^2}{4t_D} \tag{2}$$

and the dimensionless terms are defined as:

$$p_D = \frac{2\pi kh(p_i - p)}{q\mu} \tag{3}$$

$$t_D = \frac{kt}{\phi\mu c_t r_w^2} \tag{4}$$

and

$$r_D = \frac{r}{r_w} \tag{5}$$

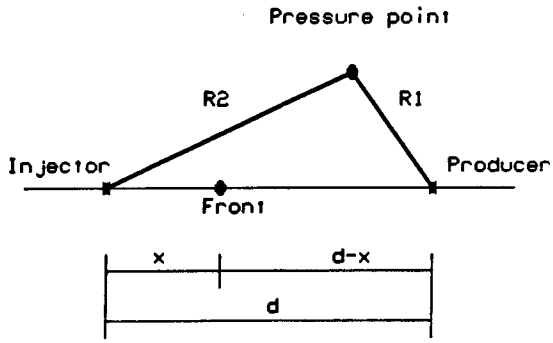

FIGURE 1: A schematic diagram of the rate-rate doublet system.

Since we are only concerned with breakthrough time, the problem can be treated in one dimension. The injecting well is placed at the origin, and the producing well is placed at a distance d away along the x-axis in the positive direction. The breakthrough time is found by tracking fluid flow along the x-axis. The distance between the injecting well and the observation point is r_{D1}, thus, $r_{D1} = x$. The distance between the observation point and the producing well is r_{D2}, thus, $r_{D2} = d - x$ (See Figure 1). The velocity of the fluid is related to the change in pressure with respect to change in distance:

$$V_x = -\frac{k}{\mu}\frac{\partial P}{\partial x} = -\frac{k}{\mu}\frac{q\mu}{2\pi k h r_w}\frac{\partial P_D}{\partial x_D} = \frac{-q}{2\pi h r_w}\frac{\partial P_D}{\partial x_D} \quad (6)$$

The calculation of $\frac{\partial P_D}{\partial x_D}$ is based upon the formula for the derivative of the Exponential Integral:

$$\frac{dE_i(u)}{du} = \frac{1}{u}e^u\,du \quad (7)$$

If Equation (1) is fitted to our one dimensional problem stated above, we obtain the following result:

$$P_D = -\frac{1}{2}\left\{E_i\left[-\frac{\left(\frac{x}{r_w}\right)^2}{4t_D}\right] - E_i\left[-\frac{\left(\frac{d-x}{r_w}\right)^2}{4t_D}\right]\right\} \quad (8)$$

Differentiating Equation (8) yields:

$$\frac{\partial P_D}{\partial x_D} = -\left[\frac{1}{\frac{x}{r_w}}\right]e^{-\frac{\left(\frac{x}{r_w}\right)^2}{4t_D}} - \left[\frac{1}{\frac{d-x}{r_w}}\right]e^{-\frac{\left(\frac{d-x}{r_w}\right)^2}{4t_D}} \quad (9)$$

The particle velocity, V_x, is calculated using Equations (6) and (9).

Steady State Breakthrough Time

Calculation of steady state breakthrough time is achieved by letting time go to infinity. In that case, the exponentials in Equation (9) go to unity, and a steady state condition is achieved:

$$\frac{\partial P_D}{\partial x_D} = -\frac{1}{\frac{d-x}{r_w}} - \frac{1}{\frac{x}{r_w}} \quad (10)$$

Equation (10) implies that the velocity domain is only a function of space, hence an analytical steady state breakthrough time solution may be derived. From mechanics,

$$V_x = \frac{dx}{dt} \quad (11)$$

which implies

$$t_{bt} = \int_0^d \frac{dx}{V_x} \quad (12)$$

Substituting Equation (6) and Equation (10) into Equation (12) yields:

$$t_{bt} = \frac{2\pi h}{q}\int_0^d \left[\frac{1}{\frac{1}{d-x} + \frac{1}{x}}\right]dx \quad (13)$$

which simplifies to:

$$t_{bt} = \frac{2\pi h}{qd}\int_0^d \left[xd - x^2\right]dx \quad (14)$$

Finally, the steady state breakthrough time is (*Brigham* 1979):

$$t_{bt} = \frac{2\pi h d^2}{6q} \quad (15)$$

This resulting breakthrough time for steady state is compared to the transient value.

RESULTS

An algorithm was developed to calculate transient breakthrough time based on three parameters: initial tracer injection time, t_i ; rate of injection, q ; and distance between wells, d. The algorithm sets $x = r_w$ and $t = t_i$ as the starting condition for particle tracking. The velocity is then calculated from x and t using equations (6) and (9). The increment of time, dt, is calculated from the velocity so that $dx = V_x dt$ will be a constant throughout the algorithm. The increment of time, dt, is then added to t and dx is added to x and the process repeats until x is greater than $d - r_w$.

Figure 2 presents a graph where initial injection time is graphed along the x-axis, and q varies for each curve. As expected, each curve asymptotically approaches a constant breakthrough time, the steady state value. The higher curves appear flatter than the bottom curves because of the logarithmic scale.

The difference between steady state and transient breakthrough times depends on various input parameters when the tracer is injected at the same time the two wells are activated. Figure 3 shows graphs of errors between steady state and transient breakthrough times. In this figure, d is graphed along the x-axis to show that the error does not depend on d. All of the curves are very flat. The error does depend on q, though, since there is a distinct curve for each different value of q.

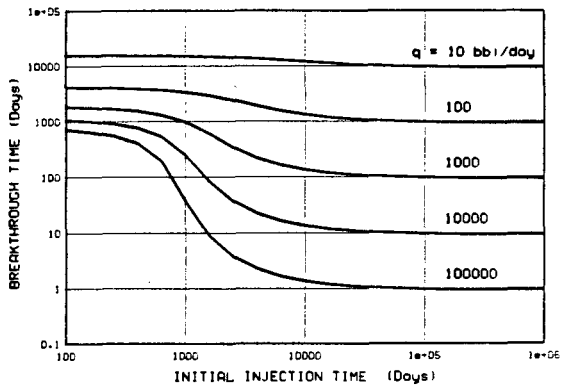

FIGURE 2: Graph of transient breakthrough time with t_i graphed along the x-axis and q varying from 10 to 100000 bbl/day. d is constant at 100 feet.

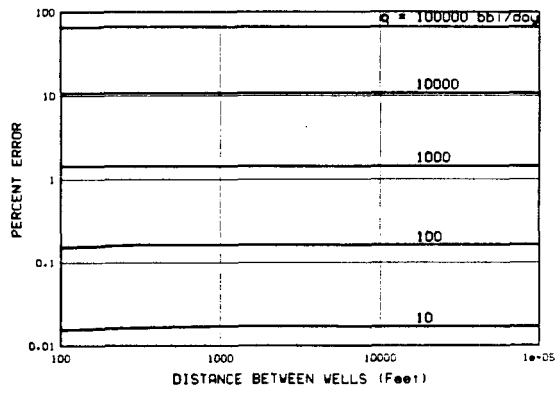

FIGURE 3: Graph of error between steady state and transient breakthrough time. d is graphed along the x-axis, and q varies 10 from 100000 to bbls/day. t_i is constant at 0.0001 days (practically zero).

In Figure 4, we put q along the x-axis, since it is obviously a key parameter that the error depends on. For each different curve, the value of $\mu c_t/hk$ is different. Changing the value of $\mu c_t/hk$ results in four (almost) straight lines that may be translated horizontally. In Figure 5, these four lines have been merged into one line by graphing the value $q\mu c_t/hk$ (10^{-7}) along the x-axis, which achieves a horizontal translation of each curve on a log-log scale. Thus, the error between the transient and the steady state breakthrough time depends on one parameter, $q\mu c_t/hk$.

Figure 5 may be used for designing and interpreting tracer tests when the tracer is injected at the same time the wells are activated. The onset of steady state (for example, when the pressures are less than 1% of the steady state pressures) is not a function of the flow rate. The particle of tracer is less attracted to to the production well during the transient flow period in comparison to the steady state period. Hence, the higher the flow rate the larger the deviation of the transient breakthrough time form the steady state breakthrough time.

Finally, in Figure 6, the distance of the front from the injecting well is graphed vs. the time from tracer injection. This graph shows a different curves for different injection

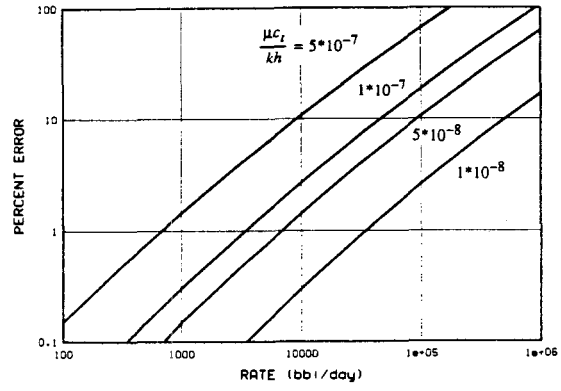

FIGURE 4: Graph of error between steady state and transient breakthrough time. q is graphed along the x-axis. The different curves represent different values of the quantity $\frac{\mu c_t}{hk}$.

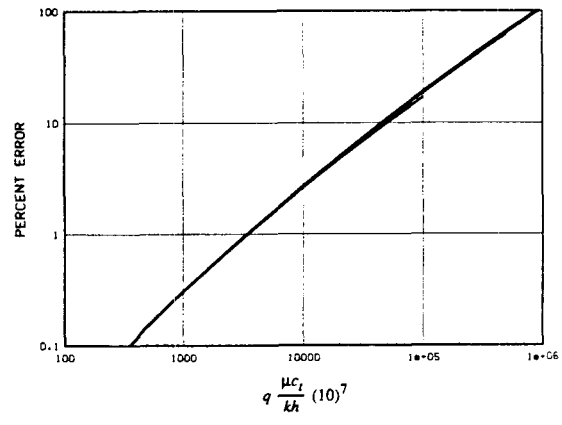

FIGURE 5: The same four curves as in Figure 4, except with $q\frac{\mu c_t}{hk}$ (10^7) graphed along the x-axis.

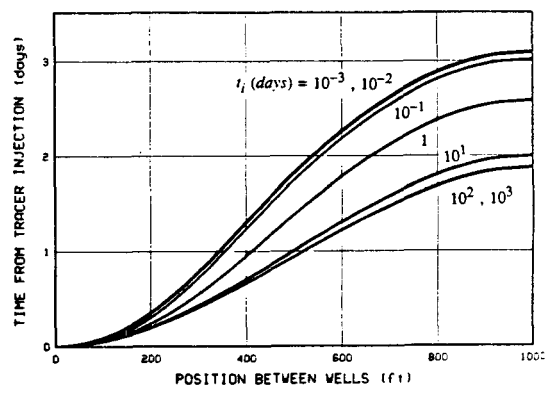

FIGURE 6: The position of the front vs. time from injection for an error parameter set to 50%. The seven different curves represent different initial injection times, from completely transient to steady state.

times. We set the error parameter to give a 50% error when initial injection time is zero. The uppermost curves are the zero injection time curves, with a breakthrough time nearly 50% longer than that of the lowermost curves, which approach steady state. Note that the error is even larger before breakthrough because conditions approach steady state as the particle approaches to the producing well. At half the distance to breakthrough, the error is nearly 100%.

CONCLUSION

1. A single correlating parameter, $q\mu c_t/hk$, describes the effects of the transient flow period on tracer breakthrough time.
2. For systems with high values of $q\mu c_t/hk$, transient breakthrough times will be markedly longer than steady state breakthrough times at early initial injection of the tracer.
3. For all systems, the transient breakthrough times approach the steady state breakthrough times when the tracer is injected after the wells were activated and the system allowed to approach steady state conditions.

ACKNOWLEDGEMENTS

Financial support was provided by the Stanford Geothermal Program, DOE Contract No. DE-AT02-80SF11459, and by Stanford University.

NOMENCLATURE

c_t = compressibility
d = distance between wells
h = formation thickness
k = permeability
p = pressure
p_D = dimensionless pressure, $2\pi kh(p_i-p)/q\mu$
q = rate of injection/production
r = radius
r_d = dimensionless radius, r/r_w
r_w = wellbore radius
t = time
t_{bt} = breakthrough time
t_D = dimensionless time, $kt/\phi\mu c_t r_w^2$
V = velocity
x = distance along x-axis
ϕ = porosity
μ = viscosity

REFERENCES

Brigham, W.E.: Stanford University Advanced Reservoir Engineering Class, Stanford University, 1979.

Carslaw, H.S. and Jaeger, J.C.:*Conduction of Heat in Solids*, 2nd ed. Oxford University Press, 1960.

Kruseman, G.P., and De Ridder, N.A.: "Analysis and Evaluation of Pumping Test Data," International Institute of Land Reclamation and Improvement, Wageningen, The Netherlands (1970).

Mueller, T.D., and Witherspoon, P.A.: "Interference Effects Within Reservoirs and Aquifers," *J. Pet. Tech.* (Oct. 1965) 1803-1812.

Ramey, H.J., Jr., Kumar, A. and Gulati, M.S., "*Gas Well Test Analysis Under Water Drive Conditions*", American Gas Association, Arlington, VA, (1973).

Sageev, A., and Horne, R.N.: "Pressure Transient Analysis of Reservoirs with Linear or Internal Linear Boundaries," SPE 12076, Presented at the 58th Annual Technical Conference and Exhibition, San Francisco, California (Oct. 1983).

Sageev, A., and Horne, R.N.: "Interference Between Constant Rate and Constant Pressure Reservoirs Sharing a Common Aquifer," *SPEJ* (June, 1985) 419-426.

Stallman, R.W., "Nonequilibrium Type Curves Modified for Two-Well Systems", U.S. Geol. Surv., Groundwater Note 3, (1952).

Stehfest, H.: "Algorithm 368, Numerical Inversion of Laplace Transforms," *Communications of the ACM*, D-5 (Jan. 1970) 13, No. 1, 47-49.

Theis, C.V., "The Relationship between the Lowering of Piezometric Surface and Rate and Duration of Discharge of Wells using Groundwater Storage," Trans., AGU, 2, 519, (1935).

Van Everdingen, A.F. and Hurst, W.: "The application of the Laplace Transformation Flow Problems in Reservoirs," *Trans.*, AIME (Dec. 1949) 186, 305-324.