The authors present a three-dimensional model for reservoir simulation to show some behaviors of fluid flow in a geothermal reservoir, assuming water influx and heat conduction from heat sources under the reservoir.

**Basic Equations** The following three equations describe a system, where mass transfer and heat conduction occur.

1. **Mass conservation equation**
   
   \[
   \frac{\partial}{\partial t}(\phi S_{w} \rho_{w} + \phi S_{g} \rho_{g}) = \text{div}(\rho_{w} \mathbf{u}_{w} + \rho_{g} \mathbf{u}_{g}) + q_{w} + q_{g} + q_{\text{we}}
   \]  

2. **Energy balance equation**
   
   \[
   \frac{\partial}{\partial t}\left[\phi S_{w} \rho_{w} U_{w} + \phi S_{g} \rho_{g} U_{g} + (1-\phi)(\rho C)_{f} T\right] = \text{div}(\rho_{w} U_{w} \mathbf{u}_{w} + \rho_{g} U_{g} \mathbf{u}_{g}) + \text{div} \mathbf{q} + q_{h} + q_{l}
   \]

3. **Equation of state (in case of water-steam equilibrium)**
   
   \[
   P = P_{a}(T)
   \]

where, \( \mathbf{u} = -\frac{K}{\mu} \nabla \mathbf{p}, \) \( \mathbf{q} = -T_{c} \nabla T \) and \( S_{w} + S_{g} = 1 \)

The boundary conditions for solving the above equations are \( \partial \mathbf{u} / \partial n = 0 \) and \( \partial T / \partial n = 0 \) for mass and heat flow, respectively. Both mass and heat production terms are also considered to account for water encroachment and heat flow from the boundaries.

Potential equilibrium and heat equilibrium are adopted for the initial conditions which in turn imply no mass flow and steady state heat flow.

**Difference Equations** Equations 1 through 3 are approximated by the finite difference equations as follows:

\[
\frac{\nabla}{\Delta t} \mathbf{S}(\phi S_{w} \rho_{w} + \phi S_{g} \rho_{g}) = \Delta T_{w} \Delta \mathbf{u}_{w} + \Delta T_{g} \Delta \mathbf{u}_{g} - Q_{w} - Q_{g} - Q_{\text{we}}
\]

\[
\frac{\nabla}{\Delta t} \mathbf{S}[\phi S_{w} \rho_{w} U_{w} + \phi S_{g} \rho_{g} U_{g} + (1-\phi)(\rho C)_{f} T] = \Delta H_{w} T_{w} \Delta \mathbf{u}_{w} + \Delta H_{g} T_{g} \Delta \mathbf{u}_{g} + \Delta T_{c} \Delta T - Q_{h} - Q_{l}
\]
\[-P ST + S P = -P^n\]  \quad \text{------ (6)}

The right hand sides of the eqs. 4 and 5 are expressed in terms of \(S P\) using \(\Delta \Xi = \Delta S P + \Delta P^n - P g \Delta Z\). The eqs. 4 and 5 can be written in the following matrix form with \(\delta S_w\), \(\delta T\) and \(\delta P\) as independent variables.

\[
\begin{pmatrix}
C_{11} & C_{12} & C_{13} \\
C_{21} & C_{22} & C_{23} \\
C_{31} & C_{32} & C_{33}
\end{pmatrix}
\begin{pmatrix}
\delta S_w \\
\delta T \\
\delta P
\end{pmatrix}
= 
\begin{pmatrix}
Y_1 \\
Y_2 \\
0
\end{pmatrix}
+ 
\begin{pmatrix}
R_1 \\
R_2 \\
R_3
\end{pmatrix}
\quad \text{----- (7)}
\]

where, \(Y_1 = \Delta (T_w + T_g) \Delta S P\) and \(Y_2 = \Delta (H_w T_w + H_g T_g) \Delta S P\).

By elimination, eq. 7 is transformed into

\[
\begin{pmatrix}
1 & C_{12} & C_{13} \\
0 & 1 & C_{23} \\
0 & 0 & C_{33}
\end{pmatrix}
\begin{pmatrix}
\delta S_w \\
\delta T \\
\delta P
\end{pmatrix}
= 
\begin{pmatrix}
B_{11} & 0 & 0 \\
B_{21} & B_{22} & 0 \\
B_{31} & B_{32} & B_{33}
\end{pmatrix}
\begin{pmatrix}
Y_1 \\
Y_2 \\
0
\end{pmatrix}
+ 
\begin{pmatrix}
R_1' \\
R_2' \\
R_3'
\end{pmatrix}
\]

The third row of the above equation contains only one independent variable, \(\delta P\), which satisfies the following set of finite difference equations.

\[f S P_{k-1} + d S P_{k-1} + b S P_{k-1} + a S P + c S P_{k+1} + e S P_{k+1} + g S P_{k+1} = R_3\]

These equations are solved by the direct method.\(^{62}\) Fig. 1 is a simplified flow chart that shows the program's basic logic.

**Heat Loss**

Heat flow perpendicular to the top and bottom boundaries is assumed. In calculating heat flow at the boundaries, some more blocks are added above and below the reservoir. The heat conduction equation,

\[
\frac{2}{2z} \left[ T_c \frac{\partial T}{\partial z} \right] = \rho c_p \frac{\partial T}{\partial t}
\]

is solved in such blocks with appropriate boundary and initial conditions at the newly formed boundaries. In this procedure, the heat flow at the new time step calculated with the following equation.

\[Q_L^{n+1} = Q_L^n + \alpha \delta T, \quad \alpha = A \cdot T_0 / L \cdot \text{erf} \left( \frac{L/2}{\sqrt{2 \kappa N t}} \right)\]
Water Influx Based on the solution of the diffusivity equation for the linear flow case by van Everdingen and Hurst, the water influx is

\[
Q_{we} = A \phi C_w (\Delta P + \Delta P^\prime) Q(t) - A \phi C_w \Delta P^\prime \frac{Q(t)}{\Delta t}
\]

The reservoir is divided into 10 \times 10 \times 5 blocks in \(X - Y - Z\) directions, respectively (Fig. 2). The size of the blocks and physical properties of the rock and fluid are shown in Table 1. For simplicity, the model treats an initially hot-water system and the bottom hole pressure of the well is assumed to be constant. Plots of potential distributions at various times are shown in Fig. 3 without water influx, while Fig. 4 with water influx.

**NOMENCLATURE**

\[
\begin{align*}
A &= \text{cross sectional area, cm}^2 \\
C &= \text{compressibility, vol/vol-atm} \\
C_p &= \text{specific heat, cal/g-°C} \\
\hat{H} &= \text{enthalpy, cal/g} \\
g &= \text{gravitational acceleration} \\
K &= \text{absolute permeability, darcy} \\
k_r &= \text{relative permeability} \\
T_c &= \text{thermal conductivity, cal/cm-°C-sec} \\
P &= \text{pressure, atm} \\
Q &= \text{production rate, g/sec} \\
Q_{H} &= \text{enthalpy production rate, } H_w Q_w + H_g Q_g, \text{ cal/sec} \\
Q_L &= \text{rate of heat loss to surroundings, cal/sec} \\
Q(t) &= \text{fluid influx, dimensionless} \\
S_w &= \text{saturation, fraction} \\
t &= \text{time, sec} \\
T &= \text{temperature, °C} \\
T_w &= \text{water transmissibility, } f \cdot A K_r / \mu L, \text{ g/atm-sec} \\
T_s &= \text{steam transmissibility, } g/atm-sec \\
\bar{U} &= \text{internal energy, cal/g} \\
V &= \text{bulk volume, cm}^3 \\
Z &= \text{depth, cm} \\
\Phi &= \text{potential, } P - \int \rho g \, dZ, \text{ atm} \\
\phi &= \text{porosity, fraction} \\
\delta &= \text{time difference, } S_P = P^{n+1} - P^n \\
\Delta t &= \text{time increment, } t^{n+1} - t^n \\
\mu &= \text{viscosity, c.p} \\
\rho &= \text{density, g/cm}^3 \\
k &= \text{diffusivity, } T_c / \rho C_p, \text{ cm}^2/sec \\
P'_{ST} &= \text{derivative of the saturated curve with respect to temperature} \\
\end{align*}
\]

Subscripts

\(we\) = water encroachment, \(f\) = formation

\(w\) = water, \(g\) = steam, \(i,j,k\) = grid, \(n\) = time level
REFERENCES

4) JSME Steam Tables, The Japan Society of Mechanical Engineering (1968)

Table 1. Example data of Physical Properties and Block Dimensions

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Initial temperature (1,1,1)</td>
<td>260°C</td>
</tr>
<tr>
<td>Initial temperature gradient</td>
<td>5°C/100m</td>
</tr>
<tr>
<td>Initial pressure (1,1,1)</td>
<td>70 atm</td>
</tr>
<tr>
<td>Horizontal permeability</td>
<td>100 md</td>
</tr>
<tr>
<td>Vertical permeability</td>
<td>10 md</td>
</tr>
<tr>
<td>Porosity</td>
<td>0.38</td>
</tr>
<tr>
<td>Initial water saturation</td>
<td>1.0</td>
</tr>
<tr>
<td>Water compressibility</td>
<td>1.65x10^{-4} vol/vol-atm</td>
</tr>
<tr>
<td>Formation compressibility</td>
<td>4.4x10^{-5} vol/vol-atm</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>1.53x10^{-3} cal/cm·°C·sec</td>
</tr>
<tr>
<td>NX = 10, NY = 10, NZ = 5</td>
<td></td>
</tr>
<tr>
<td>ΔX = 150m, ΔY = 150m, ΔZ = 15m</td>
<td></td>
</tr>
<tr>
<td>Initial production rate</td>
<td>1.2x10^4 g/sec</td>
</tr>
<tr>
<td>Bottom-hole pressure</td>
<td>60 atm</td>
</tr>
</tbody>
</table>
Calculate initial values (P, T, f', H and U) corresponding to initial conditions (no mass flow and steady state heat flow), using look-up tables of steam.

Calculate heat flow at the top and bottom boundaries, Q_L.

Calculate water influx, Q_w.

Calculate production rate of mass and enthalpy, Q_w, Q_g and Q_H.

Calculate transmissibilities and densities between blocks, T_w, T_g, f_w and f_g.

Calculate α term.

Calculate ∂f/∂T, ∂g/∂T, ∂f/∂P and ∂g/∂P.

Calculate coefficients C_1~C_3 for δS_w, δT and δP and residuals R_1, R_2 and R_3.

Calculate δP by the direct method using D4 ordering scheme.

Convergence?

yes

Calculate δT and δS_w.

Calculate P_{w+1}, T_{w+1} and S_{w+1}.

Print P, T and S_w.

no

t = t_{max}?

yes

End

Fig. 1 Simplified Flow Chart
Fig. 2 Geometry of 3-D Geothermal Reservoir Model

Fig. 3 Potential Distribution Behavior (without water influx)
Solid curve, after 30 days of production. Dashed curve, after 50 days of production.

Fig. 4 Potential Distribution Behavior (with water influx)
Solid curve, after 50 days of production. Dashed curve, after 100 days of production.