SHUT-IN AND FLOWING BOTTOM HOLE PRESSURE CALCULATION FOR GEOTHERMAL STEAM WELLS

Michael J. Economides
Shell Oil Company
Houston, Texas, USA

ABSTRACT

The success of pressure transient analysis often depends on the accurate measurement or estimation of the bottom hole pressure. Measurement can be accomplished by a descending probe. Estimation may be realized via a calculation method.

Although a pressure survey may be a more desirable mode, it is nevertheless time consuming and costly. In the case of a geothermal well, two additional shortcomings are obvious: high temperature and frequent presence of highly corrosive noncondensable gases. The latter may render expensive pressure "bombs" obsolete.

A calculation method for predicting bottom hole pressures based on easily obtainable wellhead parameters is therefore not only desirable, but necessary. Several correlations are presently available. This paper presents four calculation procedures for the estimation of bottom hole pressures. Two of the methods are for static pressure, suitable for buildup analysis, while the remaining two are for flowing wells.

In both cases, the first procedure is the established, classic technique followed by a novel correlation, tailored to suit wells that operate at either saturated or slightly superheated conditions. These correlations are particularly applicable to the reservoirs in The Geysers area, where the properties of the geothermal fluid closely match the assumptions in this paper.

INTRODUCTION

Reservoir engineering principles long established in oil and gas fields have proven valid in geothermal reservoirs. In spite of minor idiosyncracies, saturated or superheated steam reservoirs behave like gas condensate or pure gas reservoirs. In this vein, the classic methods for calculating bottom hole pressures for gas wells can be readily extended to steam wells. Perhaps the best known are: (a) the method for static and flowing gas columns outlined in the State of Texas Railroad Commission Back-Pressure Manual,[1] and (b) the Static and Flowing Gas Column Method, by Cullender and Smith.[2] The latter method is based upon a mechanical energy balance. The calculation procedure for the flowing bottom hole pressure that is presented in this report is also based on a mechanical energy balance. Fowler[3] and Sukkar and Cornell[4] presented a general correlation in which they utilized an integral form of the gas law deviation factor, Z, and they assumed a

constant average temperature. This is not a shortcoming of the method, since the length of the wellbore can be divided into several sections. The Sukkar and Cornell method requires, however, graphical interpolations, a somewhat cumbersome procedure. Messer et al.[5] presented a method for the calculation of the bottom hole pressures for "deep, hot, sour gas wells," which inadvertently gave rise to the method for the flowing case presented in this report.

DISCUSSION

The classic approach to the shut-in bottom hole pressure calculation originates from the pressure gradient in a gas column (description and units of all variables appear in the nomenclature section).

$$\frac{dP}{dH} = \frac{\rho}{144} \tag{1}$$

The gas law can be expressed as:

$$PV = ZNRT \tag{2}$$

from which an expression for density, ρ, can be extracted:

$$\rho = \frac{PM}{ZRT} \tag{3}$$

where M is the molecular weight. From Eqs. 1 and 3, we can easily develop an equation of the form:

$$\frac{dP}{dH} = .01165\ \frac{P}{ZT} \tag{4}$$

The constant is a result of introducing the value of the molecular weight of steam (18), the universal gas constant, R (10.73 psi ft^3/lb mole °R), and converting psf to psi. Equation 4 can be formally integrated over the range of the wellbore, yielding:

$$P_{ws} = P_{ts} e^{.01165\ H/\overline{ZT}} \tag{5}$$

where $\overline{Z}$ and $\overline{T}$ are constant average values. Equation 5 can be easily solved in the case of saturated steam via trial and error. One may assume a bottom hole pressure, P_{ws}, which will in turn furnish a bottom hole temperature (from the steam tables). Armed with the pressures and temperatures of both wellhead and bottom hole, the averages can be calculated which can then provide a value for the gas deviation factor, Z. P_{ws} can then be calculated via Eq. 5 and compared with the assumed value. The procedure can be repeated until a desirable agreement is attained. An example calculation using this approach can be found in Appendix C.

The method just described is of a general utility. It can be applied in the case of geothermal wells that operate anywhere in the domain of saturated steam or superheated steam. In the case of reservoirs such as the ones at The Geysers, where the product is either saturated or slightly superheated, the following analysis is proposed.

By simple observation (see Fig. 1), one may reach the seemingly unexpected conclusion that vapor density is a linear function of pressure in the usual range of a geothermal steam well. Appendix A contains some theoretical justification for this observation. The function is of the form:

$$\rho = \alpha + \beta P \tag{6}$$

Introduction of Eq. 6 in Eq. 1, separation of variables, and integration yields:

$$\int_0^H \frac{dH}{144} = \int_{P_{ts}}^{P_{ws}} \frac{dP}{\alpha+\beta P} = \frac{1}{\beta} \ln (\alpha+\beta P) \Bigg|_{P_{ts}}^{P_{ws}} \tag{7}$$

The constants α and β can be obtained with a least squares fit of readily available steam table data. Following such a fit of values between 50 psia and 500 psia, the calculated constants are:

$$\alpha = .01267$$

$$\beta = .00212$$

Equation 7 can be then manipulated into the following form:

$$P_{ws} = -6 + (6+P_{ts})e^{.000015\ H} \tag{8}$$

The major and obvious advantage of Eq. 8 is that it can afford direct calculation of P_{ws} without trial and error. One needs only the wellhead pressure, P_{ts}, and the depth, H. P_{ws} values for a geothermal well at The Geysers calculated by the two methods described above are the same (to the nearest psi), as shown in Appendix C.

Both methods described above presume a wellbore that is truly static. Lingering transient effects may influence the accuracy of the methods at early shut-in times due to inertia, totally disregarded in the original assumptions. The phenomenon has not escaped the attention of reservoir engineers. An oscillating front may be isolated and tested for the momentum causing force. The latter is related to the driving force in the reservoir, which in turn can be analyzed for the estimation of reservoir parameters.

However, for the purposes of this report, very early transient analysis becomes a moot point since the intent of bottom hole pressure calculation is to assist well testing techniques. In order to avoid both storage and inertia effects, all analyzable points are 1-1/2 log cycles of time removed from the cessation of the characteristic 45° line (in a log-log plot of ΔP versus time). Wholesale credence should not be attributed to calculated bottom hole pressures at very early times. The results would be useful, though, in observing the effects of the oscillating front and its duration.

DYNAMIC OR FLOWING BOTTOM HOLE PRESSURE

The so-called exponential form of the flowing bottom hole pressure calculation is a modification of the static column method which utilizes the Moody friction factor and an average $\overline{Z}$ and $\overline{T}$.[6] The most familiar form of the equation is:

$$\frac{BP_{wf}^2 + A\overline{Z}^2P_{pc}^2}{BP_{tf}^2 + A\overline{Z}^2P_{pc}^2} = e^{.0375\ L/\overline{TZ}} \tag{9}$$

where $B = H/L$ and $A = 667\ fmq^2\overline{T}^2/d^5P_{pc}^2$

Equation 9 is generally applicable to any gas column. The same equation can be modified to apply in a geothermal well:

$$BP_{wf}^2 + A = (BP_{tf}^2 + A)e^{.0233H/\overline{ZT}} \tag{10}$$

$$\text{where } A = \frac{1.719 \times 10^{-4}\ fm\overline{Z}^2\overline{T}^2W^2}{d^5} \tag{10a}$$

The procedure dictated by this correlation is again trial and error. One assumes P_{wf} obtains T_{wf}, $\overline{T}$, $\overline{P}$, and $\overline{Z}$. Equations 10 and 10a can then be used to calculate P_{wf} and compare it to the assumed value. An example calculation illustrating this approach is provided in Appendix C.

Equation 10 can be applied to a static gas column as well. If there is no flow, A = 0; by simply taking the square root of both sides of the equation, we can extract Eq. 5.

The calculation method to be described uses the same original basis as Cullender and Smith, Sukkar and Cornell, and Messer et al. It will digress, though, from the somewhat cumbersome procedures that the first two methods require, thanks to the apparent linearity of the gas deviation factor, Z, with respect to pressure over the domain of interest of a geothermal well.

All of the above-mentioned methods start from a mechanical energy balance:

$$VdP + dH + \frac{UdU}{g_c} + dWf = -dWs \tag{11}$$

where VdP = pressure - volume potential energy

dH = potential energy due to position

$\frac{UdU}{g_c}$ = kinetic energy

dWf = friction loss

dWs = shaft work

Neglecting kinetic energy and shaft work, introducing the value of the friction loss and representing velocity by U = WV/A*, the following equation emerges:

$$VdP + \frac{H}{L}\,dL + \frac{fmW^2V^2dL}{2g_cDA^{*2}} = 0 \tag{12}$$

Equation 12 can be solved for dL, yielding:

$$dL = \frac{-\frac{1}{V}\,dP}{\frac{f_mW^2}{2g_cDA^{*2}} + \frac{H}{LV^2}} \tag{13}$$

The value of the specific volume, V, can be calculated using the gas law and, in the case of steam:

$$V = 85.84\ \bar{T}Z/P \tag{14}$$

Equations 13 and 14 can be then combined and manipulated into:

$$\frac{.01165(H/L)}{\bar{T}}\,dL = \frac{-ZdP/P}{1 + \frac{4.84x10^{-4}\ fmW^2\bar{T}^2(Z/P)^2}{d^5(H/L)}} \tag{15}$$

Introducing a parameter A:

$$A = \frac{4.84x10^{-4}\ fmW^2\bar{T}^2}{d^5P_c^{\,2}(H/L)}$$

Then Eq. 15 becomes:

$$\frac{.01165(H/L)dL}{\bar{T}} = \frac{-(Z/P)dp}{1+A(Z/P)^2P_c^{\,2}} \tag{16}$$

Remembering that $P/P_c = P_r$ and that $dP/P = dP_r/P_r$, then Eq. 16 becomes:

$$\frac{.01165(H/L)dL}{\bar{T}} = \frac{Z\,\frac{dPr}{Pr}}{1+A(Z/P_r)^2} \tag{17}$$

Again, observing a plot of the gas deviation factor as a function of pressure, with temperature as a parameter, one concludes that for the domain of interest of a geothermal well, the relationship is linear and of the form:

$$Z = 1 + mP_r \tag{18}$$

Figure 2 is a plot that can be described by Eq. 18. The intercept is equal to unity, while the slope, m, is a characteristic of the flowing average temperature. Equation 18 can be combined with Eq. 17 to yield:

$$\frac{.01165(H/L)dh}{\overline{T}} = \frac{(1+mP_r)\frac{dP_r}{P_r}}{1+\frac{A(1+mP_r)^2}{(P_r)}} \qquad (19)$$

The left side of Eq. 19 can be integrated readily, whereas the right side can be formally integrated with some difficulty. Appendix B contains the result of the integration. Although the equation thus obtained is far from being linear, it can be easily programmed in a handheld calculator to obtain direct readings for the bottom hole pressure.

NOMENCLATURE

A^* = cross-sectional area of pipe, ft^2
d = diameter of pipe, inches
f_M = Moody friction factor
g_c = 32.174, conversion factor, (ft-lb mass/(ft-lb force))$(ft/sec)^2$
H = vertical distance, ft
γ = gas gravity (to air)
L = distance along tubing, ft
n = number of moles
P = pressure, psia
P_c = critical pressure, psia
P_r = reduced pressure
P_t = tubing head pressure, psia
P_w = bottom hole pressure, psia
q = flowrate at 50°F and 14.65 psia, MMscf/D
R = gas law constant
T = average temperature, °R
U = velocity, ft/sec
V' = total volume, ft^3
V_m = molar volume, scf/lb-mol (see Eq. A-8)
V = specific volume of flowing fluid, cu ft/lb mass
w = mass flowrate, lb mass/hr
W_f = energy loss due to friction, ft-lb force/lb mass
W_s = shaft work done by flowing fluid
Z = gas law deviation factor
ρ = density
λ = molal heat of vaporization

REFERENCES

1. Anon.: "Back-Pressure Test for Natural Gas Wells," State of Texas, Railroad Commission, Oil & Gas Division.

2. Cullender, M.H., and Smith, R.V.: "Practical Solution of Gas-Flow Equations for Wells and Pipelines with Large Temperature Gradients," Trans. AIME (1956), 207, 281-287.

3. Fowler, F.C.: "Calculations of Bottom-Hole Pressures," Petrol. Engr. (1947), 19, No. 3, 88-90.

4. Sukkar, Y.K., and Cornell, D.: "Direct Calculation of Bottom-Hole Pressures in Natural Gas Wells," Trans., AIME (1955), 204, 43-48.

5. Messer, P.H., Raghavan, R., and Ramey, H.J., Jr.: "Calculation of Bottom Hole Pressures for Deep, Hot, Sour Gas Wells," J. Pet. Tech. (Jan. 1974), 85.

6. Katz, D.K., and Coats, K.H.: Underground Storage of Fluids (1968), 145.

APPENDIX A

The apparent linearity between vapor density and pressure at the saturation condition can be aptly demonstrated using the Clausius/Clapeyron equation and the gas equation.

The Clausius-Clapeyron equation is:

$$\frac{dP^*}{dT} = \frac{\lambda}{T(Vg-Vl)} \qquad \text{(A-1)}$$

Where P^* = Vapor pressure
λ = Molal heat of vaporization
Vg, Vl = Specific volume of gas and liquid respectively

The gas law states that

$$P = \frac{Z\rho RT}{M} \qquad \text{(A-2)}$$

Differentiating (A-2) with respect to pressure yields:

$$1 = \frac{R}{M}\left(\rho T \frac{dZ}{dP^*} + TZ\frac{d\rho}{dP^*} + Z\rho\frac{dT}{dP^*}\right) \qquad \text{(A-3)}$$

The first term inside the parenthesis can be neglected since it has a very small value.

Combining (A-1) and (A-3) and rearranging one can solve for $\frac{d\rho}{dP}$:

$$\frac{d\rho}{dP} = \frac{M}{ZRT} - \frac{1}{\lambda} \qquad \text{(A-4)}$$

It can be easily concluded that the right hand side of the equation is roughly constant for the range of geothermal interest.

APPENDIX B

The right hand side of equation (19) is

$$\int_{Pt_r}^{Pw_r} \frac{\left\{1 + mP_r\right\}\frac{dP_r}{P_r}}{1 + A\left(\frac{1+mP_r}{P_r}\right)^2} =$$

and it can be separated into:

$$\int_{Pt_r}^{Pw_r} \frac{\frac{dP_r}{P_r}}{1+A\left(\frac{1+mP_r}{P_r}\right)^2} + \int_{Pt_r}^{Pw_r} \frac{mdP_r}{1+A\left(\frac{1+mP_r}{P_r}\right)^2}$$

The first integral can be evaluated and it yields:

$$\int_{P_{tr}}^{Pw_r} \frac{\frac{dP_r}{P_r}}{1+A\left(\frac{1+mP_r}{P_r}\right)^2} = \frac{1}{m^2A+1}\left[\ln\left\{\frac{\sqrt{A\left[\frac{1+m}{P_r}\right]^2 + 1}}{\sqrt{A}/P_r}\right\}\right.$$

$$\left. + m\sqrt{A}\tan^{-1}\left(\sqrt{A}\left(\frac{1}{P_r} + m\right)\right)\right]_{P_{tr}}^{P_{wr}}$$

The second integral is:

$$\int_{P_{tr}}^{P_{wr}} \frac{mdP_r}{1+A\left(\frac{1+mP_r}{P_r}\right)^2} = -m\sqrt{A}\left(\frac{m\sqrt{A}\,(m^2A-1)}{\left[m^2A-1\right]^2+4m^2A} - \frac{m\sqrt{A}}{\left[m^2A+1\right]}\right)\ln \frac{\sqrt{A}}{P_r}$$

$$- \frac{1}{\left[1+m^2A\right]\left[\sqrt{A}/P_r\right]} + \frac{m\sqrt{A}\ln\left[\left\{1/Pr+m\right\}^2A+1\right]}{\left[m^2A-1\right]^2+4m^2A}$$

$$+ \frac{m^2A-1}{(m^2A-1)^2+4m^2A}\tan^{-1}\left[(1/P_r+m)\sqrt{A}\right]\Bigg|_{P_{tr}}^{P_{wr}}$$

APPENDIX C: SAMPLE CALCULATIONS

Static Bottom Hole Pressure

For the sample calculation of the static bottom hole pressure, the following field data will be given: pressure history of a buildup test and the depth of the well. The product is either saturated or slightly superheated steam.

H = 6615 ft

t (sec)	p (psig)	t (min)	P (psig)
0	204	1	227
5	205	5	272
10	208	10	300
15	211	15	312
20	214	20	324 (etc.)

Equation 5 can be utilized:

$$P_{ws} = P_{ts} e^{.01165H/\overline{ZT}}$$

At time 5 min, the wellhead pressure, P_{ts}, is 272 psig or 285 psia. (Atmospheric pressure is 13.2 psia). From the steam tables, the wellhead temperature, T_{ts}, is 412°F. At first trial, assume a bottom hole pressure of 320 psia, which furnishes a bottom hole temperature of 423°F. Average pressure, $\overline{P}$, and temperature ($\overline{T}$) are 302.5 psia and 877.5°R, respectively. The gas deviation factor, Z, is .905. Using Eq. 5, the P_{ws} calculated is 314 psia, which varies from the assumed value. A second trial with P_{ws} assumed = 315 psia is successful, with P_{ws} calculated = 315 psia.

Alternately, Eq. 8 avoids the trial and error approach:

$$P_{ws} = -6 + (6+P_{ts})e^{.000015H}$$

For t = 5 min, P_{ts} = 285 psia and H = 6,615 ft

$$P_{ws} = 315 \text{ psia}$$

Flowing Bottom Hole Pressure

For the flowing bottom hole pressure calculation, the following field data are given: pressure history, depth (7,500 ft), casing diameter (9-5/8"), H/L = 1, and flowrate (100,000 lb/hr). Geothermal fluid is either saturated or superheated vapor. Equations 10 and 10a can be used to calculate the flowing bottom hole pressure for this well.

Let the wellhead pressure be 400 psia. The first task is to get a value for the Moody friction factor (f_M) that appears in Eq. 10a. A cursory look at the Moody friction factor chart would instruct that at highly turbulent flow ($Re > 10^6$), the friction factor depends only on the

relative roughness of the conduit. Such highly turbulent flow is mostly the rule in geothermal steam wells. The Reynolds number in the example can be calculated by the equation:

$$Re = \frac{6.32\ W}{d\mu}$$

where w = flowrate (lb/hr)
d = diameter (in)
μ = viscosity (cp)

$$Re = \frac{6.32 \times 100{,}000}{9.625 \times .01696} = 3.9 \times 10^6$$

The roughness of the casing is $\varepsilon = .0015$ in., and the relative roughness $\varepsilon/d = .00016$ ($d = 9.625$ in.). The Moody friction factor is then $f_M = .0135$ (from Fig. 3). The same value of f_M is obtained for any Reynolds number larger than 10^6.

It is necessary to note here that the roughness of a wellbore will increase with time as scaling occurs. The degree of scaling varies significantly among geothermal reservoirs, and it is a function of the geochemistry of each region.

At a wellhead pressure of 400 psia, the saturation temperature is 444.6°F (from the steam tables). Assuming a bottom hole pressure of 450 psia, the associated saturation temperature is 456.8°F. Therefore, $\overline{T}$ = 450°F (910°R), $\overline{P}$ = 425 psia. The $\overline{Z}$ factor is equal to .855.

We can then calculate parameter A in Eq. 10a:

$$A = \frac{1.719 \times 10^4\ f_M \overline{Z}^2 \overline{T}^2 W^2}{d^5}$$

$$A = \frac{1.719 \times 10^{-4} \times .0135 \times (.855)^2 \times (910)^2 \times (100{,}000)^2}{(9.625)^5}$$

$$= 1.701 \times 10^5$$

From Eq. 10:

$$P_{wf}^2 = (P_{tf}^2 + A)e^{.0233H/\overline{ZT}} - A$$

The value of P_{wf} calculated is 493 psia.

A second trial is obviously in order. Assume P_{wf} = 493 psia and T_{wf} = 465.5°F. Therefore, $\overline{T}$ = 458.1°F (918°R), and $\overline{P}$ = 445.6 psia. The $\overline{Z}$ factor is equal to .851.

Then, from Eq. 10a:

$$A = 1.715 \times 10^{5}$$

and from Eq. 10:

$$P_{wf} = 493 \text{ psia (good agreement with assumed value)}$$

Alternately, using the procedure developed in this report and using $\overline{T} = 450°F$, the value of constant $A = 6.373 \times 10^{-2}$. The slope in Eq. 18 (from Fig. 2) is $m = -1.1$.

By solving equations in Appendix B, we can obtain a value for $P_{wf} = 492$ psia.

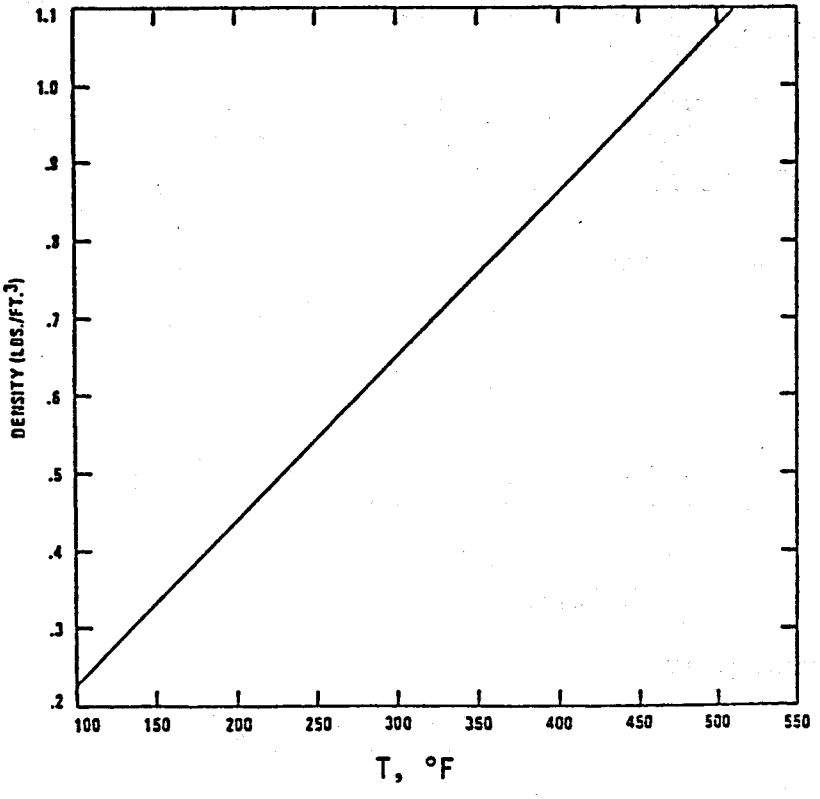

FIG. 1: VAPOR DENSITY OF SATURATED STEAM

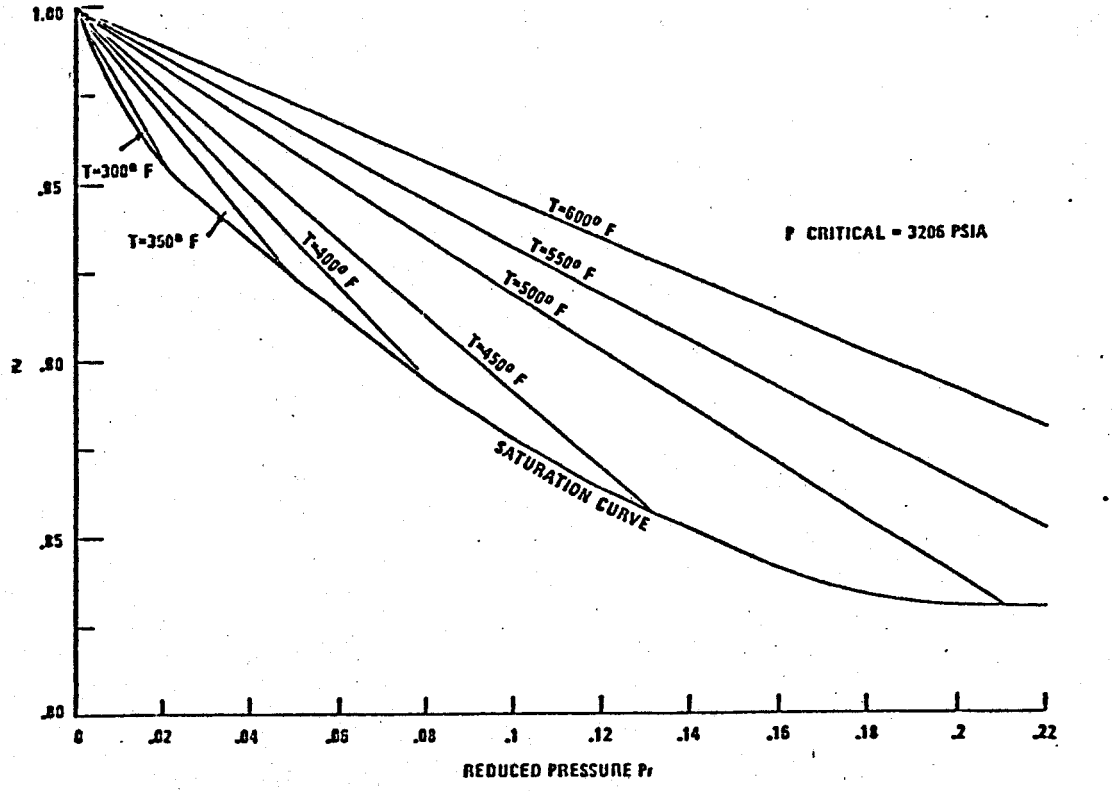

FIG. 2: COMPRESSIBILITY FACTOR FOR SATURATED AND SUPERHEATED STEAM

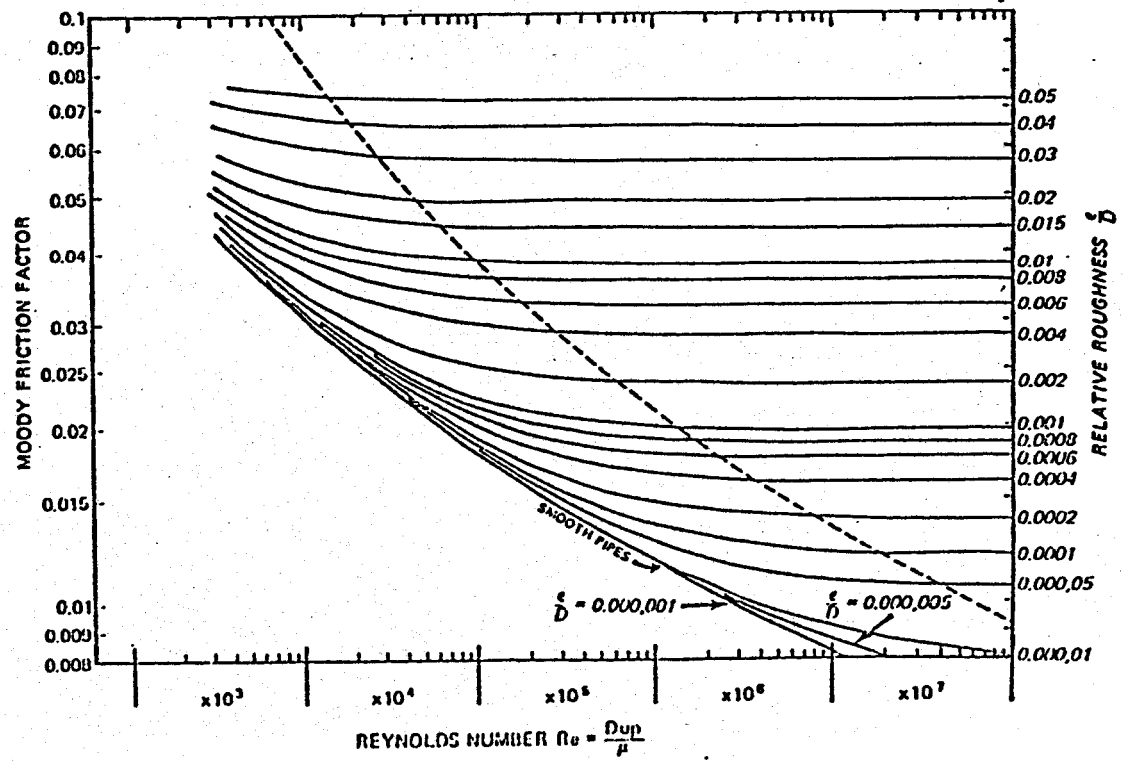

FIG. 3: FRICTION FACTOR AS A FUNCTION OF REYNOLDS NUMBER WITH RELATIVE ROUGHNESS AS A PARAMETER