APPLICATION OF FOURIER AND WAVELET ANALYSES TO PRESSURE DATA FROM THE TE AROHA HYDROTHERMAL SYSTEM

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SUMMARY – Pressure data fiom two separate wells at Te Aroha are analysed using Fourier and wavelet techniques. The long term aim is to examine the possibility of identifying characteristic pressure fluctuations to benchmark the existing state of unexploited hydrothermal systems.

1. INTRODUCTION

Subsurface fluid reservoirs demonstrate fluctuations in reservoir temperature and pressure with time. In general these will be either cyclic due to tidal effects, seasonal changes or other regular disturbances, or random due to the nature of the flow through the reservoir. It may be possible to use the random fluctuations to characterise the flow through the reservoir and hence the existing state of natural systems. The first step in examining this is to check the sensitivity of available instrumentation and develop the analytical techniques.

The Te Aroha hydrothermal system was chosen for this study because it is an easily accessible unexploited geothermal system with several unused wells. Moreover the wells are low temperature and therefore safe and easy to use, but discharge by geysering in a pseudo-regular manner.

Te Aroha is located near the central east coast of the North Island of New Zealand. Its thermal water has been used for bathing and therapeutic treatment since 1884 (Healy 1959). There are three wells capable of discharging, of which one is known as the Mokena Geyser because it is left fully open and discharges like a geyser with a period of approximately 30 minutes. It was drilled in 1938 and has apparently geysered ever since. The discharge is collected for bathing. Although there has been no continuous discharge or pumped production, the geyser discharge is essentially a low flow rate continuous discharge when considered over a time period much longer than the discharge period. The wells are drilled to 105 metres or less and have maximum temperatures of 98 ^{0}C or less. The two wells other than the Mokena Geyser are known as the Wilson Street bore, which was drilled in 1995 but has remained closed except for use for experiments by the Geothermal Institute Diploma students, and the Domain Trust bore, which is older and smaller in bore and output, and has also remained closed. All wells discharge neutral pH sodium-bicarbonate water with a high dissolved solids of about 12 g/kg (Henderson 1938).

The Mokena Geyser was completed with 4" casing to 28 m depth and 3" from 28 m to 67 m. The discharge has a maximum temperature of 83 ^{0}C and an average discharge rate of 0.7 kg/s. The well geysers fiom a 1" pipe on the wellhead up to 5 metres in height. Deposition of aragonite occurs both within the well and around the wellhead. The well is reamed at about 6 monthly intervals to maintain flow capacity.

The Wilson St well is located about 200 metres south of the Mokena. When opened it discharges with a period of approximately 15 minutes at and an average rate of 0.1 kg/s long term equivalent, at temperatures up to 75 ^{0}C. It is cased to 60 m and has maximum depth of 79 m.

At the outset it was thought that the disturbances to the reservoir caused by the Mokena Geyser might be detectable at the (closed) Wilson Street bore, or if not then at the Domain Trust bore which is closer to the Mokena Geyser. Even if not detectable, the wells provide samples of random pressure variations in an undisturbed reservoir.

2. EXPERIMENTAL METHOD

The pressure transducers used were Geokon vibrating wire transducers which have surface readout to a datalogger or computer. The principle of operation is that a circular diaphragm deflects according to the pressure change. A taut wire stretches fiom the centre of the diaphragm to a fixed point, so that its tension vanes as the diaphragm deflects. The wire is electronically plucked and the frequency of vibration is measured and related to the diaphragm deflection and hence the pressure outside the instrument. The instruments used are suitable for a maximum temperature of 120 °C and maximum pressure of 7 bars abs.. Their resolution is approximately 0.04 bar. Their temperature response is rather slow (several minutes to reach thermal equilibrium), but the instruments have been used in isothermal conditions at the bottom of the wells. Their response to transient pressures has not yet been fully quantified but is of order of tens of Hz.

Pressure data was measured at *65.8* metres below the casing head flange (CHF) in the Wilson Street bore with the well shut, and at 61.5 metres below CHF in the Mokena Geyser with the well open. Samples of measurements are shown in Figures 1 and **2.**

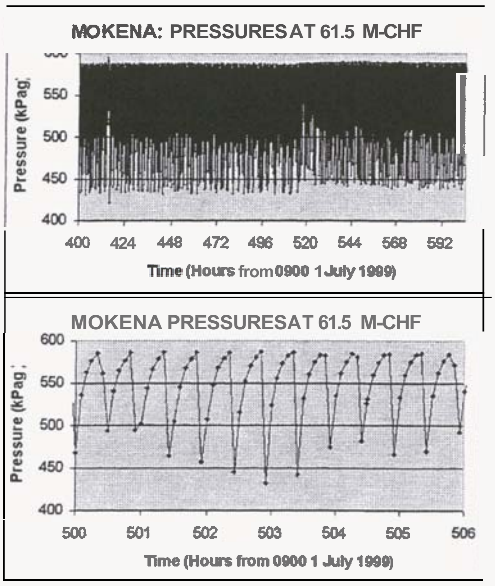

Figure 1 - Well pressures in the Mokena Geyser.

3. ANALYSIS METHODS

3.1 Fourier analysis methods

The traditional Fourier series analysis is for a periodic function f(t). It is based on the idea that such a function can be represented by a

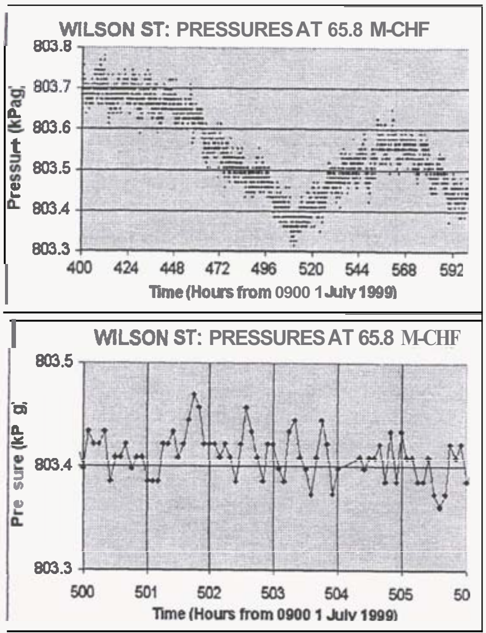

Figure 2 - Well pressures in Wilson St. well.

superposition of sinusoids. The function is thus represented as :-

$$f(t) = \sum_{n=1}^{\infty} (a_n \cos w_n t + b_n \sin w_n t) \quad (1)$$

Where w is the angular frequency and t is time.

The result of an analysis of this type is the specification of values of a_n and b,. The presence of a particular frequency component in **a signal** can be deduced by this type of analysis. For example, if the Wilson Street well was in hydraulic communication with the Mokena well, the pressure variations at the production zone(s) of the Mokena well due to the geysering would have a frequency component representing the **30** minute periodic discharge. This method of analysis would reveal this component.

An important parameter in the analysis of random signals is the autocorrelation function. If we define a random variable x having a Gaussian probability distribution, the probability of a particular value of x is $p(x)$, which is defmed by the usual bell-shaped curve. If we wished to predict how **x** varies with time and produce this as a graph of **x** versus **time,** there would be an infinite number of possible graphs – an ensemble. An average can be defined as

$$E(x) = \int x\, p(x).dx \qquad (2)$$

For one of these graphs we can define a particular average

$$E_\tau(x) = \int x_1.x_2.p(x_1,x_2).dx \qquad (3)$$

where x_1 and x_2 are the values of **x** at times t_1 and t_2 in a particular signal. This is called the ***autocorrelation function.*** If the process is stationary, then the autocorrelation function does not depend on the actual values of t_1 and t_2 but only on the difference $t_1 - t_2 = \tau$. **A** particular class of stationary processes is ***ergodic,*** which means that the statistical properties of the process deduced from analysing the ensemble, notably the ensemble averages, are the same as the temporal averages determined fiom a single measurement of x versus time. Combining these ideas we see that

$$E_\tau(x) = R(\tau) \qquad (4)$$

The autocorrelation function $R(\tau)$ is an easily computed function. The determination of the spectral density of a stationary ergodic random function can be determined fiom the autocorrelation function. Since the function is random, there is no single frequency component representation of the signal – the frequency components present at any time follow a Gaussian (or other) distribution. They may nevertheless represent a real random process that has physical significance. The power spectral density of the signal is the Fourier transform of the autocorrelation function. In practice the signal is Fourier transformed directly rather than via the autocorrelation function use the Fast Fourier Transform algorithm. If the signal to be analysed consists of N samples, where N is a power of **2,** then the Fourier Transform can be performed with a fractional reduction in the number of calculations of about $1/N\log_2 N$ (Newlands, 1992) where N is the sample size.

The Fourier Transform is :-

$$F(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} x(*)\, e^{-i\omega t}\, dt \qquad (5)$$

It is not effective at modeling frequency information that may occur only over part of the signal as the transform assumes that the frequencies obtained from the transform are uniform over the full length of signal. In order to locate frequencies that occur over parts of the signal it is necessary to use windowed analysis in which the signal is analysed in a series of smaller sections.

32 Wavelet Transform

Wavelet analysis provides a way of decomposing a signal into its constituent parts using a compact waveform. **The** signal is broken up into scaled and translated versions of the wavelet. Analysis can be performed with either a continuous (CWT) or discrete wavelet transform (DWT). The CWT is continuous in that the analysing wavelet is shifted smoothly over the signal domain. The CWT can also operate at every scale fiom that of the original signal to a specified maximum.

The DWT operates on scales and translation positions which are dyadic (i.e. power of two). Computational effort **is** substantially reduced with DWT. Processing is further enhanced if a fast wavelet transform algorithm (Mallat, 1989) **is** used. One-dimensional analysis involves signal decomposition by a family of orthogonal analysing signals. Orthogonal wavelets are derived fiom scaling functions.

The wavelet transform coefficients depend on scale and amplitude. The magnitude of these coefficients represents a form of correlation between the signal and wavelet and are calculated by:

$$C(a,b) = a^{-0.5}\int_{-\infty}^{\infty}\Psi\left(\frac{x-b}{a}\right)f(x)\,dt \qquad (6)$$

Where Ψ **is** the wavelet function and ***a,b*** are dyadic scale and translation factors respectively.

Data was recorded at intervals ranging from **2** seconds to 5 minutes. The period of continuous data collection was limited by data logger memory capacity of 7999 points and the ability to travel to the site to download **data.** The data range selected for analysis was 400 to 600 hours from the zero **datum** of 0900 hours on 1 July 1999. During *this* **time data was** collected continuously at **5** minute intervals.

4. DISCUSSION

4.1 Fourier Analysis

Fourier analysis for Mokena (Figure **3)** demonstrates pressure cycling at a frequency of about 410 cycles per 200 hours or a cycling period of about 29 minutes.

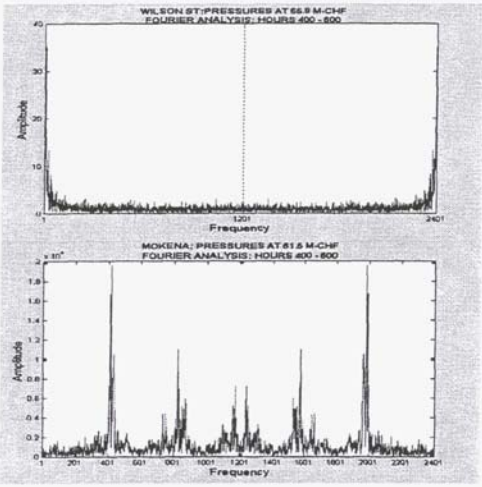

Figure 3 - Fourier analysis of Wilson St. and Mokena data

Other features occur at frequencies of 910 cycles or period of 13 minutes and 1150 cycles or period of 10 minutes. There **are no** obvious features in the Wilson St data analysis in Figure **3.**

4.2 Autocorrelation Analysis

Auto-correlation analysis for the Mokena data (Figure **4)** shows high correlation coefficients at lag cycles of about 400 data points or **33** hours.

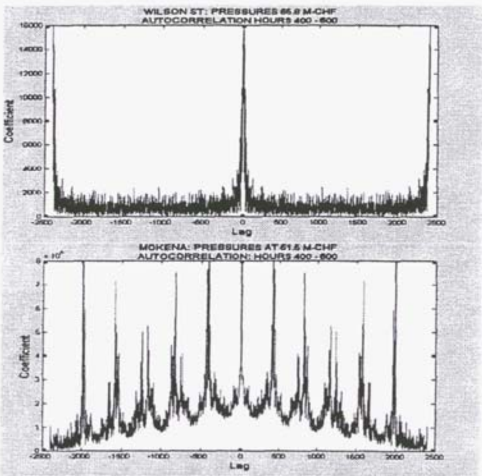

Figure 4 – Correlograms for Wilson St. and Mokena data.

There .is little of obvious significance in Wilson St well analysis.

4 3 Wavelet Analysis

Wavelet analysis in this study uses one of the family of orthogonal and compactly supported wavelets. These include the wavelet types of Daubechies (dbN), symlets (symN), and coiflets (coifN).

General properties of this family are that they are based on a scaling function that has a given number of vanishing moments and the analysis is orthogonal. Both the scaling function and the wavelet function are compactly supported. These properties are conducive to both continuous and discrete wavelet transform analysis due to the numerical efficiency generated as a result of orthogonality.

The wavelet used to analyse Te Aroha data is the order **5** "coif5" wavelet (Figure 5) developed by Coifman et al. (1992). This wavelet was chosen because of its ability to model sharp points in the data and its analytical efficiency.

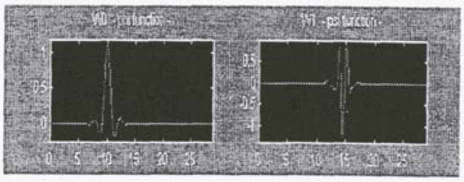

Figure 5 – Profile of 'coifs' wavelet

The scaling and wavelet functions have a support width of 6N-1, and length of 6N. They also have 2N-1 and 2N vanishing moments respectively.

A 12 level wavelet decomposition was performed on both the **Wilson** St (Figure *6)* and Mokena (Figure 7) pressure data. A detailed interpretation is outside the scope of this paper. The finest details (highest frequency) of the signal are shown in Details 1 and the lowest frequency in Detail *6* of Figures 8 and **9).**

For the Wilson St well the plots of details at levels **6** and lower are not statistically significant as pressure fluctuations are below gauge resolution. The amplitude peaks in Figure 9 demonstrate characteristic fiequencies **of** the pressure cycles in the Mokena well.

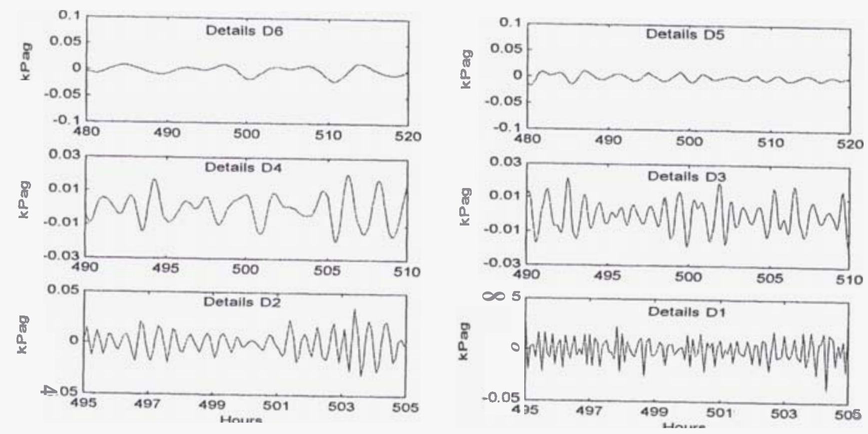

Figure 6 – Wavelet decomposition: Wilson St. data

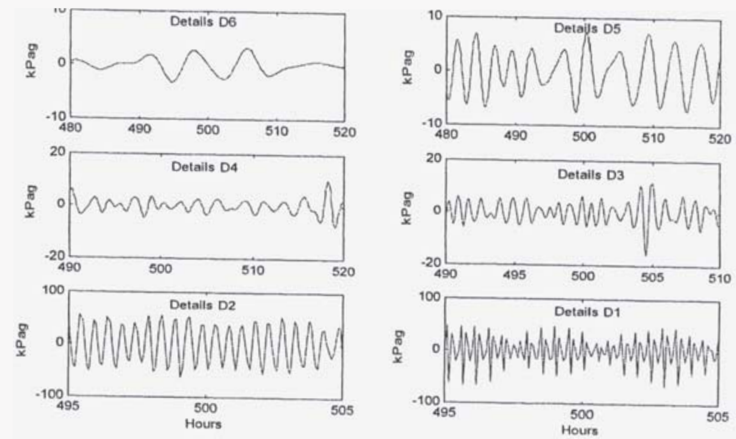

Figure 7 – Wavelet decomposition: Mokena data

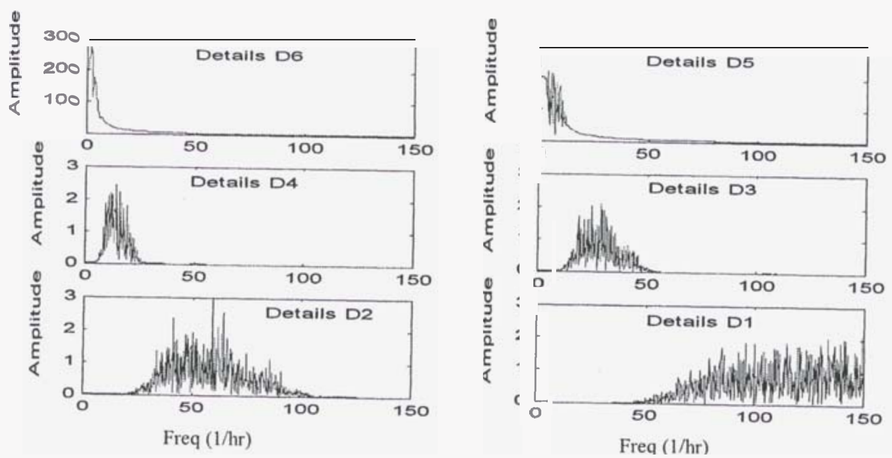

Figure 8 – Wavelet decomposition: Wilson St. data

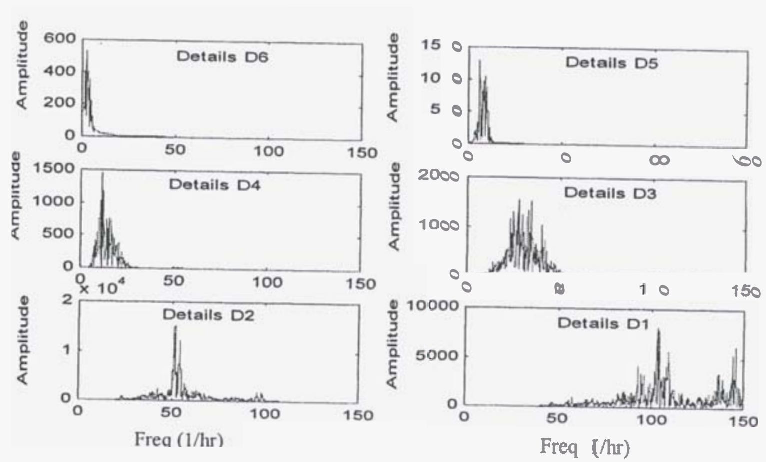

Figure 9 – Wavelet decomposition: Mokena data

5. CONCLUSIONS

The analysis techniques reviewed reveal that the combination of conventional Fourier analysis with autocorrelation and wavelet analysis should provide greater clarity in the detection characteristic pressure fluctuations. Much more detailed analysis will be required to examine the significance of the fluctuations noticed.

6. ACKNOWLEDGEMENTS

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