Modeling Hydrothermal Systems with OpenFOAM
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ABSTRACT
This paper examines the applicability of OpenFOAM for the simulation of hydrothermal systems. OpenFOAM is a free and open source C++ toolbox for the development of numerical solvers in continuum mechanics. It offers some major advantages such as object-oriented code, which enables developers to work at a high level of abstraction, extensive options for parallelization and the potential to construct tailor-made solvers for specific problems. In this project a solver has been constructed which is capable of accounting for the hydrology and thermodynamics of two-phase flow in porous media. The IAPWS-IF97 thermodynamic formulation is used for the equation of state, where the model is both capable of handling phase change and the phases traveling at different velocities. This solver is then validated against a thermal analogy of Henry's seawater diffusion problem and a test case involving natural convection of two phases. These problems increase the confidence in the validity of the solver and the accuracy of the solution. The conclusion is that OpenFOAM is a viable option in the simulation of geothermal reservoirs and offers some advantages over commonly used simulation tools.

1. INTRODUCTION
Numerical simulation of geothermal reservoirs has become standard practice in the past few decades. There are several reasons for why numerical modeling is important in the modern management of geothermal reservoirs. Acquisition of data on site can often be very expensive and technically difficult. Laboratory studies, which aim to represent the behavior of the reservoirs, can never fully represent the complex and multiscale nature of the systems. In addition to that, the spatial and temporal scales relevant to the prediction of the behavior of hydrothermal systems are often much larger than would be possible to determine directly by measurement (Ingebritsen et al., 2010, O’Sullivan, 2001).

The earliest efforts to apply numerical models to geothermal reservoirs were made in the early 1970's, but the usefulness of numerical modeling did not begin to gain acceptance by the geothermal industry until after the 1980 Code Comparison Study (Stanford Geothermal Program, 1980). Since that study was performed, the experiences gained in carrying out site-specific studies as well as generic reservoir modeling studies have led to a constant improvement in the capabilities of numerical reservoir models.

Numerical modeling of hydrothermal systems is often defined by which components of the system are taken into account. Traditionally they have been divided into hydrological, thermal, mechanical and chemical components. Those components are coupled together in a way that is inherently multiscale in nature, such that their temporal and spatial scales vary by several orders of magnitude (Ingebritsen et al., 2010). Because of the complex nature of those couplings, models involving all four components are rare.

The equations that describe hydrothermal systems are relatively complex but they can nevertheless be solved analytically for a highly idealized set of initial and boundary conditions. Such cases usually only involve one of the four components, where the Theis problem is an example thereof (Theis, 1935). Some analytical solutions also exists for two components, such as the description of a boiling front moving through a porous medium (Pruess and Celatis, 1987) and the advance of a diffused salt water wedge in a confined aquifer (Henry, 1964) which will be the main focus of this paper. These analytical solutions are very important in validating numerical models that are supposed to handle more complicated problems.

The current generation of numerical simulators is in most cases able to account for multi-phase, multi-component flow. The most versatile ones are software packages such as Finite Element Heat and Mass Transfer (FEHM) (Keating et al., 2002) and the Transport of Unsaturated Groundwater and Heat (TOUGH) family of codes (Pruess, 1991). These solvers have been applied to a wide variety of problems, such as CO2 sequestration, geothermal studies and other environmental issues (Ingebritsen et al., 2010).

Other solvers are more specialized, such as the Complex Systems Modeling Platform (CSMP++) (Matthäi et al., 2007) and Fully Implicit Seafloor Hydrothermal Event Simulator (FISHES) (Lewis and Lowell, 2009). They have been developed specifically to allow simulation of high-temperature multiphase flow of NaCl-H2O fluids. Other codes such as FALCON (Podgorny et al., 2011), developed at Idaho National Labs have focused on the tightly coupled process of fluid rock interaction.

In this paper the applicability of using a free and open source package named OpenFOAM for modeling hydrothermal systems is examined. OpenFOAM is a highly customizable set of C++ libraries and tools for the solution of problems in continuum mechanics. It is also gaining considerable popularity in academic research and among industrial users, both as a research platform and a black-box CFD and structural analysis solver (Jasak et al., 2007).

The object orientation and operator overloading of C++ has enabled the developers of OpenFOAM to build a framework for computational fluid dynamics that enables modelers to work at a very high level of abstraction (Weller et al., 1998). This makes it possible to manipulate the set of partial differential equations that describe the problem and customize the solver itself for each class of cases that needs to be solved. This will enable researchers with sufficient knowledge about the relevant dynamics of each problem to construct efficient and accurate solvers for it. This is the main motivation for using OpenFOAM, rather than currently existing models.
The main focus of this paper will be to benchmark the OpenFOAM solver that has been developed with a thermal analogy of Henry’s saltwater diffusion problem. Henry’s problem is one of the best known and most commonly used benchmark problems in hydrogeology (Simpson and Clement, 2003). This is mainly due to the analytical solution that exists and was derived by Henry in 1964 (Henry, 1964). Despite the widespread use of the analytical solution as a benchmark problem, it has remained controversial and some authors have even contested its validity (O’Sullivan and Croucher, 1995). Ségol later revised the analytical solution and found the updated solution to be closer to numerical simulations (Ségol, 1994). Even though this solution exists another version of the problem has become more widespread as a benchmark problem. This updated version changes the one fifth of the seawater boundary into a zero gradient boundary condition with respect to concentration. This allows convective transport over the boundary, which reduces the numerical difficulties in solving the problem as well as being more physically realistic (Frind, 1982). This is the version that will be used to benchmark the solver in this paper.

2. THEORY

In the following analysis the governing equations for two phase flow in porous media are derived along with their proposed numerical solution.

2.1 Governing Equations

2.1.1 Pressure Equation

If the volume weighted average density is given as
\[ \rho_0 = (1 - \alpha) \rho_l + \alpha \rho_v \]  
(1)
the continuity equation can be expressed as
\[ \frac{\partial}{\partial t} (\varphi \rho_0) + \nabla \cdot (\rho_0 \vec{q}_l + \rho_v \vec{q}_v) = 0 \]  
(2)

where \( \alpha \) is the vapor saturation, \( \rho_l \) is the density of saturated liquid and \( \rho_v \) is the density of saturated steam. The superficial velocities \( \vec{q}_l \) and \( \vec{q}_v \) are given by Darcy’s law
\[ \vec{q}_l = -\frac{k_{rl}}{\mu} \kappa (\nabla p + \rho_l \vec{g}) \]  
(3)
\[ \vec{q}_v = -\frac{k_{rv}}{\mu_v} \kappa (\nabla p + \rho_v \vec{g}) \]  
(4)

where \( \mu \) is the dynamic viscosity, \( \kappa \) is the intrinsic permeability tensor and \( k_{rl} \) and \( k_{rv} \) are the relative permeabilities of the respective phases. If it is assumed that the porosity \( \varphi \) is constant, the accumulative term in the continuity equation can be written as
\[ \varphi \frac{\partial \rho_0}{\partial t} = \varphi \left( (1 - \alpha) \frac{\partial \rho_l}{\partial t} + \alpha \frac{\partial \rho_v}{\partial t} + (\rho_v - \rho_l) \frac{\partial \alpha}{\partial t} \right) \]  
(5)

Since the goal is to solve the continuity equation for pressure, we can approximate how density is dependent on pressure by applying the Taylor expansion
\[ \rho_l = \rho_{l0} + \frac{\partial \rho_l}{\partial p} \bigg|_{p=p_0} (p - \rho_{l0}) \]  
(6)
\[ \rho_v = \rho_{v0} + \frac{\partial \rho_v}{\partial p} \bigg|_{p=p_0} (p - \rho_{v0}) \]  
(7)

The full pressure equation, which is used in the solver, is then retrieved by inserting equations 3, 4, 6 and 7 into equation 2. This equation is then solved for pressure in each time step.

2.1.2 Energy Equation

The energy per unit volume in porous media can be given as
\[ E = (1 - \varphi) \rho_r c_r (T - T_0) + \varphi (1 - \alpha) \rho_l u_l + \varphi \alpha \rho_v u_v \]  
(8)

where the subscript \( r \) denotes the properties of the porous matrix and \( u \) is the internal energy of each component. In this case \( E \) is not a one-to-one function with respect to internal energy. Later in the process the internal energy of the fluid is solved from the energy content in a cell. Since a one-to-one function is desirable in that case, we choose to express the energy content per mass unit rather than volume unit. If that quantity is expressed as \( e = E/\varphi \rho_\alpha \) the following expression for energy content per unit mass of fluid is retrieved
\[ e = \frac{1 - \varphi}{\varphi} \frac{\rho_r}{\rho_\alpha} c_r (T - T_0) + \frac{(1 - \alpha) \rho_l u_l + \alpha \rho_v u_v}{\rho_\alpha} \]  
(9)
the energy equation can then be written as

$$\varphi \frac{\partial (\rho_e e)}{\partial t} + \nabla \cdot (\rho_e h \bar{q}_e) + \nabla \cdot (\rho_e U \bar{q}_e) = \nabla \cdot (\Gamma \nabla T)$$  \hspace{1cm} (10)$$

where $h$ is enthalpy, $\Gamma$ is the combined conduction coefficient for liquid water and steam and $T$ is the temperature.

### 2.2 Numerical Solution

#### 2.2.1 Solution Method

In the beginning of each time step the vapor saturation $\alpha$ is evaluated in each cell along with the physical properties from the equation of state. The relative permeabilities are then evaluated using the Corey curve. The continuity equation is solved for pressure, where equations 3, 4, 6 and 7 have been inserted into equation 2. After the pressure equation has been solved, the superficial velocities are reevaluated, using equations 3 and 4.

The energy equation (10) is then solved for $e$, where its time derivative is treated as the only implicit term even though the other variables in the equation are functions of $e$. The reason for this is that in geothermal systems pressure changes travel much faster in the system than temperature changes. This means that the time step is mostly determined by pressure changes. It should therefore be sufficient for the energy equation to treat those quantities explicitly.

#### 2.2.2 Treatment of Different Phases

In order to be able to determine the vapor saturation $\alpha$ from the energy per unit mass $e$, equation 9 can be rearranged such that

$$\alpha = \frac{\rho_e \varphi (e - u_{ls}) - (1 - \varphi) \rho_c c_e (T - T_0)}{\varphi (e (\rho_{ls} - \rho_{ls}) - (\rho_{ls} u_{ls} - \rho_{ls} u_{lv}))}$$  \hspace{1cm} (11)$$

where the subscript $ls$ denotes that the physical property is evaluated at the saturated liquid state and $vs$ the saturated vapor state. This equation is therefore only a function of the pressure and the energy content per mass unit, $e$. If the value of $\alpha$ which is retrieved from equation 11 is on the interval $0 \leq \alpha \leq 1$ all the properties subscripted with $l$ or $v$ are evaluated at their respective saturated states.

Equation 11 can also be used to determine whether the current cell consists entirely of liquid or vapor. If the value for $\alpha$ is determined to be less than 0, the cell is in a pure liquid state, which means that the physical properties should not be evaluated at the saturated states. The equation for energy per unit mass then becomes

$$e = \frac{1 - \varphi \rho_c c_e (T - T_0) + u_l}{\varphi \rho_l}$$  \hspace{1cm} (12)$$

which has to be solved for $u_l$ in order to be able to correctly determine the internal energy of the cell. The same is true for cells in pure vapor states, where the equation 9 becomes

$$e = \frac{1 - \varphi \rho_c c_e (T - T_0) + u_v}{\varphi \rho_v}$$  \hspace{1cm} (13)$$

This problem can be posed as a root finding problem, where $u$ can either be $u_l$ or $u_v$ and $T$ and $\rho$ are functions of that internal energy for a constant pressure.

$$f(u) = e - \left(1 - \varphi \rho_c c_e (T - T_0) - u\right)$$  \hspace{1cm} (14)$$

In order to be able to apply a Newton-Rhapson iteration, the derivative of that function is also needed

$$f'(u) = \left(-\frac{1 - \varphi \rho_c c_e}{\varphi \rho} \frac{\partial T}{\partial u} - 1\right)$$  \hspace{1cm} (15)$$

where $v$ is the specific volume of the fluid. For a given energy per unit mass $e$ the internal energy $u$ is then found by applying the following iteration

$$u_{n+1} = u_{n} - \frac{f(u_n)}{f'(u_n)}$$  \hspace{1cm} (16)$$

where the properties $T$, $\rho$, $\partial T/\partial u$ and $\partial v/\partial u$ are evaluated from the equation of state between each iteration. Once the relative change of $u$ between iterations becomes less than $10^{-10}$ the temperature $T$, density $\rho$, viscosity $\mu$ and the derivative of the density with respect to pressure $\partial \rho/\partial p$ are evaluated. This procedure is then repeated for every cell in the domain.

#### 2.2.3 Relative Permeabilities

The relative permeability of a phase is a measure on the permeability of that phase. The water saturation is normalized such that
\[ S_{wn} = \frac{S_w - S_{wtr}}{1 - S_{vtr} - S_{vtr}} \]  

(17)

this is done since the liquid becomes immobile when the water saturation is below \( S_{wtr} \) and the vapor becomes immobile when the water saturation is above \( S_{vtr} \). If a value of \( S_{wn} \) is retrieved that is below zero the relative permeabilities are set as \( k_{rl} = 0 \) and \( k_{rv} = 1 \), if the value of \( S_{wn} \) is above one they are set as \( k_{rl} = 1 \) and \( k_{rv} = 0 \).

\[ k_{rl} = S_{wn}^4 \]  

(18)

\[ k_{rv} = (1 - S_{wn})^2 (1 - S_{wn}^2) \]  

(19)

The relative permeabilities as functions of water saturation \( S_w \) can be seen on Figure 1.

Figure 1: The relative permeabilities as functions of water saturation using the Corey relation

If \( 0 < S_{wn} < 1 \) then both phases are mobile and the relative permeabilities follow the Corey curve, such that

\[ \mathbf{n} \cdot (\rho_l \mathbf{q}_l + \rho_v \mathbf{q}_v) = \mathbf{m} \]  

(20)

where \( \mathbf{n} \) is the normal vector to the boundary.

If the superficial velocity is only coupled to pressure through Darcy’s law, as given in equations 3 and 4, this condition is fulfilled if

\[ \nabla p = -\frac{\rho_l^2 k_{rl} \mu_u + \rho_v^2 k_{rv} \mu_v}{\rho_l k_{rl} \mu_v + \rho_v k_{rv} \mu_l} (\mathbf{n} \cdot \hat{g}) + \frac{\mu_l \mu_v}{\kappa (k_{rl} \mu_l + k_{rv} \mu_v)} \]  

(21)

which can be implemented as a Neumann boundary condition on the pressure on the domain.

3. CASE STUDIES

3.1 Henry’s Salt-Water Intrusion Problem

In this case the goal is to validate the solver by constructing a problem driven by thermal buoyancy, analogous to the salinity driven problem first posed by Henry in 1964 (Henry, 1964).

In order to do that it is necessary to introduce the relationship between the variables already presented and those that are more commonly used in single-phase groundwater hydrology. This includes the pressure, which is often written in terms of the equivalent freshwater head \( \psi \), defined as

\[ \psi = \frac{p}{\rho_0 g} + z \]  

(22)

where \( \rho_0 \) is the reference freshwater density, \( g \) is the length of the gravitational acceleration vector and \( z \) is the elevation above datum. Another commonly used quantity in single-phase groundwater hydrology is the hydraulic conductivity \( K \), which is defined as follows

\[ K = \kappa \frac{\rho_0 g}{\mu} \]  

(23)

where in an isotropic aquifer \( K \) and \( \kappa \) can be simplified to scalar values \( K \) and \( \kappa \).
In Henry’s problem the fluid flow is driven by buoyancy effects due to different concentrations of salinity in the fluid. In this case the density can be given by the Boussinesq approximation

$$\rho = \rho_0 (1 + \gamma c) \quad (24)$$

where $c$ is the relative dimensionless salinity which can be given in terms of the absolute salinity $c'$ and the maximum salinity $c$, such that $c = c'/c$.

The constant $\gamma$ is derived from the maximum density of the fluid $\rho_{\text{max}}$

$$\gamma = \frac{\rho_{\text{max}}}{\rho_0} - 1 \quad (25)$$

By substituting equations 22, 23 and 24 into equation 3 and assuming single phase flow, the darcian velocity can be written as

$$\vec{q} = -K \left( \nabla \psi + \gamma c \hat{z} \right) \quad (26)$$

where $\hat{z}$ is a unit vector in the positive $z$ direction.

When solving this problem (Frind, 1982) used the continuity equation

$$\rho(\alpha + \varphi \beta) \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{q}) = 0 \quad (27)$$

which the darcian velocity from equation 26 was then substituted into to give the pressure equation

$$S_s \frac{\partial \psi}{\partial t} = \nabla \cdot (K (\nabla \psi + \gamma c \hat{z})) \quad (28)$$

where $S_s$ is the specific storage. If the fluid and the bulk aquifer material are assumed to be incompressible with regards to pressure, as is done in Henry’s problem, the specific storage becomes zero, making equation 28

$$\nabla \cdot (K (\nabla \psi + \gamma c \hat{z})) = 0 \quad (29)$$

The advection-diffusion equation for the solute concentration is given in (Frind, 1982) as

$$\frac{\partial c}{\partial t} + \nabla \cdot (\varphi^{-1} \vec{q} c) = \nabla \cdot (D \nabla c) \quad (30)$$

In 1987 Voss and Souza (Voss and Souza, 1987) noticed a discrepancy in the value of the diffusion coefficient used by several analyst compared to the original value used by Henry (Henry, 1964). They argued that since the porosity does not multiply the dispersion coefficient in equation 30, it must be implicit in the diffusion coefficient. The relationship between the values of $D$ used in equation 30 with the original value used by Henry $D_{\text{Henry}}$ should therefore be (Ségol, 1994)

$$D = \frac{1}{\varphi} D_{\text{Henry}} \quad (31)$$

If the porosity is constant over the domain, equation 30 could then also be written as

$$\varphi \frac{\partial c}{\partial t} + \nabla \cdot (c \vec{q}) = \nabla \cdot (D_{\text{Henry}} \nabla c) \quad (32)$$

3.1.1 Analogous Thermally Driven Problem

By inserting the velocity pressure coupling from equation 2 into the continuity equation (2) and assuming single-phase flow the following is retrieved

$$\varphi \frac{\partial \rho}{\partial t} = \nabla \cdot \left( \frac{\rho \kappa}{\mu} (\nabla \rho + \rho \vec{q}) \right) \quad (33)$$

In order to get an equation in terms of the same variables as equation 29 the relations from equations 22 and 23 are inserted into equation 33, giving

$$\varphi \frac{\partial \rho}{\partial t} = \nabla \cdot (\rho K (\nabla \psi + \frac{\rho}{\rho_0} \hat{z})) \quad (34)$$

Several approximations are then made. The first is that the flow is steady state, making $\partial \rho/\partial t = 0$ in equation 33. Then it is assumed that a Taylor approximation can be made of the density in the term for gravitationnal acceleration, such that
\[
\rho \approx \rho_0 \left( 1 + \frac{1}{\rho_0} \frac{\partial \rho}{\partial u} \Delta u \right)
\]  
(35)

which inserted into equation 34 and assuming that \( \rho_0 \) is constant gives

\[
\nabla \cdot \left( K (\nabla \psi + \frac{1}{\rho_0} \frac{\partial \rho}{\partial u} \Delta u \vec{z}) \right) = 0
\]  
(36)

For the analogy between equations 10 and 32, the equivalence of saying that the porous rock matrix cannot store the solute is to say that the heat capacity of the rock is zero, \( c_r = 0 \). Since Henry’s problem is also only a single phase problem, this makes the total energy per unit mass in the system given by equation 9, simply the internal energy of the liquid, \( e = u_l \). Equation 10 then becomes

\[
\frac{\partial (\rho u)}{\partial t} + \nabla \cdot (\rho \vec{u} \vec{q}) = \nabla \cdot (\Gamma \nabla T)
\]  
(37)

The chain rule can be applied to the laplacian in equation 37 giving

\[
\nabla \cdot (\Gamma \nabla T) = \nabla \cdot \left( \Gamma \frac{\partial T}{\partial u} \nabla u + \Gamma \frac{\partial T}{\partial p} \nabla p \right)
\]  
(38)

where the second term drops out if it is assumed that the temperature is much more dependent on the internal energy than the pressure.

Since there is only liquid in the system the enthalpy can be approximated by the internal energy \( h \approx u \). Finally by assuming that the density is constant in equation 37, the energy equation is revealed

\[
\frac{\partial u}{\partial t} + \nabla \cdot (u \vec{q}) = \nabla \cdot \left( \Gamma \frac{\partial T}{\partial u} \nabla u \right)
\]  
(39)

which is analogous to equation 32.

### 3.1.2 Matching the Equations

In order to construct an analogous problem, the physical parameters in both the continuity equations (29 and 36) have to be matched as well as the parameters in the salinity concentration equation (32) and the energy equation (39). In order for equations 29 and 36 to be analogous, the following has to hold true

\[
\gamma c = \frac{1}{\rho_0} \left. \frac{\partial \rho}{\partial u} \right|_{\omega = \omega_0} \Delta u
\]  
(40)

It is then possible to determine the boundary condition analogous to the seaside boundary condition in Henry’s problem (\( c = 1 \))

\[
\omega_1 = \omega_0 + \Delta \omega = \omega_0 + \frac{\rho_0 \gamma}{\rho \frac{\partial \rho}{\partial u}}
\]  
(41)

The second equation that has to hold true, in order for the two problems to be analogous is

\[
D_{\text{Henry}} = \left. \frac{\Gamma}{\rho_0} \frac{\partial T}{\partial u} \right|_{\omega = \omega_0}
\]  
(42)

which is a result of matching equations 32 and 39. If the value for the internal energy has been determined, the thermal conductivity \( \Gamma \) can be tuned to give analogous problems.

### 3.1.3 Boundary Conditions and Numerical Values

**Figure 2**: System definition for the thermal analogy of the modified Henry's problem
Figure 2 shows the boundary conditions of the modified Henry’s problem. It consist of a homogenous isotropic aquifer that is bounded above and below by an impermeable boundary. It is recharged by a constant influx of hot water at the left side, but at the right side it is exposed to a body of colder water. Since the change from hot to cold water is rather abrupt (or freshwater to seawater in the salinity analogy), Ségol modified the problem to include a portion of the right boundary that allowed convective mass flux.

Figure 2 shows the system definition of the thermal analogy of Henry’s seawater intrusion problem. The boundary conditions are constant internal energy at the left side and lower internal energy at the right side. The portion that allowed convective transport in the seawater problem is now considered to have zero gradient, as well as the top and the bottom of the aquifer.

In this case $e_0$ was selected to be 700 kJ/kg and $p_0$ to be 1 MPa. The parameters used by Frind (Frind, 1982) were used to construct the analogous problem. The original parameters can be found in Table 1 and the analogous parameters, calculated from section 3.1.2 can be seen in Table 2.

### Table 1: Parameters of the modified Henry’s problem as given by Frind

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concentration (left side)</td>
<td>$c_l$</td>
<td>0.0</td>
<td>-</td>
</tr>
<tr>
<td>Concentration (right side)</td>
<td>$c_r$</td>
<td>1.0</td>
<td>-</td>
</tr>
<tr>
<td>Hydraulic conductivity</td>
<td>$K$</td>
<td>$1.0 \times 10^{-2}$</td>
<td>m/s</td>
</tr>
<tr>
<td>Porosity</td>
<td>$\phi$</td>
<td>0.35</td>
<td>-</td>
</tr>
<tr>
<td>Diffusion coefficient</td>
<td>$D_{Henry}$</td>
<td>$6.6 \times 10^{-6}$</td>
<td>m$^2$/s</td>
</tr>
<tr>
<td>Inlet velocity</td>
<td>$q_x$</td>
<td>$6.6 \times 10^{-5}$</td>
<td>m/s</td>
</tr>
<tr>
<td>Height of aquifer</td>
<td>$z_0$</td>
<td>1.0</td>
<td>m</td>
</tr>
<tr>
<td>Height of isolation</td>
<td>$z_1$</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>Width of aquifer</td>
<td>$x_l$</td>
<td>2.0</td>
<td>m</td>
</tr>
</tbody>
</table>

### Table 2: Parameters of the thermally analogous problem as given by section 3.1.2

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>Internal energy (left side)</td>
<td>$e_l$</td>
<td>700.0</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>Internal energy (right side)</td>
<td>$e_r$</td>
<td>605.0</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>Reference pressure</td>
<td>$p_0$</td>
<td>1.0</td>
<td>MPa</td>
</tr>
<tr>
<td>Permeability</td>
<td>$\kappa$</td>
<td>$1.857 \times 10^{-10}$</td>
<td>m$^2$</td>
</tr>
<tr>
<td>Thermal conductivity</td>
<td>$\Gamma$</td>
<td>25.91</td>
<td>W/(mK)</td>
</tr>
<tr>
<td>Reference density (left side)</td>
<td>$\rho_{0l}$</td>
<td>923.0</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Reference density (right side)</td>
<td>$\rho_{0r}$</td>
<td>901.9</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>Mass flux</td>
<td>$\dot{m}$</td>
<td>$5.952 \times 10^{-2}$</td>
<td>kg/s</td>
</tr>
</tbody>
</table>

### 3.1.4 Numerical Simulation

The results from the numerical simulation of the modified Henry’s problem, using the parameters from Table 2 can be seen in Figure 3. In order to make the comparison with the existing literature on Henry’s problem easier, the internal energy has been normalized, such that $e_n = (e - e_r)/(e_l - e_r)$. The figure shows five isotherms at 0.01, 0.25, 0.5, 0.75 and 0.99. It can be seen how the cooler water diffuses into the aquifer where it meets a constant inflow of hot water. Due to this effect the hot water is directed over the cooler regions of the aquifer where it finally exits at the boundary at the upper right side.
Figure 3 seems to be rather consistent with the existing literature. However the isothermal lines seem to extend slightly further into the aquifer than in the literature. The reason for this behavior is that in this analysis the equation of state has been evaluated at every time step, updating the relevant values of the viscosity and the density. This might result in a lower hydraulic conductivity, which might make the cold boundary diffuse further into the aquifer.

3.2 A Cavity with Natural Convection

In the previous section the single-phase capabilities of the model have been demonstrated. In order to demonstrate the two phase capabilities of the model a case has been set up where phase change takes place with water both in liquid and vapor phase.

3.2.1 Boundary Conditions and Numerical Values

Figure 4 shows the system definition of the two-phase natural convection problem in porous media. The cavity is completely closed with zero mass flux at the top and the bottom, as defined in section 2.3.1 and zero pressure gradient normal to each side. For the total energy, the top and the bottom are isolated, while constant energy levels are defined on each side. At the left side, the energy should be low enough to condense any steam that touches it; while at the right side it should be high enough to produce superheated steam. The numerical values for the problem can be seen in Table 3.

<table>
<thead>
<tr>
<th>Property</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Energy per mass (left side)</td>
<td>$e_l$</td>
<td>1115</td>
<td>kJ/kg</td>
</tr>
<tr>
<td>Energy per mass (right side)</td>
<td>$e_r$</td>
<td>7813</td>
<td>MJ/kg</td>
</tr>
<tr>
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<td>J/(kgK)</td>
</tr>
<tr>
<td>Density of soil</td>
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<td>kg/m$^3$</td>
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<tr>
<td>Thermal conductivity</td>
<td>$\Gamma$</td>
<td>2.680</td>
<td>W/(mK)</td>
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</table>
3.2.2 Numerical Simulation

Figures 5 and 6 show the steady state solution to the two-phase natural convection problem. Figure 5 shows the void fraction $\alpha$ color-coded from blue to red, where blue indicates fully liquid water and red indicates steam entirely in the vapor phase. The figure also shows the streamlines, which are color coded for velocity. They show that the highest velocity in the liquid phase is at the upper left corner of the cavity.

Figure 6 shows the temperature color-coded for temperature in Kelvins. It can be seen how the constant temperature at the sides diffuses into the cavity close to the walls and is then mostly convected through it. In the figure the streamlines are also plotted in order to properly visualize the flow. As would be expected the steam condenses at the left side, turns into a liquid phase, which advects to the bottom and then to the right side. There it boils, rises to the top and repeats the same process again.

![Figure 5: The steady state solution to the cavity problem where $\alpha$ is color-coded, along with the streamlines.](image)

![Figure 6: The steady state solution to the cavity problem, where $T$ is color-coded along with the streamlines.](image)

4. DISCUSSION

This paper has described the governing equations and the numerical solution strategies for a solver for two phase flow in porous media. In order to benchmark it, a thermal analogy of the well known Henry’s seawater diffusion problem was derived. The solution seemed to be rather consistent with the current literature. However the front of the cool water at the right side boundary seems to diffuse slightly further into the bottom of the aquifer. The reason for this might be the fact that the physical properties of water are not constant, but calculated from the IAPWS-IF97 thermodynamic formulation. Most benchmark problems however assume constant physical properties. This might decrease the hydraulic conductivity of the aquifer leading to the cool water diffusing further into it.

The ability of the solver to handle phase change has also been demonstrated. This was done by constructing a problem involving natural convection at temperatures that led to steam condensing at one side and the water boil at the other side. This resulted in a steady state solution that seemed to give physical results.

In order to gain full confidence in the solver further work is needed. The methodology that holds the greatest promise in that matter is systematic intercomparison of independently developed simulators. The main reason for this is that it is not possible to adequately test multiphase, multicomponent simulators against analytical solutions or experimental results (Ingebritsen et al., 2010). The work presented in this paper might be one step in that direction.
5. CONCLUSIONS

This paper has described the applicability of the OpenFOAM framework to take on problems involving hydrothermal systems. A thermal analogy of the well known Henry’s seawater diffusion problem was derived and used as a benchmark problem for the solver. It seemed to agree rather well with previous solutions to the problem. In order to test the phase change abilities of the solver another test case was defined. It was able to solve that problem, giving a physically realistic solution. This increases our confidence in the numerical solver and encourages continued usage of the OpenFOAM framework in geothermal reservoir modeling.

6. ACKNOWLEDGEMENTS

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REFERENCES


