Estimation of the Stress Magnitudes in Basel Enhanced Geothermal System

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ABSTRACT
The in-situ state of stress plays a major role in determining the response of the rock mass to hydraulic stimulation injections used to develop heat-exchangers in low-permeability EGS reservoirs. As such, stress and its heterogeneity must be specified in any geomechanical model of the stimulation process. This paper presents the results of an evaluation of stress magnitudes in the granitic EGS reservoir in Basel, Switzerland. The profile of minimum principal horizontal stress, Shmin, is constrained by hydraulic tests, but the magnitude of the maximum horizontal principal stress, SHmax is uncertain. Here we derive estimates for SHmax by analysing breakout width data from an acoustic televiewer log run the 5 km deep borehole BS-1. Some 81% of the borehole below the granite top at 2.42 km is affected by breakouts, which is favourable for examining the depth trends of the estimates. A primary objective of the analysis was to evaluate the impact of four different failure criteria on the SHmax magnitude estimates. The criteria where Rankine, Mohr-Coulomb, Mogi-Coulomb, and Hoek-Brown 3D. All were parametrized using stress data from a single multi-stage triaxial compressive test on a core plug taken from near the well bottom. A numerical approach was employed to derive SHmax magnitude from the estimated breakout widths, taking into account all stress components at the borehole wall including the remnant thermal stress arising from the cooling of the borehole wall by the drilling. Previous studies of breakout width have shown that large, small-scale fluctuations are associated with fractures, which reflect variations in strength or stress, or both. At larger scales, breakout width tends to decrease with depth. Assuming there is no significant systematic change in the stress characteristics of the rock along the length of the hole, for which there is no evidence, the large-scale trend has the consequence of implying a small gradient of the SHmax profile. This result is independent of the failure criterion, and also of the profile of Shmin used in the analysis. The absolute values of SHmax depend upon the failure criterion used. Criteria that consider the strengthening effect of the intermediate stress (Mogi-Coulomb and Hoek-Brown 3D) yield profiles that violate frictional limits on the strength of the crust above 4 km, whereas the profiles of the Mohr-Coulomb and Rankine criteria do not (the latter two are essentially identical for the case where pore pressure and wellbore pressure are equal and in the range of Shmin and SHmax relevant for our analyses). The Mohr-Coulomb/Rankine criteria profiles indicate a trend in SHmax from favoring strike-slip faulting above 4200 m to strike-slip/normal faulting below. This is reasonably consistent with focal mechanisms recorded during the reservoir stimulation which show a mix of strike-slip and normal faulting throughout the depth range considered.

1. INTRODUCTION
The hydraulic stimulation of low-permeability EGS reservoirs is necessary for such systems to produce commercially interesting flow rates. The in-situ state of stress plays a major role in the response of the rock mass to hydraulic stimulation injections. Thus, it is a key parameter that must be specified in any geomechanical model of the stimulation process. In most situations, the vertical stress magnitude, Sv, at reservoir depth can be taken as the weight of the overburden derived from density logs. The minimum horizontal stress magnitude, Shmin, can be estimated from hydraulic tests performed in isolated intervals in the open-hole of the reservoir, or in short intervals drilled immediately below casing points. In deep boreholes, wellbore failure is common, and this provides a good indication of the orientation of Shmin. However, the magnitude of the maximum principal horizontal stress, SHmax, is particularly difficult to constrain.

One approach that is now commonly applied is to estimate SHmax from the width of breakouts (Barton et al., 1988). However, such approach relies heavily on the appropriateness of the failure criteria used in the analyses. Different failure criteria lead to significantly different SHmax estimates. For example, criteria that consider the strengthening effect of the intermediate principal stress, give significantly higher SHmax magnitude estimates compared to those that consider only the minimum and maximum principal stress. Irrespective of the failure criterion used, obtaining a representative estimate of the in-situ strength characteristics of the rock is difficult. Such an estimate is usually based on mechanical tests on a few core samples. The paucity of data limits our ability to quantify strength variability, and hinders the assessment of the representativeness of the estimate to in-situ conditions. In addition, core damage due to the formation of micro-cracks when coring, and the relaxation of the in-situ stresses is likely to lead to an underestimation of the in-situ strength (Martin and Stimpson, 2004). An alternative approach is to estimate in-situ strength using physical properties obtained from borehole logs (e.g. Chang et al., 2006). However, the correlations underlying this approach often do not bring robust constraints on the strength if they are not intensively calibrated to the site specific rock types. Despite these potential limitations, analyses of borehole failure provide some constraints on SHmax magnitude, particularly when the intensity or style of borehole failure varies with depth (Valley and Evans, 2007). Matching these depth dependent variations, as for example a transition with depth from drilling induced tension fractures to breakouts as observed in the deep boreholes of Soulitz-sous-Forêts (Valley and Evans, 2007), allow usually to set useful bounds in the SHmax magnitude or at least to evaluate the stress magnitude gradients.

This paper presents SHmax profiles in the granitic section of the 5 km deep Basel-1 well from breakout width derived using four different failure criteria. The focus is to demonstrate the sensitivity of the SHmax estimates to the failure criteria used. The resulting profiles are compared to the variations in stress suggested by the style of focal mechanisms of micro-earthquakes induced in the reservoir during the stimulation injections.
2. CONSTRAINTS ON DEPTH TRENDS OF STRESS COMPONENTS AT THE BASEL GEOTHERMAL SITE

In 2006, a 5 km deep borehole was drilled below the city of Basel (Switzerland) with the intention of developing an Enhanced/Engineered Geothermal System (EGS) within a granite body whose weathered top lies beneath 2,426 m of sedimentary cover (Häring et al. 2008). After an extensive reservoir characterization phase, the open hole section between 4,632 m and 5,009 m was subject to a hydraulic stimulation injection in December 2006 as the first stage in creating a heat exchanger within the granite. This stimulation injection produced seismicity that was felt in the city of Basel, which eventually resulted in the abandonment of the project.

The average orientation of the maximum horizontal principal stress in the basement of the southern part of the Upper Rhine Graben and the Swiss foreland region centered on Basel is well defined from failure analyses in deep boreholes (Evans and Roth, 1998; Valley & Evans, 2009) and stress inversion of focal mechanisms (Kastrup et al., 2004). The results are consistent with the observation of borehole failure in the 5 km deep BS1 borehole that indicate a mean SHmax orientation of 143°±14° (Valley and Evans, 2009). Focal mechanisms in the area show a mix of strike-slip and normal faulting, with strike-slip being dominant in the southern end of the Upper Rhine Graben (Kastrup et al., 2004). Häring et al. (2008) proposed an initial characterization of the stress magnitude in the Basel EGS reservoir that is consistent with a strike slip regime. Their characterization is based various indicators, including a single RACOS (Rock Anisotropy Characterization on Samples; Braun, 2007) measurement on a single core sample recovered from 4,911 m. The principle of RACOS measurement is similar to residual strain analyses, but in the case of the RACOS approach, P- and S-wave velocities are measured and used in the analyses. The details of the analysis, however, are not disclosed, and thus validity cannot be assessed.

A profile for vertical stress in the open-hole section below 4,623 m was derived by Häring et al. (2008). The density profile of the sedimentary section was estimated from density logs run over sections of the 1 km distant well Otterbach-2, supplemented by average formation densities given by Schärli and Kohl (2002). Combining this with the density log from Basel-1, they estimated a mean density for the rock overlying the open hole section of 2,538 kg/m³, giving an Sv profile of:

\[
Sv = \rho g z = 24.9 z \ \text{MPa/km}
\] (1)

The mean density of the granite section below 2,426 m BG is 2,683 kg/m³, somewhat higher than the mean value above, giving a steeper gradient of 26.3 MPa/km for the granite section. Taking into account the density data for the sedimentary section used by Häring et al. (2008) gives a Sv profile for the granitic section of: \(Sv = 26.3 z - 4.0 \ \text{MPa/km}\). The estimate of Eq. (1) is a convenient simplification for our analysis. It is exact at 2,857 m depth, overestimates \(Sv\) by 0.6 MPa at the top of the granite, and underestimates \(Sv\) by 3.0 MPa at the hole bottom (5,004 m).

The profile of Shmin is best estimated from hydrofracture tests. In deep wells, in the oil and gas sector, conventional hydrofracture or ‘minifract’ tests are rarely conducted owing to operational difficulties in hydraulically isolating short test intervals. Rather, Shmin estimates are usually extracted from FITs (Formation Integrity Tests), LOTs (Leak-off Tests), or XLOTs (Extended Leak-off Tests). These are small-volume injection tests performed on a short (~10 m) section of hole drilled ahead of a casing shoe after cementing a casing to test the integrity of the cementing and determine the pressure at which the injectivity of the formation begins to increase due to the opening of fractures. It is usually not known whether a natural or an induced fracture opens - and thus whether the opening pressure is a direct measure of the minimum principal horizontal stress or is greater. Only XLOT tests feature pressurization cycles, which are necessary to demonstrate that the increase in permeability is due to mode-1 fracture opening, as required for the pressure to be representative of the normal stress on the fracture that opens. FITs are usually the most ambiguous in this regard, and may show permeability increases due to shearing at pressures much lower than the level of the minimum principal stress. FITs, unlike XLOTs, may yield underestimates for Shmin, but both can yield overestimates. An FIT test was performed at 2,602 m BG immediately after cementing the 10.75” liner and showed flattening at a downhole pressure of 34 MPa. If this is taken as a direct measure of Shmin at 2,602 m BG, and the Shmin profile is assumed to be linear and passes through the origin (i.e. Shmin is zero at the surface), then the value of Shmin at the top of the open hole section at 4,632 m would be 60.6 MPa. The Shmin profile would be similar to that found at Soultz inasmuch as Shmin would be about 52% of the vertical stress (Valley & Evans, 2006). However, since the wellbore pressure at 4,632 m reached 74.4 MPa towards the end of the stimulation injection, the extrapolated Shmin profile from the FIT measurement would imply that the borehole pressure exceeded Shmin at the casing shoe by 13.8 MPa. Such a large excess is implausible, particularly as there are well-developed drilling-induced tension fractures at the top of the open hole that would surely extend as hydrofractures under such a high overpressure. For this reason, the best estimate of Shmin at 4,632 m (the 7-5/8” liner shoe) is taken as 74.4 MPa, the maximum pressure reached during the stimulation injection. Evidence at Soultz suggests that the maximum pressure attained at the top of the open hole section represents a reasonable measure of Shmin at that depth, although this is probably not a universal rule (Valley & Evans, 2006). Assuming a linear Shmin trend that passes through the origin, this Shmin measure at 4,632 m leads to the following linear trend:

\[
\text{Shmin} = 16.06 z \ \text{MPa/km}
\] (2)

This is slightly less than the value of 17.1 MPa adopted by Häring et al (2008), which is influenced by a single RACOS measurement on core taken from near 4,900 m (Braun, 2007). Considering a linear fit through both the FIT test result at 2,602 m and the maximum pressure reached at the liner shoe (4,632 m) leads an even larger gradient with a large negative offset at zero depth:

\[
\text{Shmin} = 19.90 z - 17.78 \ \text{MPa/km}
\] (3)

In the following analyses, the pore pressure will be considered to be given by hydrostatic conditions with a fluid density of 1,000 kg/m³.

\[
P_p = \rho _p g z = 9.81 z \ \text{MPa/km}
\] (4)
The SHmax profile is the most difficult to constrain and is the focus of this paper. Specifically, we will examine SHmax estimates derived from the width of borehole breakouts. This analyses starts by considering the stress concentrations arising at the borehole wall.

3. STRESS STATE AT THE BOREHOLE WALL

The objective of the analysis is to examine the relationship between the width of breakout failure at the BS-1 borehole and the far-field stresses for different failure criteria. We assume that borehole failure initiates at the borehole wall, and thus only the stress state at the wall is relevant for determining breakout width (not true if other aspects of the failure of the geometry are considered). The borehole BS-1 is taken as vertical, since the maximum deviation from verticality is 8°. We also assume that one principal stress is vertical and thus co-axial with the borehole axis. An extension of the analysis to the more general case where the borehole is not aligned with a principal stress axis could be made using the approach of Hiramatsu and Oka (1968).

The total stress state at the circular borehole wall can be expressed in cylindrical coordinates by the principal stress components of the tangential stress, $S_\theta$, the radial stress, $S_r$, and the axial stress, $S_z$. Assuming far-field stress magnitudes, $S_v$, SHmax and Shmin, and a wellbore fluid pressure of $P_w$, the magnitudes of the components at an angle $\theta$ from the SHmax direction are:

\[
S_r = P_w + S_r^{\Delta T}
\]

\[
S_\theta = Shmin + SHmax - 2(SHmax-Shmin)\cos(2\theta) - P_w + S_\theta^{\Delta T}
\]

\[
S_z = S_v - 2v(SHmax-Shmin)\cos(2\theta) + S_z^{\Delta T}
\]

where $v$ is Poisson’s ratio, and $S_r^{\Delta T}$, $S_\theta^{\Delta T}$, and $S_z^{\Delta T}$ are possible thermal stress components in the radial, tangential and axial directions arising from any difference in rock temperature, $\Delta T$, at the borehole wall from ambient temperature. The thermal stress components at the borehole wall are given by Stephens and Voight (1982) as:

\[
S_r^{\Delta T} = 0
\]

\[
S_\theta^{\Delta T} = S_z^{\Delta T} = \frac{\alpha \Delta T}{1-v}
\]

where $\alpha$ is the Young’s modulus, $\alpha$ in the coefficient of linear expansion, and $\Delta T$ is positive for heating and negative for cooling.

The question of which of the three principal stresses at the borehole wall correspond to the maximum, intermediate or minimum stress for any far field stress condition (i.e. any combination of Shmin, SHmax, Sv and Pw) and any location around the hole ($\theta$) is not trivial. However, when considering the circumferential locations favorable for breakout-failure (sectors centered at $\theta = 90$ or 270°), $S_\theta - S_v$ or $S_\theta - S_z$ is valid in most situation. An exception arise when SHmax and Shmin are almost equal and of low magnitudes compared to $S_v$: in this situation, $S_z$ can become the maximum principal stress. Special care must thus be taken when considering borehole failure analyses in such a stress regime.

4. FAILURE CRITERIA

In this section we describe the various failure criteria used the analysis. Extended reviews of failure criteria used in wellbore failure analysis can be found in Colmenares and Zoback (2002) and Zhang et al. (2010). The criteria were parametrized to the case of the Basel monzogranite using multi-stage confined compression tests performed on a single core plug by Braun (2007). The 34 mm diameter sample was 70 mm in length, and was itself cored from the ~100 mm diameter core recovered from the BS-1 hole. The sample is from a depth of 4902 m BG, and consists of monzogranite with a composition of approximately 50% quartz, 25% plagioclase, 10% potassium feldspar and 15% ferromagnesian feldspar. The grain size in the plug sample is about 3-5 mm, although large K-Feldspar (up to 5 cm long) are present in the BS1 core. It was tested under axial loading with increasing confinement in steps of 5, 10, 30, 50 and 70 MPa. Confinement was increased when signs of yield were identified on the axial or radial strain records. The axial stress at yield for each confining stress level are listed in Table 1. The elastic properties determined during the first loading step with 5 MPa confinement and ignoring initial closure effects are $E = 39$ GPa for the Young’s modulus and $\nu = 0.22$ for the Poisson’s ratio. Similar values were obtained in subsequent steps at higher confining pressure. The measured static Young’s modulus is unrealistically low for a fine-grained monzogranite and is not consistent with the average dynamic modulus of ~80 GPa derived from sonic and density logs. Static Young's modulus would be expected to be only 15 to 20% lower than dynamic modulus (Eissa and Kazi, 1988), which in this case would be about 65 GPa, but not less. A possible explanation for this discrepancy could be the presence of core damage, although in this case one would expect the effect of damage on modulus would diminished with increasing confinement which is not the case for this test. This discrepancy in modulus remains at the moment unexplained. A modulus of 65 GPa will be used in the analyses. The coefficient of linear thermal expansion estimated from mineralogy is approximately 1e-5 K⁻¹.

A number of factors must be born in mind when considering the degree to which the data in Table 1 reflect the strength characteristics of the intact rock under in-situ conditions. Significant core damage in the form of disking or incipient disking with 2-3 cm spacing was observed along almost the entire 10 m length of the 10 cm diameter core extracted from near the bottom of the BS-1 borehole. Although it is likely that the plug used in the strength testing was selected to be as far as possible from an incipient
Valley and Evans
disking fracture, the spacing and ubiquitous nature of the incipient disking would make it difficult to find a zone to extract a 34 mm plug that did not contain microscopic damage associated with the disking process. In any case, relaxation of in-situ stress would induce microscopic damage that would serve to reduce the strength and Young's modulus of the sample, and increase it's Poisson's ratio. Strength reductions due to stress relaxation of up to 30% have been reported (Martin and Stimpson, 1994), and disking-related damage could also add to this. Moreover, the size of the sample tested is significantly smaller than recommended (Beniawski and Bernede, 1979), although this is more likely to have the opposite effect of producing an overestimate of the strength and modulus compared to tests on standard size (Darlington et al., 2011). A further concern is that the multistage procedure used in the testing can generate bias if the loading system is too slow to react to the onset of yield and significant damage occurs before stabilizing the sample by increasing the confinement. If such occurs, the strength estimate at higher confinement tends to be low, leading to underestimation of the internal friction angle. Finally, only a single test was performed, and thus there is no demonstration of reproducibility or assessment of the variability of strength characteristics along the hole. In view of these limitations, the strength data given in Table 1 are viewed as tentative.

| Table 1: Results from multi-stage strength testing of a single plug from of the Basel core from Braun (2007). |
|-----------------|---|---|---|---|---|
| \( \sigma_2 = \sigma_3 \) (confinement) [MPa] | 5  | 10 | 30 | 50 | 70 |
| \( \sigma_1 \) (axial load at estimated yield point) [MPa] | 169.7 | 221.7 | 337.3 | 442.5 | 557.3 |

For all failure criteria considered, the weakening effect of pore pressure as described by an effective stress law must be included. The effective stresses, \( \sigma \) are computed from total stress, \( S \), as:

\[
\sigma = S - \beta P_p
\]

where \( \beta \) is a coefficient in the effective stress law that depends of the failure mode. For compressive failure we consider a coefficient \( \beta = 1.0 \), as found by Brace and Martin (1968) to be valid, even for low-porosity crystalline rocks. However, we consider that the rock at the borehole wall is likely to have a significantly higher porosity than the undisturbed granite owing to severe changes in stress experienced during drilling.

### 4.1 Rankine criterion
This is the simplest of the considered criteria in which the strength of the borehole wall is assumed to be a constant, independent of the intermediate or minimum effective stresses, \( \sigma_2 \) or \( \sigma_3 \). With this criteria, failure occurs when the maximum effective principal stress, \( \sigma_1 \), exceed a strength threshold, \( C_0 \):

\[
\sigma_1 \geq C_0^R
\]

The strength threshold \( C_0^R \) is typically considered to be equal to the uniaxial compressive strength. The value of the uniaxial compressive strength for the Basel granite is discussed below.

### 4.2 Mohr-Coulomb criterion
In the Mohr-Coulomb criterion, rock strength increases linearly with the minimum effective principal stress. This criteria can be expressed in terms of principal stresses as following:

\[
\sigma_1 \geq C_0^R + q \sigma_3
\]

where \( C_0^R \) is the uniaxial compressive strength, and \( q \) is a material constant that can be related to the internal friction angle, \( \phi \), through \( q = \tan(\phi/2) \). Plots of the Mohr-Coulomb criterion in \( (\sigma_2 - \sigma_1) \)and \( (\sigma_3 - \tau) \) spaces for the Basel data given in Table 1 are shown in Fig. 1. The data follow linear trends except for the data point at the lowest confining pressure of 5 MPa which indicates a lower \( \sigma_1 \) failure stress. Taken on face value, this suggests either a curved criterion (e.g. Hock-Brown criterion) or a bilinear criterion. However, the possibility that it reflects sample damage cannot be ruled out. If one discards the 5 MPa confinement data point, the extrapolated strength at zero confinement (i.e. the uniaxial compressive strength) is \( C_0 = 167 \) MPa (blue dashed line on Fig. 1a), and the internal friction angle is 44°. However, if the lowermost data point is considered valid, then the linear fit through the two lowest confinement data points (red line on Fig. 1a) or a curved criterion through all data, give \( C_0 = 118 \) MPa. The implied internal friction angles are 55.5° for confinement smaller than 10 MPa, and 44.0° for confinement higher than 10 MPa.

### 4.3 Mogi-Coulomb criterion
The Mogi-Coulomb criterion, proposed by Al-Ajmi and Zimmerman (2005) is a linearized version of Mogi's criterion (Mogi, 1971) that relates the octahedral shear stress, \( \tau_{oct} = \frac{1}{2} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \) to the mean stress, \( \sigma_{m.2} = \frac{\sigma_1 + \sigma_3}{2} \). Expressed in \( \sigma_{m.2} - \tau_{oct} \) space, the failure criterion is:

\[
\tau_{oct} \geq a + b \sigma_{m.2}
\]
Figure 1: Strength data of Table 1 for the Basel granite with best linear fits in a) $\sigma_3 - \sigma_1$ and b) $\sigma_n - \tau$ spaces.

Figure 2: Strength data of Table 1 for the Basel granite plotted in $\sigma_{m.2} - \tau_{oct}$ space (circles). See text for description of the linear fits.

The parameters a and b are material constants that are related to the Mohr-Coulomb parameters of internal friction, $\phi$, and cohesion, $c$, by $a = \frac{2\sqrt{3}}{3} c \cos(\phi)$ and $b = \frac{2\sqrt{3}}{3} \sin(\phi)$ . The strength test data from Table 1 are plotted in $\sigma_{m.2} - \tau_{oct}$ space in Fig. 2. The black line denotes the best linear fit to the test data points (circles). Also shown are the two failure lines derived from the Mohr-Coulomb parameters given in Fig. 1: red for the fit to the two low confinement data (red line in Fig. 1), and blue for the fit to all points except the lowest confinement point (blue line in Fig. 1). The latter is essentially identical to the directly-derived curve.

### 4.4 Three dimensional Hoek-Brown criterion

The Hoek-Brown failure criterion is an empirical, non-linear failure criterion that is based upon empirical (Hoek and Beniawski, 1965) and theoretical considerations (Griffith, 1920). For intact rock, it has the form:

$$\sigma_1 \geq \sigma_3 + C_o \sqrt{\frac{\sigma_3}{C_o m_i} + 1}$$

where $m_i$ and $C_o$ are parameters. The strength data from Table 1 are plotted in $(\sigma_3 - \sigma_1)$ space in Fig. 3a, together with the best-fitting curve of the form of Eq. (14). Zhang and Zhu (2007) proposed an extension of this criterion in order to account for the intermediate principal stress. Expressed in $\sigma_{m.2} - \tau_{oct}$ space, it takes the form:

$$\frac{9}{2C_o} \tau_{oct}^2 + \frac{3}{2\sqrt{2}} m_i \tau_{oct} \sigma_{m.2} \geq C_o$$

(15)
and can be understood as a non-linear version of the Mogi criterion. The data from Table 1 are plotted in \((\sigma_{m,2} - \tau_{oct})\) space in Fig. 3b. The fitted curve is obtained by introducing the parameters \(C_o\) and \(m_3\) determined in \((\sigma_3 - \sigma_1)\) space (Eq. 14) in Eq. (15) and this produce a very satisfactory fit. A least square best fit to the data directly in the \((\sigma_{m,2} - \tau_{oct})\) space leads to only slightly different fitting parameters.

5. SHMAX ESTIMATE FROM BREAKOUT WIDTH

Barton and Zoback (1988) adopted a Rankine failure criterion, and assumed that the width of a breakout that formed for a given combination of far-field stresses, \(\text{SHmax} & \text{Shmin}\), and interval wellbore pressure, \(P_w\), would be defined by all points around the borehole wall where the effective tangent stress, \(\sigma_0 = S_0 - P_p\), reaches or exceeds the Rankine failure threshold, \(C_o^R\). They derived an equation for computing \(\text{SHmax}\) from breakout half width, \(\theta_b\), given an independent estimate for \(\text{Shmin}\). After modification to include the presence of a thermal component, \(S_0^{\Delta T}\) (Eq. 9) to the tangent stress at the wellbore wall arising from a possible temperature perturbation from ambient (e.g. Valley, 2007), the relation becomes:

\[
\text{SHmax} = \frac{C_o^R + P_p + P_w - S_0^{\Delta T} - \text{Shmin}(1 - 2\cos(2\theta_b))}{1 + 2\cos(2\theta_b)}
\]

This approach makes several questionable assumptions. Firstly, the effect of evolving borehole geometry on tangent stress at the borehole wall during breakout development is not considered. The validity of this assumption is not clear and may depend of the mode of failure. Secondly, it is assumed that \(S_0\) is the maximum principal stress for all value of \(\theta\), which is not always strictly the case. As discussed in Section 3, this assumption is almost always true for \(\theta = 90°\) and 270°, but is often not true for \(\theta = 0°\) and 180°. Eq. (16) is thus not valid for very wide breakouts. Another reason why Eq. (16) is not valid for wide breakouts is due to its asymptotic behavior: as breakout width, \(2\theta_b\) approaches 120°, the denominator of Eq. (16) approaches zero and thus \(\text{SHmax}\) becomes undetermined. Moreover, for widths close to 120°, small variations in width produce large changes in the estimate of \(\text{SHmax}\), leading to poor constraints on \(\text{SHmax}\). For these two reasons, it is not recommended to use Eq. (16) when the breakout width is larger than 100°.

A slightly different expression can be derived using a Mohr-Coulomb criterion (Eq. 12), where strength is a function of the minimum effective principal stress, \(\sigma_3\). Here, the maximum principal stress is taken as \(S_0\) (Eq. 6) and the minimum principal stress as \(S_T\) (Eq. 5). The estimate of \(\text{SHmax}\) is obtained in the same manner as for the Rankine criterion by solving for \(\text{SHmax}\) at the breakout edge:

\[
\text{SHmax} = \frac{C_0^R + P_p + P_w - S_T^{\Delta T} - \text{Shmin}(1 - 2\cos(2\theta_b))}{1 + 2\cos(2\theta_b)}
\]

where \(q\) is the frictional strength component as defined in Eq. (12). This relation suffers from the same limitations as Eq. 16 and should not be used for breakout width larger than 100°. It is important to note that Eq. (17) reduces to Eq. (16) when \(P_w = P_p\), because then the minimum effective principal stress at the borehole wall becomes zero, and the borehole wall strength is constant and equal to \(C_o\). In our analyses, we take \(P_w = P_p\), and so the results for the Mohr-Coulomb criterion are identical to the Rankine criterion, with \(C_0^R = C_o\).
Analytical solutions for SHmax are not easily derived when considering criteria where the intermediate effective principal stress, $\sigma_2$, influences the strength (e.g. Mogi-Coulomb or Hoek-Brown 3D). However, the solution can be computed numerically. In addition, when computing the solution numerically, the relative magnitude of $S_0$, $S_z$ and $S_r$ can be checked before introducing them in the failure criterion, which removes one of the limitations discussed above.

The SHmax-Shmin solution spaces defined from numerical computations of the breakout width using each the four failure criteria parameterized using the data in Table 1 are presented in Fig. 4. Curves are shown for a range of breakout widths. The Rankine and Mohr-Coulomb criteria lead to identical results except at very high SHmax magnitudes. This is due to the fact that with very high SHmax magnitudes, the minimum principal stress at the borehole wall is the axial component ($S_z$) and is negative. With the Mohr-Coulomb criteria, a negative minimum principal stress leads to a weakening of the borehole wall to values less than $C_o$, an effect which is not captured by the Rankine criteria. This effect is although not captured by Eq. (17), due to the assumption in this equation that the minimum principal stress is always the radial stress. The 120° contour line is vertical, indicating complete insensitivity to SHmax which reflects the asymptotic behavior of Eq. (16) and (17). For the true triaxial criteria (Mogi-Coulomb and Hoek-Brown 3D), the effect of the intermediate stress is to translate the initiation of failure (0° width line) to a higher SHmax magnitude compared to the Mohr-Coulomb case.

Fig. 5 shows the dependence of breakout width on SHmax magnitude for the case Shmin $= 75$ MPa, corresponding the estimated conditions at open-hole depth in the Basel reservoir. For the Mogi-Coulomb and Hoek-Brown 3D criteria, failure initiates for SHmax magnitudes of 148 MPa and 153 MPa respectively, whereas initiation for the Rankine and Mohr Coulomb criteria occurs 32-37 MPa lower at 116 MPa. At a breakout width of 95°, the predicted SHmax difference for the two pairs of criteria increases to about 60 MPa. The curves of Fig. 5 are very steep at breakout initiation which means that a subsequent small increase in SHmax of only 2.5 MPa results in a 20° increase in predicted breakout width (true for all criteria). This suggests that breakout width of less than 20° are not stable, in agreement with the fact that such narrow breakouts are never observed.

The curves of Fig. 5 also show that if Shmin and the breakout width are known, the magnitude of SHmax can be uniquely determined, at least for breakout widths in the range 0° to 100°, either using Eq. (16) and (17) or by numerically searching for the matching SHmax magnitude. In the following sections, this approach is applied to the profile of breakout widths in the BS-1 well to obtain profiles of SHmax.

Figure 4: SHmax-Shmin solution space at the top of the open hole section (4632 m depth BG) for a range of breakout widths ($2\theta_{B}$) obtained numerically for the following failure criteria parameterized by the strength characteristics measured on the plug from the Basel-1 core: a) Rankine criterion, b) Mohr-Coulomb criterion, c) Mogi-Coulomb criterion and d) Hoek-Brown 3D criterion. The assumed parameter values at 4632 m are: $S_v = 115$ MPa, $P_p = P_w = 45$ MPa. A thermal stress of -16.7 MPa is included to reflect a persisting 20°C cooling of the hole after drilling and a Poisson’s ratio of 0.22. The bold lines denote limits on SHmax imposed by assuming that crustal strength is limited by a Coulomb friction criterion with a friction coefficient of 1.0.
Figure 5: Breakout width as a function of SH\text{max} for the four strength criteria parameterized with the strength data from Table 1. A value for Sh\text{min} of 75 MPa was used, as appropriate for the top of the open hole section of the Basel reservoir. The curves define the combinations of SH\text{max} and breakout widths prevailing along profiles A-A’ in Fig. 4 a) to d)

6. ESTIMATES OF BREAKOUTS WIDTH AT THE BASEL SITE

Borehole failure was identified on a Schlumberger ultrasonic borehole televiewer log (UBI) run in crystalline rock between 2569 and 4992 m shortly after drilling (Valley and Evans, 2009). The borehole diameter is 9-7/8” to 4841 m depth and 8-1/2” below. The borehole is sub-vertical, the maximum deviation reaching 8° towards the NW at 4600 m. The ultrasonic reflectivity and travel time logs are shown in Figs. 6a–b. Breakouts were identified along 81% of the logged section, and are almost continuous except for a large 152 m gap from 2747 to 2899 m depth, and some other less-significant gaps at 3820 – 3856 m, 4185 – 4221 m and 4582 – 4631 m (Valley and Evans, 2009).

Figure 6: a) Ultrasonic reflectivity. b) Ultrasonic travel time. c) Angular widths of the NE (green) and SW (blue) breakout limbs. d) Breakout width profile from averaging both limbs. e) to i) Histograms of breakout width for successive 500 m depth intervals and j) all data.
Only axial spalling occurring in two localized, diametrically-opposite directions were considered as potential breakouts. The breakouts were measured on successive 40 cm sections over which the borehole geometry was averaged. The angular width of both limbs of the breakouts were measured independently. Measurements were made through visual inspection of borehole cross-sectional geometry. As shown in Fig. 6c, the widths of each pair of breakout limbs are reasonably consistent except for a discrepancy near 4050 m meter depth where the SW breakout side is more developed. For further analysis, the widths of both breakout limbs were averaged to generate the depth profile of breakout width shown in Fig. 6d. Histograms of breakout width for successive 500 m depth slices are presented on Figs. 6e,f. Considering all measurements, the width is 79.2° ± 17.4° (mean ± 1 std. dev.). The smallest measured width is 20° with a single measurement exception at 15°, consistent with the theoretical consideration of Fig. 4b that breakout with width smaller than about 20° are not stable. The largest measured width is 150°. Breakout width tends to decrease with depth from an average of 93.3° in the 2500-3000 m range to 67.0° in the 4500 to 5000 m range.

7. STRESS ESTIMATE FROM BREAKOUT WIDTH OBSERVATIONS

The breakout width profile shown in Fig. 6d was used to determine a profile of SHmax assuming the profiles of Sv and Shmin are given by Eq. (1) and (2), respectively. A thermal stress component of \( S_0^{\text{AT}} = S_2^{\text{AT}} = -16.7 \text{ MPa} \) was used since temperature logs indicate the crystalline borehole section remained 20°C cooler than ambient at the time the UBI log was run. The four different failure criteria described in Section 4 were considered in the analysis. The magnitude of SHmax at each depth were derived numerically by iteratively adjusting the SHmax value and by solving the forward problem (computing the expected breakout width) until a good match with the observed breakout width was found. A minimization routine by Forsythe et al. (1976) that is implemented in the MATLAB™ Optimization Toolbox was used. When the observed breakout width exceeded about 100° (i.e. getting close to the asymptote at 120° mentioned in Section 5), a sensible match could not be achieved and generally no solution was obtained. All retained solutions match with the observed breakout width within 1e-10° or better. At depths where no breakouts were observed, the computed SHmax level for breakout initiation was taken as an upper bound for SHmax. The SHmax profiles resulting from these computations for the four failure criteria are presented on Fig. 7a-d, together with bounds on SHmax imposed by assuming that crustal strength is limited by a Coulomb friction criterion with friction coefficients, \( \mu \), of 0.6 and 1.0.

Figure 7: Result of the computation of SHmax from breakout width (see main text for details on the computation procedure) for four failure criteria: a) low confinement Mohr-Coulomb/Rankine with \( C_0^R = C_0 = 117.7 \text{ MPa} \); b) high confinement Mohr-Coulomb with \( C_0^R = C_0 = 167.3 \text{ MPa} \); c) Mogi-Coulomb (\( a = 21.26, b = 0.67 \) and d) Hook-Brown 3d (\( C_0 = 109.0 \text{ MPa}, m_3 = 27.7 \)). The Mohr-Coulomb and Rankine criteria are identical because \( P_w = P_p \). Light blue dots: results of each individual SHmax computation. Solid black line: filtered result using a 3-point (1.2 m) moving average. Red line: upper bound on SHmax computed at locations where no breakouts were observed. Green line: assumed Shmin magnitude. Blue line: assumed Sv magnitude. Black dashed line: limits on SHmax imposed by assuming the crust strength is limited by a Coulomb friction criterion with friction coefficients, \( \mu \), of 0.6 and 1.0.
8. DISCUSSION

A common feature of all the computed SHmax profiles is the very low or even slightly negative gradient with depth. A best linear fitting depth trends to the four SHmax profiles have gradients of 1 MPa/km and -4 MPa/km for the Mohr-Coulomb/Rankine criteria with low and high uniaxial compressive strength respectively (a and b), -2 MPa/km for the Mogi-Coulomb criterion (c), and 7 MPa/km for the Hoek-Brown 3D criterion (d). Such low SHmax gradients are a consequence of the observation that breakout width decreases with depth. Changing the slope of the Shmin profile within reasonable bounds from that of Eq. (2) does not change this result. This is shown in Fig. 8 where the SHmax profile was computed for the high-confinement Mohr-Coulomb/Rankine criterion for two extreme Shmin scenarios: one where the Shmin gradient was taken as zero (Fig. 8a); and the other for the steep gradient obtained by fitting the profile through the Shmin estimate from FIT test result at 2602 m as well as the maximum pressure reach at the liner shoe (4632 m) during the stimulation (Eq. 3, Fig. 8b). In both cases, very low or even negative SHmax gradients are obtained.

Although the gradients of the SHmax profiles predicted for the four failure criteria are similar, the absolute magnitudes differ significantly. Specifically, the two criteria that consider the strengthening effect of the intermediate principal stress (cases c and d) lead to SHmax estimates that are 60 to 80 MPa higher than for case (a) and 25 to 50 MPa higher than for case (b). Moreover, the SHmax magnitude estimates for (c) and (d) above 4200 m exceed the upper limit set by assigning a Coulomb friction strength to the crust with a friction coefficient of 1.0. Profile (b) derived from the Mohr-Coulomb/Rankine criterion with the higher CO does not violate this upper limit.

The stress regime implied by the SHmax profiles for the Mogi-Coulomb (c) and Hoek-Brown-3D (d) criteria is essentially strike-slip for the entire depth range (a possible exception applies for sections without breakouts where only an upper bound for SHmax can be derived). SHmax profile b) for the Mohr-Coulomb criterion with higher CO trends from a strike-slip regime above 4200 m to mixed strike-slip/normal faulting below 4200 m. SHmax profile (a) for the Mohr-Coulomb criterion with lower CO crosses over from a predominantly strike-slip regime above 4100 m to a predominantly normal faulting regime below 4500 m. These results can be compared with the depth trends of faulting style derived from focal mechanisms analyses. A catalog of 28 focal mechanisms determined by Deichmann and Ernst (2009) using the polarity of the first P-wave arrivals for the stronger events recorded during the stimulation of the BS-1 borehole is extended to 118 focal mechanisms by Terakawa et al. (2012). This data set is dominated by strike-slip faulting style with some normal component but no or very little thrust component. No clear depth trend is recognizable on this data set. A larger focal mechanism set including 639 events is derived by Kraft and Deichmann (2014) using P- and S-wave amplitudes in addition to P-wave first motion polarities to constrain the focal mechanism. This dataset contains a mix of normal and strike-slip focal mechanism as well as a few thrust events. No depth trend is present concerning the balance between normal and strike-slip mechanisms, suggesting that SHmax magnitude is not greatly different from Sv. However, mechanisms with a thrust component are confined to the upper part of the granite above the liner shoe at 4632 m depth, suggesting that shmin is closer to σv in the upper part of the profile than the lower part. Such behavior could be explained by a relatively small gradient for Shmin, similarly to the small gradient derived for SHmax. Depending upon the failure criterion used. However, in reality, the variation in breakout width is likely to reflect both stress and strength variations along the borehole. Fabbri (2011) and Sikaneta and Evans (2012) found changes in breakout width in the BS-1 borehole often correlated with locations where fracture or fracture zones cut

Figure 8: Result of the computation of SHmax from breakout width for high confinement Mohr-Coulomb (CO = 167.3 MPa, q = 5.6) with a) Shmin at a constant value (null gradient) equal to the value of 74.4 MPa determined at the casing shoe during the hydraulic stimulation and b) Shmin = 19.90 z - 17.78 MPa/km, i.e. best fit through both the FIT test result at 2602 m and the maximum pressure reach at the liner shoe (4632 m) during the stimulation (see Section 2 and Eq. 3).
the well, but note that the variations could reflect reduced strength due to alteration and damage, as well as perturbation in stress magnitudes associated with fractures. In the The SHmax profile that is most consistent with independent constraints is the one derived using a Mohr-Coulomb/Rankine failure criterion with the strength at the borehole wall given by the lower of the two estimates of uniaxial compressive strength $C_0$ derived from the strength data of Table 1. This is the only profile that do not violate bounds on admissible SHmax values imposed by widely-accepted frictional limits to the strength of the crust, and it is also fully consistent with the depth trend of faulting style derived from focal mechanisms of micro-earthquakes induced in the reservoir during stimulation. This SHmax profile, like all others, presents a very small depth gradient. A linear fit for this estimate is given by:

$$\text{SHmax} = 1.04 z + 115 \text{ MPa/km}$$

(18)

An obvious feature of all SHmax profiles is the persistent small-scale fluctuations in stress magnitude that arise from fluctuations in breakout width. Taken on face value, the breakout-width fluctuations would imply SHmax variations of several tens of MPa.

9. CONCLUSIONS

The dataset from the 5 km deep borehole BS-1 is exceptional since almost continuous breakouts along 2.5 km are observed. This dataset seems to be particularly appropriate to constrain the maximum principal horizontal stress magnitude, SHmax, by inverting the breakout width. However, the results show that breakout width inversion by itself provides only weak constraints on the SHmax magnitudes. This is because: (1) only one strength test was performed on a single core sample so it is uncertain whether the data is representative of the strength of intact granite along the borehole. (2) there remains uncertainty as to which failure criteria is most appropriate for the formation of breakouts in low porosity crystalline rocks. Using criteria that take in consideration the strengthening effect of the intermediate stress leads to SHmax magnitudes significantly higher than obtained using a standard Rankine or Mohr-Coulomb criteria. However, applied to the BS-1 data, these criteria give SHmax magnitude estimates that violate limits imposed by a frictional strength with a coefficient of friction $\mu = 1.0$, whereas the Mohr-Coulomb/Rankine criteria which ignore the intermediate principal stress do not. (3) the breakout formation process in crystalline rocks is not sufficiently well understood, and basic assumptions commonly made when inverting stress from breakout width may not be valid: for example, the assumption that the initial and final width of the breakouts are identical, and that the tangent stress around the wellbore is not significantly affected by the evolving geometry of the borehole cross-section during breakout development.

Previous analyses have shown that small-scale fluctuations in breakout width are associated with fractures, and reflect stress or stress perturbations, or both. At larger scales, breakout width in the BS-1 well tends to decrease with depth. Assuming there is no significant systematic change in the strength characteristics of the rock along the length of the hole, for which there is no evidence, this has the consequence of implying a small gradient of the SHmax profile. This result is independent of the failure criterion, and also of the profile of Shmin used in the analysis. As noted above, the absolute values of SHmax depend upon the failure criterion used, and that the Mohr-Coulomb/Rankine criteria that do not consider the strengthening effect of the intermediate stress yield profiles that are consistent with frictional limits on the strength of the crust. The profiles indicate a trend in SHmax from favoring strike-slip faulting above 4200 m to strike-slip/normal faulting below. This is reasonably consistent with focal mechanisms recorded during the reservoir stimulation which show a mix of strike-slip and normal faulting throughout the depth range considered.

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