Empirical Correlation for Optimal Turbine Inlet Temperature and Pressure for Geothermal Sub- and Supercritical Organic Rankine Cycles (ORC)

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ABSTRACT

Geothermal ORCs are classified as low temperature power-plant which has relatively low thermal efficiency. Hence, it is important to design such systems on its optimum operating condition, e.g. turbine inlet temperature and pressure (TITP) to maximize the energy recovery. In the present study, empirical correlations are developed to calculate the optimal TITP of sub- and supercritical ORC for geothermal applications, i.e. hybrid (flash-binary) power-plant and medium-enthalpy wells. In geothermal ORCs, injection temperature parameter is an important design prerequisite due to restriction to mineral scaling. Therefore, the ratio of hot-brine temperature (or equivalently, the heat input) to critical temperature of the working fluids and the injection temperature, are used to correlate the optimal TITP. The correlations are derived from the optimal TITP data of 6 typical working fluids at maximum specific net-power resulted from GeSi (Geothermal Simulation, an in-house program). In order to evaluate the accuracy, the correlations are tested by using the simulation results of other 15 pure working fluids at brine temperature of 120 °C – 180 °C and injection temperature of 70 °C – 160 °C. The prediction of the optimal TITP using the correlations is within 5% error. These correlations are very convenient for pre-design in fast and robust manner, especially to predict performance of new working fluids in specific working conditions. As a case study, hybrid power-plant application in Indonesia is used. With average brine temperature of 453.15 K (180 °C) it is found, for n-pentane, the optimal turbine inlet temperature/pressure is 434.15 K (161 °C)/1.9 MPa, which yields thermal efficiency of 15.5%.

1. INTRODUCTION

Binary cycles (Organic Rankine Cycles) are commonly used to utilize low-grade temperature heat from brine in geothermal power. Basically ORC is the same as ordinary Rankine cycle but organic Rankine cycle used an organic fluid as working fluid instead of water. The organic fluid has a lower evaporation temperature than water, which is suitable for heat recovery at low temperatures. These plants are classified as low temperature power-plant which has relatively low thermal efficiency. Hence, it is important to design such systems on its optimum operating condition, e.g. turbine inlet temperature and pressure (TITP) to maximize the energy recovery.

ORC should be operated at optimum condition to optimize the utilization of low-grade heat from the brine. Several studies has been carried out to determine the optimum conditions of the cycle. Wei [2] founded in his research that the maximum power can be obtained by utilizing the waste heat as much as possible. Then He [3] also conducted a study to determine the optimum evaporator temperature for sub-critical cycle. The condition of fluid at turbine inlet is assumed always at saturated vapor for dry fluids. The optimum point is determined by optimizing curve area on the T-s diagram that performed iteratively in EES (Engineering Equation Solver). Vetter [4] have also developed a simulation program to determine the optimum conditions of cycle numerically by varying the pressure and temperature at turbine inlet. Net power for any variations in temperature and pressure compared and selected the greatest net power as optimum conditions.

Reduced pressure parameters (Pr=P/Pcrit) and reduced temperature parameter (Tr=T/Tcrit) commonly used to express general relation like in General Compressibility Diagram (Z vs. Pr) that is applicable for a variety of gases. Based on the condition, it is also expected a correlation between the optimum conditions of ORC (optimum temperature, T1 and pressure, P1 at turbine inlet) and brine temperature (Tg). So in this study will be investigated the correlation between the dimensionless parameter P1/Pcrit, T1/Tcrit and Tg/Tcrit. By using the correlation, the optimum temperature (T1) and the optimum pressure (P1) at turbine inlet can be determined easily based on brine temperature (Tg) and critical state of working fluid (Tcrit and Pcrit).

2. THERMODYNAMIC MODELING

2.1 Organic Rankine Cycle

In Figure 1 is shown a simple scheme of organic Rankine cycle that consists of evaporator, turbine, condenser and pump. Geothermal fluid provides the heat source for the process. The heat is extracted in the heat exchanger using working fluid. Selection of the working fluid depends on its thermo-physical properties. After [9] the working fluid should inter alia meet the following criteria:

- Low critical pressure and temperature (compared to water)
- Low specific volume
- High thermal conductivity
- Non corrosive, toxic or flammable and stable
In addition, low ozone depletion potential (ODP) and a low greenhouse warming potential (GWP) are important requirements for the suitability of the working fluid. Depending on the gradient of the vapor line a distinction is made between dry (retrograde) and wet fluids. Water as a wet fluid has a negative dew line slope, but many organic media are retrograde e they have a dew line with at least a partial positive slope as butane in Figure 1.

Type of cycle depends on the pressure of working fluid in evaporator when the heat is given. If the fluid pressure is smaller than critical pressure \((P_1<P_{crit})\) while heat is given, then the working fluid will undergo evaporation from the liquid phase to a vapor while it passed through two phases region which makes a process called sub-critical cycle (cycle by the dashed lines in Figure 1: 1-2-3-4-1). The super-critical cycle (cycle: 1*-2*-3*-4*-1 in Figure 1), the pressure of working fluid is above critical pressure \((P_1>P_{crit})\) while heat is given, so that the working fluid does not passed through two phases region but past the critical point directly.

The first and second laws of thermodynamics can be applied to determine the performance of organic Rankine cycle. The amount of work produced and heat added to the system can be determined by using energy balance. The following is a calculation of organic Rankine cycle for each component.

a. Process 1-2: process of isentropic expansion in turbine. The maximum power that can be produced by the turbine is:

\[
W_t = \dot{m} \cdot (h_1 - h_2)
\]

(1)

b. Process 2-3: process of isobaric cooling in condenser. The rate of heat rejected from condenser is:

\[
\dot{Q}_w = \dot{m} \cdot (h_2 - h_1)
\]

(2)

c. Process 3-4: process of isentropic compression in pump. Power required by pump to raise the pressure of working fluid is:

\[
W_p = \dot{m} \cdot (h_3 - h_4)
\]

(3)

d. Process 4-1: process of isobaric heating in evaporator. Heat rate received on evaporator is:

\[
\dot{Q}_e = \dot{m} \cdot (h_1 - h_4)
\]

(4)

e. Net power output (\( P_{net} \))

\[
P_{net} = W_t - W_p = (h_1 - h_4) - (h_3 - h_1)
\]

(5)

f. Thermal efficiency

\[
\eta = \frac{W}{Q_e} \times 100 \%
\]

...(6)

All the above processes are ideal processes, which are considered no losses. In the actual condition, these losses always occur and can not be avoided. The losses led to increase the entropy in compression and expansion process. Because of the entropy increasing in compression and expansion process, so the isentropic efficiency of pump and the isentropic efficiency of turbine can be determined by Equation (7) and Equation (8).

\[
\eta_{\text{is,pump}} = \frac{h_{s,1} - h_1}{h_1 - h_4} = \dot{V}_s \frac{(P_4 - P_1)}{h_1 - h_4}
\]

(7)

\[
\eta_{\text{is,tur}} = \frac{h_1 - h_4}{h_1 - h_{s,4}}
\]

(8)

### 2.2 Specific net power output

In order to compare different working fluids this paper focuses on the specific net power output of the thermodynamic cycle. This is the relevant variable for geothermal application and waste heat utilization. If the energy is extracted from a closed loop (e.g. CHP systems) or from a valuable energy source, one has to compare the efficiency of the processes. Furthermore, the net power output considering the electricity demands of the cycle pump and condenser is is taken into account to evaluate the thermodynamic systems. The net power output of the cycle is the product of thermal efficiency and the heat supplied to the organic fluid

\[
P_{net} = \eta \cdot \dot{Q}_{1,net}
\]

(9)
where $P_{net}$, $\eta_{th}$, and $Q_{inj,geoc}$ are net power output, thermal efficiency, and heat input, respectively. As it can be seen by the formula above, the net power output is dependent on two factors that affect each other. The heat input to the cycle is not a fixed value, but, like thermal efficiency, depends on live vapor parameters and cycle design. This is because of the sensitive heat source and the varying exit temperature of the geothermal fluid. In order to objectively compare various cycle designs with different workings fluids, an index number is used. This is the specific net power output the net power output that can be achieved with 1 kg/s geothermal fluid mass flow rate under given conditions

$$P_{net,spec} = \frac{P_{net}}{m_{geo}} [kW / kg]$$

where $P_{net,spec}$, $P_{net}$, and $m_{geo}$ are specific net power output, net power output, and brine mass flow rate.

3. RESULTS AND DISCUSSION

Table 1 shows the default parameters of the simulation of the ORCs using six typical working fluids. Pressure losses in the heat exchangers, pipes, or mechanical losses were not included in the calculations. The brine temperatures and injection temperatures are standard value for state-of-the art of medium-enthalphy geothermal application worldwide.

**Table 1 Power-plant simulation parameters**

<table>
<thead>
<tr>
<th>ORC process parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Condensation temperature</td>
<td>40 °C</td>
</tr>
<tr>
<td>Pump efficiency</td>
<td>0.75</td>
</tr>
<tr>
<td>Turbine efficiency</td>
<td>0.8</td>
</tr>
<tr>
<td>Minimal temperature difference in the evaporator</td>
<td>5 K</td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Geothermal fluid parameters</td>
<td></td>
</tr>
<tr>
<td>Mass flow rate</td>
<td>1 kg/s</td>
</tr>
<tr>
<td>Pressure</td>
<td>1.4 Mpa</td>
</tr>
<tr>
<td>Brine temperature</td>
<td>120 - 180 °C</td>
</tr>
<tr>
<td>Injection temperature</td>
<td>70 - 160 °C</td>
</tr>
</tbody>
</table>

By varying temperature and pressure at turbine inlet (TITP), the maximum possible specific net power output was evaluated. The brine temperature and injection temperature is kept constant with 1 kg/s of brine mass flow rate. Simulations were carried out to calculate 174 working conditions with variation in brine temperature of 120 °C and injection temperature of 70 – 160. The minimum temperature difference between brine temperature and injection temperature was 20 °C, which is economically sound for practical application.

3.1 The empirical correlation of optimum TITP

Temperature and pressure at turbine inlet are the main parameter to determine the optimum conditions of cycle. Based on the optimum conditions of each working fluid that obtained from simulation, then the relationship can be investigated from characteristics of working fluid at turbine inlet: vapor temperature ($T_1$), vapor pressure ($p_1$), critical temperature ($T_{crit}$), critical pressure ($p_{crit}$) and brine temperature ($T_b$). The results of investigation shows that at optimum conditions, a dimensionless parameter $T_1/T_{crit}$ and $T_b/T_{crit}$ for all working fluid is spread on a particular line as shown in Figure 9. With a quadratic equation approach then the relationship of $T_1$, $T_b$ and $T_{crit}$ can be written in the form of the following correlation:

$$f = p_{crit} + p_{\alpha} \left( \frac{T_1}{T_{crit}} \right) + p_{\beta} \left( \frac{T_1}{T_{crit}} \right)^2 + p_{\gamma} \left( \frac{T_1}{T_{crit}} \right)^3 + p_{\delta} \left( \frac{T_b}{T_{crit}} \right) + p_{\epsilon} \left( \frac{T_b}{T_{crit}} \right)^2 + p_{\zeta} \left( \frac{T_b}{T_{crit}} \right)^3 +$$

$$p_{\eta} \left( \frac{T_1}{T_{crit}} \right) \cdot \left( \frac{T_{w,x}}{T_{w,x}} \right) + p_{\theta} \left( \frac{T_1}{T_{crit}} \right)^2 \cdot \left( \frac{T_{w,x}}{T_{w,x}} \right) + p_{\iota} \left( \frac{T_1}{T_{crit}} \right)^3 \cdot \left( \frac{T_{w,x}}{T_{w,x}} \right)$$

where $f = \left( \frac{T_1}{T_{crit}} \right)$ for TIT, $f = \left( \frac{T_{inj}}{T_{crit}} \right)$ for TIP, and the polynomial coefficients are listed in Table 1 as following:

**Table 2 Optimum turbine inlet temperature and pressure (TITP) correlation coefficients**

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>TIT</th>
<th>Coefficient</th>
<th>TIP</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_{\alpha}$</td>
<td>0.9506</td>
<td>$p_{\alpha}$</td>
<td>-8.002·10^{-3}</td>
</tr>
<tr>
<td>$p_{\beta}$</td>
<td>0.6863</td>
<td>$p_{\beta}$</td>
<td>-1.981·10^{-2}</td>
</tr>
<tr>
<td>$p_{\gamma}$</td>
<td>0.1239</td>
<td>$p_{\gamma}$</td>
<td>1.23·10^{-3}</td>
</tr>
<tr>
<td>$p_{\epsilon}$</td>
<td>0.5541</td>
<td>$p_{\epsilon}$</td>
<td>2.113·10^{-3}</td>
</tr>
<tr>
<td>$p_{\zeta}$</td>
<td>1.218·10^{2}</td>
<td>$p_{\zeta}$</td>
<td>-8.662·10^{-3}</td>
</tr>
<tr>
<td>$p_{\eta}$</td>
<td>0.1196</td>
<td>$p_{\eta}$</td>
<td>-1.29·10^{-2}</td>
</tr>
<tr>
<td>$p_{\theta}$</td>
<td>-7.57·10^{-3}</td>
<td>$p_{\theta}$</td>
<td>-2.799·10^{-3}</td>
</tr>
<tr>
<td>$p_{\iota}$</td>
<td>2.67·10^{2}</td>
<td>$p_{\iota}$</td>
<td>1.951·10^{-3}</td>
</tr>
</tbody>
</table>
The empirical correlation above was using normalized value of input variables, which are normalized ratio of brine to critical temperature and normalized injection temperature. Therefore, each variable should be transformed to normalized value using:

\[
\frac{T_g}{T_{crit}} = \left( \frac{T_g}{T_{crit}} - 1.006 \right) / 0.263 \times 10^{-1} \quad \text{and} \quad \frac{T_{inj}}{T_{crit}} = \left( \frac{T_{inj}}{372} - 1 \right) / 22.01
\]  

(32)

The correlation was derived from 174 simulation results (blue dots) with variation in brine temperature of 120 – 180 °C and injection temperature of 70 – 160 °C. It was observed that they have consistent trend in relation with ratio of brine temperature to critical temperature and injection temperature. Hence, they are fitted using third order polynomial function with respect to these two parameters. The fitting result has R-squared of 0.9794 and RMSE (Root Mean Squared Error) of 0.01732 as can be seen in Figure 2.

![Figure 2 Fitting result of optimum turbine inlet temperature (TIT): surface fit (a); contour of the fitting (b)](image)

For the optimum turbine inlet pressure, similar trends are observed. Using third order polynomial, the data are fitted results empirical correlation for TIP with R-squared of 0.9821 and RMSE of 0.07639. The surface fitting results can be seen in Figure 3.

![Figure 3 Fitting result of optimum turbine inlet temperature (TIP): surface fit (a); contour of the fitting (b)](image)

The correlation surface seems has quite good agreement with symmetrical trend. However, validation compared to simulation results were carried out in section 3.2 to prove the empirical results.

### 3.2 Empirical results versus simulation results

As mentioned earlier that there is a certain correlation between the optimum conditions (vapor temperature and pressure at turbine inlet), critical state of the working fluid (critical temperature and critical pressure) and brine temperature. These correlations can be used to predict optimum temperature and pressure at turbine inlet. However, the correlation must be tested to others working fluid to prove whether these correlations generally accepted. Fifteen organic working fluids with critical temperature near brine temperature were selected. The results of verification for all the selected working fluid can be seen in Table 3. The comparison of
optimum temperature and pressure at turbine inlet were obtained from correlation and simulation at random \( T_g = 120 \rightarrow 180 \) °C and \( T_{inj} = 70 - 160 \) show that all optimum point of cycle (optimum temperature and pressure at turbine inlet) have quite good agreement. There are some inaccuracies in TIP prediction which results a maximum relative error of 25.64%. Nonetheless, the specific net power-output predictions are still in accuracy within 2%.

**Table 3** Comparison between correlation and simulation results

<table>
<thead>
<tr>
<th>Working fluid</th>
<th>( T_g ) [°C]</th>
<th>( T_{inj} ) [°C]</th>
<th>TIT [K]</th>
<th>TIP [MPa]</th>
<th>Spec. net-power [kW]</th>
</tr>
</thead>
<tbody>
<tr>
<td>propane</td>
<td>175</td>
<td>99</td>
<td>445</td>
<td>443</td>
<td>0.40%</td>
</tr>
<tr>
<td>R124</td>
<td>180</td>
<td>158</td>
<td>453</td>
<td>448</td>
<td>1.14%</td>
</tr>
<tr>
<td>isobutene</td>
<td>152</td>
<td>115</td>
<td>410</td>
<td>420</td>
<td>-2.44%</td>
</tr>
<tr>
<td>butene</td>
<td>134</td>
<td>86</td>
<td>371</td>
<td>373</td>
<td>-0.52%</td>
</tr>
<tr>
<td>trans-butene</td>
<td>141</td>
<td>92</td>
<td>381</td>
<td>381</td>
<td>0.07%</td>
</tr>
<tr>
<td>R114</td>
<td>151</td>
<td>102</td>
<td>403</td>
<td>401</td>
<td>0.68%</td>
</tr>
<tr>
<td>R236ea</td>
<td>139</td>
<td>106</td>
<td>393</td>
<td>394</td>
<td>-0.04%</td>
</tr>
<tr>
<td>cis-butene</td>
<td>161</td>
<td>121</td>
<td>418</td>
<td>427</td>
<td>-2.11%</td>
</tr>
<tr>
<td>R21</td>
<td>180</td>
<td>132</td>
<td>440</td>
<td>448</td>
<td>-1.77%</td>
</tr>
<tr>
<td>neopentane</td>
<td>182</td>
<td>158</td>
<td>452</td>
<td>450</td>
<td>0.43%</td>
</tr>
<tr>
<td>R125</td>
<td>122</td>
<td>74</td>
<td>390</td>
<td>390</td>
<td>-0.10%</td>
</tr>
<tr>
<td>R11</td>
<td>180</td>
<td>136</td>
<td>437</td>
<td>445</td>
<td>-1.94%</td>
</tr>
<tr>
<td>R245ca</td>
<td>165</td>
<td>140</td>
<td>424</td>
<td>425</td>
<td>-1.0%</td>
</tr>
<tr>
<td>R123</td>
<td>158</td>
<td>121</td>
<td>410</td>
<td>409</td>
<td>0.31%</td>
</tr>
<tr>
<td>R141b</td>
<td>180</td>
<td>155</td>
<td>439</td>
<td>445</td>
<td>-1.18%</td>
</tr>
</tbody>
</table>

When the optimum point of each working fluid is plotted with surface of the empirical correlation then it would seem that all the optimum point scattered around correlation line, especially the area around \( T_g/T_{crit} = 1 \) as shown in Figure 4.

![Figure 4 Residuals of 174 data and validation data from different 15 working fluids: TIT (a); TIP (b)](image)

Figure 12 showed the net power output as a function \( T_g/T_{crit} \) ratio, it is clearly seen that a higher net power output is obtained from the working fluid with critical temperature close to brine temperature \( T_g/T_{crit}=1 \).

Based on the results of verification can be concluded that the temperature correlation in Equation (11) - (12) can be used to predict the optimum temperature and pressure at turbine inlet. But, for more accurate results need to be checked a few points around prediction point with simulation programs to ensure that the prediction point was right.

### 3.3 Choice of working fluids: Practical application

Hybrid power-plant application in Indonesia example was for practical application. The working conditions are taken from the literature for Lahendong, North Sulawesi field (Bambang, 2006). The brine temperature out from the flash power-plant is 453.15 K (180 °C). The minimum injection temperature is 160 °C to avoid scaling. The mass flow rate of brine is measured as 11.44 kg/s. Five working fluid candidates are taken such as isobutane, butane, pentane, isopentane, and propane. The results are shown in Table 4 below.

As can be inferred from Table 3, pentane results the best net power output with subcritical-process in the cycle. It can be observed that for hybrid power-plant application, supercritical cycle does not convey real benefit since the injection temperature is limited. The plant with higher efficiency will have better net power output.
Table 4 Results of empirical correlation: Lahendong case study

<table>
<thead>
<tr>
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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>pentane</td>
<td>441</td>
<td>2.07</td>
<td>157.7</td>
<td>15.8%</td>
<td>subcritical</td>
</tr>
<tr>
<td>2</td>
<td>isopentane</td>
<td>443</td>
<td>2.43</td>
<td>154.2</td>
<td>15.4%</td>
<td>subcritical</td>
</tr>
<tr>
<td>3</td>
<td>butane</td>
<td>452</td>
<td>4.69</td>
<td>149.6</td>
<td>15.0%</td>
<td>supercritical</td>
</tr>
<tr>
<td>4</td>
<td>isobutane</td>
<td>454</td>
<td>5.58</td>
<td>141.1</td>
<td>14.1%</td>
<td>supercritical</td>
</tr>
<tr>
<td>5</td>
<td>propane</td>
<td>439</td>
<td>9.33</td>
<td>121.2</td>
<td>12.1%</td>
<td>supercritical</td>
</tr>
</tbody>
</table>

4. CONCLUSIONS
Based on the simulation results and discussion, it can be concluded some of the following:

1. Simulation for six typical organic working fluids with 1 kg/s mass flow rate at brine temperatures of 120 – 180 °C and injection temperature of 70 – 160 °C has been carried out to derive empirical correlation for optimum turbine inlet temperature and pressure (TIP).
2. It has been obtained correlation to predict the optimum temperature (T1) and pressure (p1) at turbine inlet. The correlation has been tested on 15 organic working fluids. It was obtained that all the optimum point spread around correlation line with relative error for optimum TIT, TIP, and specific net power output within 2.5%, 26%, and 2%.
3. For brine temperature 453.15 K (180 °C), the optimum condition of cycle with pentane as working fluid is obtained at turbine inlet temperature, T1 = 434.15 K (161 °C) and pressure P1 = 1.9 MPa, with thermal efficiency (\(\eta_{th}\)) of 15.8%.
4. The empirical correlations obtained can be used for rough estimation of TITP and specific net power output. Within typical value ranges, it valid regardless of pinch-point, condensation temperature, and regeneration option. This correlation is very useful in pre-design stage to predict the optimum temperature and pressure at turbine inlet for organic Rankine cycle.

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