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On the Inverse Problems of Geothermic

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ABSTRACT

The formulation and solution of the problem of calculating the value of heat flow on the base temperature observations in wells - the inverse problem of geothermic under sedimentation conditions are given. This problem reduces to a quadratic programming problem with linear constraints on the unknowns. The implementation of the algorithm for solving the inverse problem of geothermic under sedimentation conditions was carried out on the basis of the FEFLOW computer program. The application of the program is demonstrated using model examples and solving the problem of predicting the water content of the Khoja-Obigarm hydro-geothermal system (Tajikistan).

1. INTRODUCTION

Geothermal problems can be described by mathematical models, i.e., some set of partial differential equations together with initial and/or boundary conditions defined in a particular domain. Models in computational geothermal quantitatively predict what will happen if the crust and mantle deform slowly over geologic time, often with complications in the form of simultaneous heat transfer (e.g., thermal convection in the mantle), phase changes in the Earth's deep interior, complex rheology (viscosity, plasticity, non-Newtonian fluids), melting and migration of melts, chemical reactions (e.g., thermochemical convection), motion of solid, lateral forces, etc.

A mathematical model relates the causal characteristics of a geothermal process to its consequences. The causal characteristics of the simulated process include, for example, the parameters of the initial and boundary conditions, the coefficients, and the right side of the differential equations, as well as geometric parameters, and areas of determination. The purpose of the direct problem is to determine the relationship between the causes and effects of the geothermal process, and therefore to formulate a mathematical problem for a given set of parameters and coefficients. The inverse problem of geothermal is the opposite of the direct problem. The inverse problem is posed when there is no information about the causal characteristics, but there is information about the effects of the geophysical (more specifically, geothermal) process. Inverse problems can be classified as follows: inverse time problems (e.g., to reconstruct the development of a geodynamic process); coefficient problems (e.g., determination of coefficients, right sides of model equations), geometric problems (e.g., determination of the location of heat sources in a region or geometry of boundaries), and many others.

Inverse problems often turn out to be poorly posed or incorrect in J. Hadamard's terminology [Hadamard J., 1902]. A mathematical model for a geophysical problem should be well-established in the sense that it should have the properties of (1) existence, (2) uniqueness, and (3) stability of the solution of the problem. Tasks for which at least one of these properties is not performed are called poorly defined. If, for example, a problem does not have property (3), then its solution is almost impossible to compute because the calculations are polluted by inevitable errors. If the solution of a problem is not continuously dependent on the initial data, then, generally speaking, the computed solution may have nothing to do with the true solution. In the works of A.N. Tikhonov and his followers, methods for solving incorrect problems are proposed. The essence of A.N. Tikhonov's method is the construction of regularizing families of problems, the solution of which in the limit gives the solution of the initial incorrect problem [Tikhonov A.N., Arsenin V.Ya., 1986]. The application of A.N. Tikhonov's method to a wide class of geodynamic problems is described in [Ismail-Zade A. at all., 2016].

2. STUDY AREA

The Khoja-Obigarm thermal deposit of slightly mineralized nitric thermal waters is located 48 km north of the capital of Tajikistan, Dushanbe (Figure 1-3).

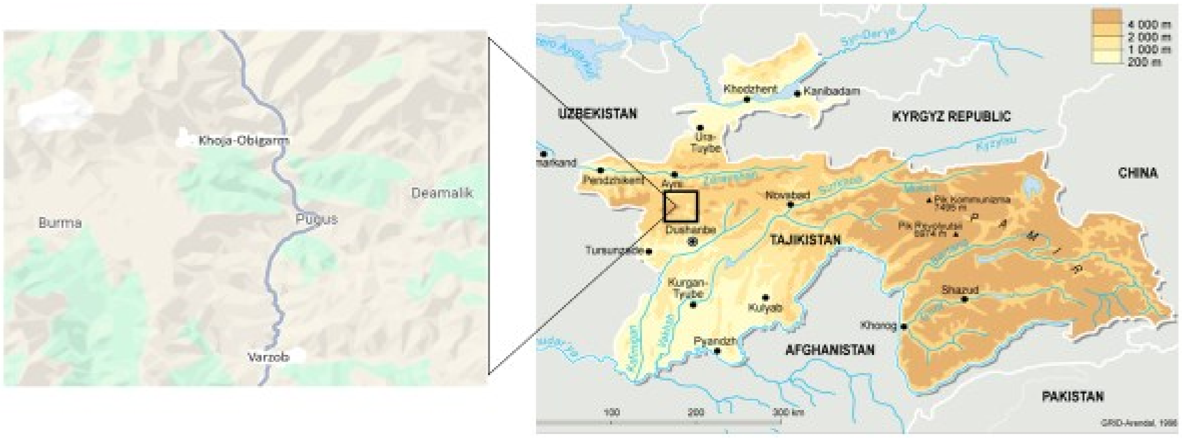

Figure 1: Location of Khoja-Obigarm deposit with model boundary.

Figure 2: ***a*****) Balneological resort Khoja-Obigarm; b) Surrounding mountains**

Figure 3: ***Manifestations of water and steam***

The deposit and the accompanying balneological resort are localized in the central part of the southern slope of the Hissar Range, in the gorge of the Khoja-Obigarm River, at an altitude of 1740-1960 m. The deposit is confined to a tectonic zone, along which there is an upward movement of thermal waters. In geological and structural terms, the field is located on the northern wing of the Khoja-Obigarm graben syncline. The geological structure of the area mainly involves intrusive rocks of the southern part of the North Varzob intrusive massif, represented by granites, granodiorites, and granite porphyries of the middle and early Quaternary age. The intrusions are overlain by a cover of Quaternary sediments (Figure 4,5). The source Khoja-Obigarm (well 14-bis) is located at an altitude of 1835 m above sea level, the water temperature reaches 98 C, and mineralization 0.4/l. The pH value is 8.5, the flow rate is 1.5 l/s. The main gas is N2 (98%). According to N.M. Churshina [Churshina N.M., Krat V.N., 1973] the reserve is 2160 m3/day. Hydrogeological conditions of the deposit are complex and are determined by the geological and structural features of the site. According to the conditions of occurrence and circulation, these are fracture-vein thermal waters, which are confined to a thick zone of granite crushing associated with the Khoja-Obigarm fault in the place of its junction with the north-north-west strike [Barabanov L.N., Disler V.N., 1963].

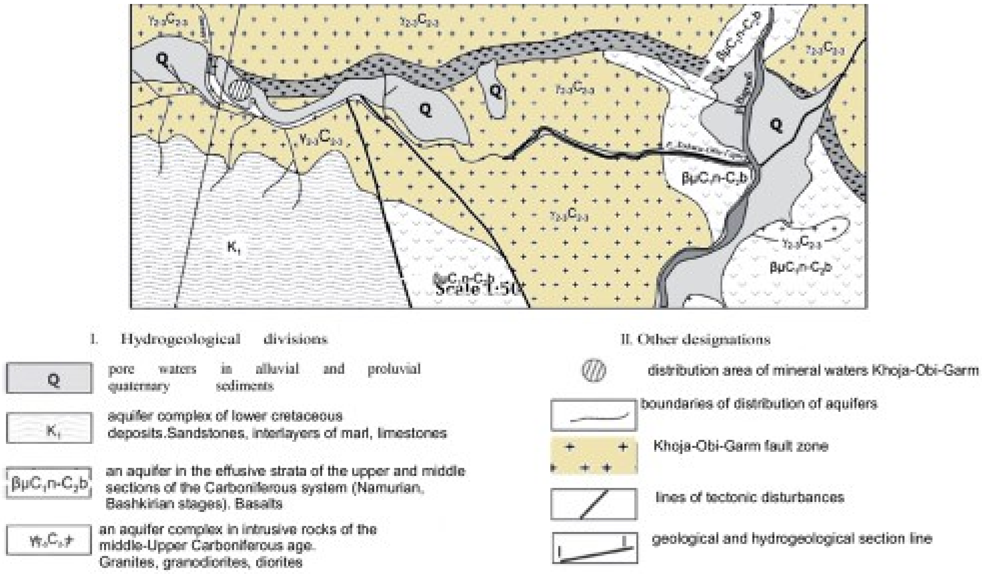

Figure 4: Geological and hydrogeological map of the Khoja-Obigarm deposit.

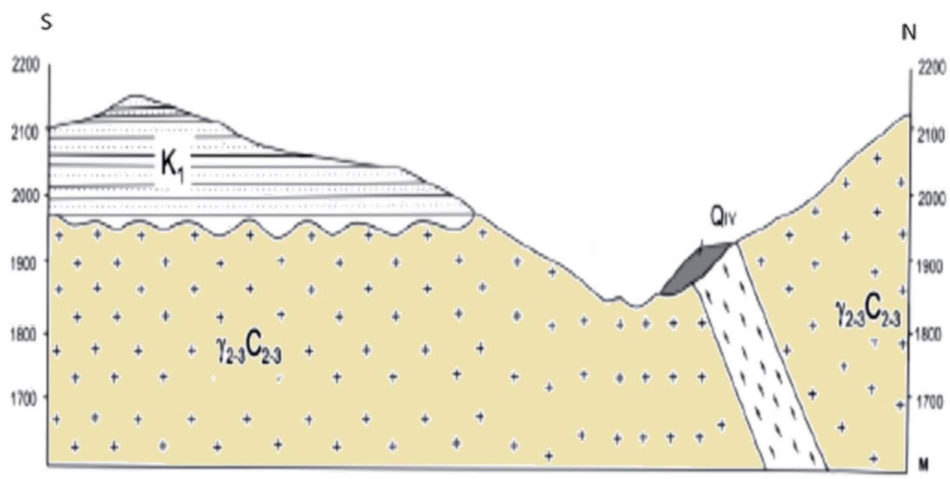

Figure 5: Hydrogeological section diagram of the Khoja-Obigarm deposit.

3. FORMULATION OF THE PROBLEM OF PALEOCLIMATE RECONSTRUCTION

In this and the next two paragraphs of the paper, the inverse problem of geothermal is formulated and various properties of its solutions are analyzed. More specifically, we are talking about the task of reconstructing the paleoclimate on the example of the Khoja-Obigarm nitrogen thermae. Information about the change in the temperature of the earth's surface in the geological past is carried by the temperature of the earth's interior, which is determined by the deep flow of heat and the flow of solar heat. If you change the temperature change from the depth in the borehole (vitrinite paleothermometry) and remove the influence of the deep heat flow, you get an anomalous temperature determined only by the temperature of the earth's surface. Thus, we come to the inverse problem of determining past changes in the temperature of the earth's surface from borehole measurements of the temperature distribution with depth. A great contribution to the theory and practice of inverse geothermal problems was made by the school of Academician A.N. Tikhonov. He and his followers showed that with the layered structure of the medium and the known thermal parameters of the medium, it is possible to unambiguously determine the change in the temperature of the earth's surface in the past from modern measurements of the temperature distribution as a function of depth in the wells [Strakhov V.N., 1970], [Starostenko V.I., 1978], [Ilolov M., Rakhmatov J.S., 2024], [Dmitriev V.N., 2012].

Two fundamental problems arise in the interpretation of geothermal data. These are (1) the need to calculate the anomalous temperature in the borehole with reasonably good accuracy, and (2) to use of a priori information about the behavior of the solution to ensure the stability of the solution.

Let the coefficients of thermal diffusivity $a^2(z)$, thermal conductivity $\lambda(z)$ and Heat Sources $f(z)$ change only with depth $0 < z < \infty$. Then the temperature distribution $T(z, t)$ is the solution of the following initial-boundary value problem

$$\begin{cases} \frac{\lambda(z)}{a^2(z)} \cdot \frac{\partial T}{\partial t} = \frac{\partial}{\partial z}\left(\lambda(z)\frac{\partial T}{\partial z}\right) + f(z), -t_0 \le t \le 0, 0 \le z \le H, \\ T(z, -t_0) = T_{st}(z), 0 \le z \le H, \\ T(0,t) = T_s(t), T_s(t_0) = T_{st}(0), \\ \lambda \frac{\partial T}{\partial z}\Big|_{z=H} = q, [T]|_{z_n} = 0, \left[\lambda \frac{\partial T}{\partial z}\right]\Big|_{z_n} = 0, \end{cases} \quad (1)$$

where is q-heat flux at depth H, $T_{st}(z)$-steady-state initial temperature distribution at $t = -t_0$. The last two conditions mean continuity at the points $z = z_n$, Where the odds are gaping $a^2(z)$ or $\lambda(z)$.

The inverse problem of paleoclimate reconstruction is to determine the change in the temperature of the earth's surface $T(0,t) = T_s(t)$ at the past $-t_0 \le t \le 0$ according to the known modern distribution of temperature with depth, obtained from measurements in the well. Values $a(z), \lambda(z), q$ and $T_{st}(z)$ in problem (1) are considered known, and the solution to the inverse problem (1) is determined from an additional condition

$$T(z, 0) = T^e(z). \quad (2)$$

The existence and uniqueness of the inverse problem (1) - (2) are established in [Ilolov M., Rakhmatov J.S., 2024], [Dmitriev V.N., 2012]. In recent decades, several publications have appeared on the inverse problems of paleoclimate reconstruction for specific data in different regions of the world [Isaev V.I. at all., 1995]. In this regard, there is a need for research on the stability of the obtained solutions and the development of new, more effective methods for solving the inverse problem of paleoclimate reconstruction. In this paper, we consider a method of solving problems based on the reduction of problems to an integral equation of genus and the solution of this equation by the method of regularization [Glasko V.B., 1970] to the first difficulty in solving the problem of paleoclimate reconstruction is the determination of the initial temperature distribution $T_{st}(z)$. We will assume that the distribution of the thermal conductivity coefficient $\lambda(z)$, heat sources $f(z)$ and deep heat flow q highest changed during the time we studied $-t_0 < t < 0$. In addition, suppose that there was a sufficiently large period $-t_1 < t < -t_0$, precede the one under study, during which the average long-term temperature of the earth's surface was constant. Under these assumptions, we can assert that the initial temperature is the solution of the following stationary thermal problem:

$$\begin{cases} \frac{d}{dt}\left(\lambda(z)\frac{dT_{st}(z)}{dz}\right) = -f(z), 0 < z < H, \\ T_{st}(H) = T_H, \frac{dT}{dz}(H) = p = \frac{q}{\lambda(H)}, \\ [T_{st}]|_{z_n} = 0, \left[\lambda \frac{dT_{st}}{dz}\right]\Big|_{z_n} = 0. \end{cases} \quad (3)$$

The solution to problem (3) is of the form

$$T_{st}(z) = T_H - \int_z^H \left(\lambda(H)p + \int_\xi^H f(\xi)d\xi\right)\frac{d\xi}{\lambda(\xi)}. \quad (4)$$

In (4) along with the well-known $\lambda(z)$ и $f(z)$, enter the unknown T_H and p. It follows from (4) that when $z \in [H_z - \Delta H, H]$ function $T_{st}(z)$ behaves in an almost linear manner, it behaves in a linear manner i.e., $T_{st}(z) \approx T_H - p(z - H)$. Therefore, the unknown parameters T_H and p are derived from the asymptote of the experimental observed temperature distribution in the region $H - \Delta H < z < H$:

$$\int_{H-\Delta H}^H (T^e(t) - T_H - p(z-H))^2 dz = \min_{P,T_H}. \quad (5)$$

This means that the experimental curve must have a linear section at the end of the curve $H - \Delta H < z < H$. This section should be long enough to determine with good accuracy T_H and p. Defining $T_{st}(z)$, we can introduce an abnormal temperature

$$u(z,t) = T(z,t) - T_{st}(z) \quad (6)$$

and calculate the abnormal temperature measured in the borehole,

$$u^e(z) = T^e(z) - T_{st}(z). \quad (7)$$

In this case, the temperature of the earth's surface is equal to

$$T_s(t) = T_0 + u(0,t), \quad (8)$$

where

$$T_0 = T_{st}(0) = T_H - \int_0^H \left(\lambda(H)p + \int_\xi^H f(\xi)d\xi \right) \frac{d\xi}{\lambda(\xi)}. \quad (9)$$

Now the problem of paleoclimate reconstruction can be set for anomalous temperature

$$\begin{cases} \frac{\lambda(z)}{a^2(z)} \frac{\partial u}{\partial t} = \frac{\partial}{\partial z}\left(\lambda(z) \frac{\partial u}{\partial z} \right), -t_0 \le t \le 0, 0 \le z \le H \\ u(z, -t_0) = 0, u(0,t) = u_s(t), u_s(-t_0) = 0 \\ \left. \frac{\partial u}{\partial z} \right|_{z=H} = 0, [u] \mid_{z_n} = 0, \left[\lambda \frac{\partial u}{\partial z} \right] \Big|_{z_n} = 0. \end{cases} \quad (10)$$

Abnormal temperature of the earth's surface $u_3(t) = u(0,t)$ is determined from an additional initial condition

$$u(z,0) = u^e(z). \quad (11)$$

In case $f(z) = 0$ (when radioactive heat sources at depths of 2 to 3 km are usually small) according to (4) the initial temperature distribution is

$$T_{st}(z) = T_H - p \int_z^H \frac{d\xi}{\mu(\xi)}, z \in [0, H], \quad (12)$$

where $\mu(z) = \frac{\lambda(z)}{\lambda(H)}$-The relative change in the coefficient of thermal conductivity. Typically, the thermal conductivity coefficient is measured in the borehole. However, $\lambda(z)$ changes along the borehole quite rapidly, and it follows from formula (12) that $T_{st}(z)$ The integral value of the inverse temperature coefficient is affected. Therefore, an accidental change in $\lambda(z)$ at the point z_0 may differ significantly from the average value $1/\lambda(z)$. Figure 6 a) shows two different distributions $\lambda(z)$, having similar average values $1/\lambda(z)$, a Figure 6 b) shows their respective distributions

$$\Lambda(z) = \frac{T_H - T_{st}(z)}{PH} = \frac{1}{H} \int_S^H \frac{d\xi}{\mu(\xi)}, z \in [0, H].$$

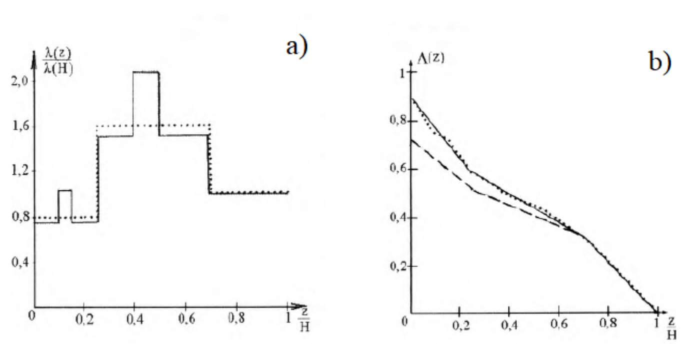

Figure 6: a) Initial Temperature Distribution $\lambda(z)$; b) Начальное распределение температуры $\Lambda(z)$

It is easy to see that when measuring at a point $\frac{z}{H} \in [0{,}1; 0{,}15]$ and $\frac{z}{H} \in [0{,}4; 0{,}5]$ we'll get $\lambda_1 = 1{,}11 \cdot \lambda(H)$ and $\lambda_2 = 2{,}08 \cdot \lambda(H)$. If we assume that these values belong to the first layer ($0 \le z \le 0{,}25$) and the second layer ($0{,}25 \le z \le 0{,}7$), respectively, we get an incorrect initial temperature distribution (shown by a dotted line in Figure 6 b). It's easy to see that in this case $T_{st}(z)$ differs significantly from the true one. The resulting error falls into an abnormal temperature and will significantly spoil the resulting solution to the inverse problem. What is the way out of this situation? First, we can take measurements $\lambda(z)$ at many points and then compute the $\Lambda(z)$ and average it by zooming in $\Lambda(z)$ piecewise linear function.

4. REDUCTION OF THE INVERSE PROBLEM TO THE INTEGRAL EQUATION

If the Green's function is known $G(z,t)$, which is the solution to the problem

$$\begin{cases} \dfrac{\lambda(z)}{a^2(z)}\dfrac{\partial G}{\partial t} = \dfrac{\partial}{\partial z}\left(\lambda(z)\dfrac{\partial G}{\partial z}\right), t \in (0,\infty), z \in (0,H), \\ G(z,0) = 0, G(0,t) = 1, \quad \left.\dfrac{\partial G}{\partial z}\right|_{z=H} = 0, \\ [G]|_{z_n} = 0, \left[\lambda\dfrac{dG}{dz}\right]\Big|_{z_n} = 0, \end{cases} \tag{13}$$

then, in accordance with Duhamel's principle, the solution of the problem for the anomalous temperature (10) is represented as

$$u(z,t) = \int_{-t_0}^{t} u_s(t')\frac{\partial G(z,t-t')}{\partial t}dt', t \in [-t_0, 0]. \tag{14}$$

Substituting (14) for condition (11), we obtain an integral equation of the first kind for the definition of $u_s(t)$ according to the well-known $u^e(z)$:

$$u^e(z) = \int_{-t_0}^{G} u_s(t')M(z,t')dt', \tag{15}$$

where the kernel of the equation is

$$M(z,t') = \left.\frac{\partial G(z,t-t')}{\partial t}\right|_{t=0} = \left.\frac{\partial G(z,t)}{\partial t}\right|_{t=-t'}. \tag{16}$$

Let's replace the variable $t' = z - t_0$ and introduce the $v(\tau) = u(t - t_0)$. Then the integral equation will be of the form

$$u^e(z) = \int_0^{t_0} v(\tau)K(z,\tau)d\tau, \tag{17}$$

where

$$K(z,\tau) = -\frac{\partial G(z,t_0-\tau)}{\partial \tau}. \tag{18}$$

Having solved equation (17), we can easily find the temperature of the earth's surface

$$T_s(t) = T_0 + v(t+t_0), t \in (-t_0, 0). \tag{19}$$

Equation (15) is of the first kind, which leads to the instability of the solution. Therefore, to solve this equation, it is necessary to apply the Tikhonov regularization method [Zhdanov M.S., 2007]. An important point of the computational algorithm is the calculation of the kernel of the integral equation. To do this, according to (18), it is necessary to develop an algorithm for calculating the Green's function that satisfies the problem (13).

Such a method for a wide range of geophysical problems was developed in [Demonova A.Y., 2017], and this method allows for a quick calculation of the Green's function.

5. SOLVING THE INVERSE PROBLEM

As was shown in Sections 3 and 4, the inverse problem of paleoclimate reconstruction is reduced to the solution of the integral equation of the first kind (17), which we will now write in the form

$$A[v] = \int_0^{t_0} K(z,\tau)v(\tau)d\tau = u^e(t), z \in [0,H]. \tag{20}$$

The solution of equation (20) is carried out by the method of regularization

$$\inf_v\{\|A[v] - u'(z)\|^2 + \alpha\Omega(v), \tag{21}$$

where the stabilizer is defined in terms of the order of smoothness of the function $v(t)$ In the form of

$$\Omega(\mathrm{v}) = \int_{0}^{t_0} p(\tau)(v'(\tau))^2 d\tau\ . \tag{22}$$

The (22) includes the weight function $p(\tau) > 0$, which determines the level of constraint of the derivative $v(\tau)$ at different time intervals. The fact is that the solution of the inverse problem gives a smoothed temperature of the earth's surface. The nearest temperature averages oover5-10 years, and when moving into the past, the interval increases significantly. It is this property that takes into account the coefficient $p(\tau)$. Function $p(\tau)$ should be monotonically decreasing when $\tau \in [0, t_0]$, which reduces the requirement for smoothness in the surroundings t_0.

Taking the variation of the functional (21) and equating it to zero, we get the Euler equation for the variational problem (21), which for the stabilizer (22) is an integro-differential equation of the form

$$\alpha \frac{d}{dt}\left(p(t)\frac{dv(t)}{dt}\right) + \int_{0}^{t_0} M(t,\tau)v(\tau)d\tau = u(t), t \in [0, t_0], \tag{23}$$

where

$$M(t,\tau) = \int_{0}^{H} K(z,t)K(z,\tau)d\tau, \tag{24}$$

$$u(t) = \int_{0}^{H} K(z,t)u^e(z)dz. \tag{25}$$

Equation (23) is easily solved numerically. Regularization parameter α characterizes the requirement for smoothness of function $v(t)$. In case of large α We get a badly distorted solution. Diminutive increases the detail of the solution, but also increases the instability. The weight function is usually defined in an analytical form:

$$p(t) = \frac{t_0^2}{t_0^2 + \beta t^2}, \beta \geq 0, 0 \leq t \leq t_0 \tag{26}$$

or

$$p(t) = 1 - \gamma\left(\frac{t}{t_0}\right)^2, 0 < \gamma < 1, 0 < t \leq t_0. \tag{27}$$

Factors β allow to control of the weight function. We will consider the solution of the inverse problem for the case of a homogeneous half-space. On the one hand, this makes it easier to describe the solution algorithm, but on the other hand, it is the most common case, since the anomalous field is usually different in the first homogeneous layer. Below the value of the abnormal temperature is comparable to a measurement error. For half-space, the spectrum of the Green function is

$$g^0(z,\omega) = \frac{i}{\omega} exp(iqz), q = \frac{\sqrt{i\omega}}{a}, reg > 0, a > 0. \tag{28}$$

Then Green's function is of the form

$$G(z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} g^0(z,\omega)exp(i\omega t)d\omega = 1 - \frac{2}{\sqrt{\pi}}\int_{0}^{\alpha} exp(-x^2)dx,$$

где $\alpha = \frac{z}{2a\sqrt{t}}, t \geq 0, z \geq 0$.

Knowing Green's function, we can easily compute the kernel of the integral equation (20), which according to (28), is

$$K(z,t) = \frac{\partial G(z, t_0 - \tau)}{\partial t} = \frac{z\, exp(-z^2/4a^2(t_0 - \tau))}{2\sqrt{\pi}a(t_0 - \tau)}. \tag{29}$$

We can now compute the kernel of the integro-differential equation (21) as a convolution at $H = \infty$:

$$M(t,\tau) = \frac{1}{4\pi a^2 \gamma^2} \int_0^\infty exp\left(-z^2/4a^2\left(\frac{1}{t_0-\tau}+\frac{1}{t_0-t}\right)\right) z^2 dz, \tag{30}$$

where

$$\gamma = \sqrt{(t_0-\tau)(t_0-t)}.$$

Taking into account equality

$$\int_0^\infty exp(-\alpha z^2)\, z^2 dz = \frac{\sqrt{\pi}}{4(\sqrt{\alpha})^3},$$

we'll finally get

$$M(t,\tau) = \frac{a}{2\sqrt{\pi}} \cdot \frac{1}{\left(\sqrt{2t_0-t-\tau}\right)^3}, 0 \le t \le t_0, 0 \le \tau \le t_0. \tag{31}$$

The right-hand side of equation (31), according to (28), is

$$u(t) = \int_0^\infty K(z,t) u^e(z) dz =$$

$$= \frac{1}{2\sqrt{\pi} a (t_0-t)^3} \int_0^\infty u^e(z) exp\left(-\frac{z^2}{4a^2(t_0-t)}\right) z dz. \tag{32}$$

Substituting (31), (32) into the integro-differential equation (23) we get

$$\alpha(t) \cdot \frac{d}{dt}\left(p(t)\frac{dv}{dt}\right) + a^2 \int_0^{t_0} L(u,\tau) v(t) d\tau = \frac{F(t)}{t_0-t}, t \in [0, t_0], \tag{33}$$

where

$$\alpha(t) = 2\pi\alpha \cdot a\sqrt{t_0-t}, L(t,\tau) = \frac{\sqrt{t_0-t}}{\left(\sqrt{2t_0-t-t}\right)^3};$$

$$F(t) = \int_0^\infty exp\left(-\frac{z^2}{4a^2(t_0-t)}\right) u^e(z) z dz \approx$$

$$\approx \int_0^{H_0} exp\left(-\frac{z^2}{4a^2(t_0-t)}\right) u^\varepsilon(z) dz. \tag{34}$$

Note that in (34) the integration proceeds along the interval $z \in [0, H_0]$, because when $z > H_0$ abnormal temperature is zero. Magnitude H_0 It is obtained by determining the abnormal temperature in the well. At the same time, the value of the H_0 can be significantly less than the depth H, up to which the borehole measurement was carried out. This can be clearly seen from Figure 7, where $H = 800$ m and., a $H_0 = 250$ m. Time quantity t_0, determining the time up to which we can determine the temperature of the earth's surface in the past, the dependence on the t_0.

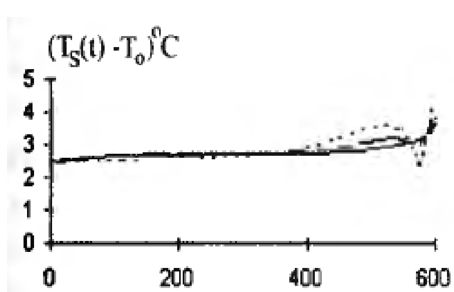

Figure 7: Paleotemperature of the Earth's Surface.

Magnitude t_0 is defined from the condition

$$\frac{H_0^2}{4a^2t_0} = 4 \;\; or \;\; t_0 = \frac{H_0^2}{16a^2}. \tag{35}$$

This means that the temperature change in the past at $t < -t_0$ does not affect the temperature changes in the well when $z > H_0$. The thermal conductivity coefficient for terrestrial rocks varies within

$$10\frac{m^2}{year} \le a^2 \le 15\frac{m^2}{year}.$$

Substituting these values in (35), we define the time t_0, up to which it is possible to find a measurement of the temperature of the earth's surface, depending on the depth H_0, up to which the abnormal temperature in the well is other than zero. The results are shown in Table 1 for 16 a^2=200.

Table 1: Abnormal temperature in the borehole.

H_0(m)	200	300	400	500	600	700	800	900
t_0(years)	200	450	800	1200	1800	2450	3200	4050

Practical measurements show that the maximum value of H_0 usually does not exceed 600 m, which gives the maximum value t_0 about 2000 years.

It should be noted that the works [Ilolov M. at all, 2021], and [Ilolov M. at all, 2023] present other aspects of topical problems of geothermal energy in the example of Central Asia (Tajikistan).

Acknowledgments

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