

# An Analytical Study of The Thermal-Hydraulic-Mechanical Processes During Cold Water Injection into EGS Reservoirs

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## ABSTRACT

During the recovery of enhanced geothermal system (EGS) reservoirs, water is cycled between engineered hydraulic well doublets. When cooled water is re-injected into the fracture system underground, the temperature in the matrix decreases, resulting in the shrinkage of matrix rock and enhancement of the permeability of fractures. Such a phenomenon is known as the thermal unloading process, the accurate modeling of which requires coupled thermal-hydraulic-mechanical simulation approaches. In this work, we brought out a novel derivation of the thermal-hydraulic-mechanical model of the matrix-fracture interaction, in order to capture the transient behavior of the fracture system in the thermal unloading process. We analytically solve the temperature field inside a matrix rock that is surrounded by fractures, using Fourier series and separation of variables. The temperature field is used to calculate the stress as well as the displacement field in the matrix rock during the thermal unloading process. Based on the analytical solution of the displacement field, we are able to obtain the deformation of the matrix rock, and therefore obtain the change in the aperture/permeability of the fracture system. Our analytical model has been successfully implemented in an in-house simulator. The proposed model can also be used to fast estimate the permeability variation during real-time injection monitoring in real practices.

## 1. INTRODUCTION

The development of enhanced geothermal system (EGS) reservoirs is through creating engineered fractures in the hot dry rocks (HDR). During the development of EGS, hot steam is produced and is used to generate electric. After the generation of electric, the cooled steam is re-injected into the hot formation. The injected cold liquid induces complex thermal-hydraulic-mechanical (THM) changes to the reservoir. Particularly, for the matrix-rock system, the reduction of *in situ* temperature causes the matrix rock to shrink, increasing the permeability of the fracture system, known as the thermal unloading process. In this work, we aim to analytically calculate the thermal unloading effect induced by cold water injection during the development of (EGS) reservoirs.

There are several approaches to model the fracture-matrix system in EGS reservoirs, namely dual porosity approach (Warren and Root, 1963; Lim and Aziz, 1995), dual permeability approach (Larsbo *et al.*, 2005), embedded discrete fracture network approach (Lee, Lough and Jensen, 2001).

In practice, the mechanical behavior of the dual-porosity systems can be modeled either in a continuum manner by using a ‘two-part’ Hooke’s law (Li *et al.*, 2014; Liu *et al.*, 2009) or by treating the matrix and the fracture separately (Wang, 2015; Wang *et al.*, 2014).

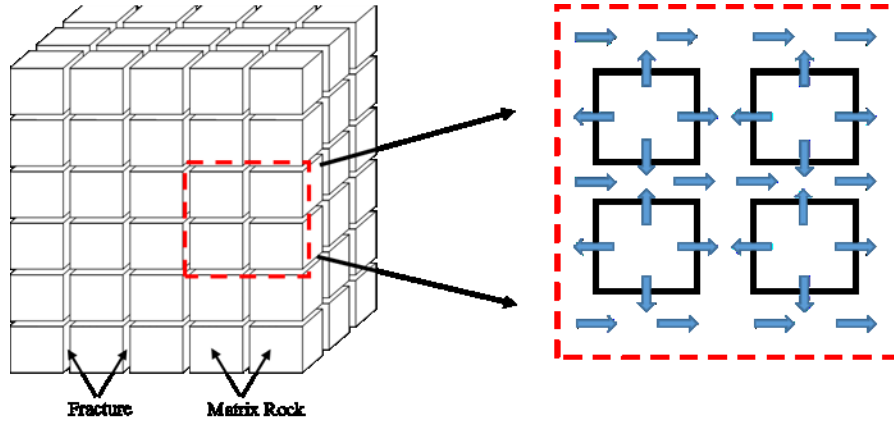
In our previous works, we have derived a semi-analytical correlation for modeling the thermal unloading process in the dual-porosity system of EGS reservoirs. The proposed semi-analytical correlation is obtained by solving the 1-D Navier’s equation for the thermo-elastic materials (Hetnarski and Ignaczak, 2004).

In the current work addressed in this paper, we move forward to extend the above model to three dimensions. The 3D solution is based on the extended Beltrami-Michell equation (Jaeger, Cook and Zimmerman, 2009). In our preceding efforts, we have successfully numerically solved the extended Beltrami-Michell equation fully coupled with the mass/energy conservation equations for poro-elastic problems (Wang *et al.*, 2017; Wang *et al.*, 2017 (1); Winterfeld & Wu, 2016). Such program framework enables all the governing equations to be solved on the same grid. Although great progress has been achieved, the thermal-hydraulic-mechanical behaviors of the fractured rock systems are still not fully understood.

As mentioned above, the cooling (also known as thermal unloading in the geoscience community) of the matrix rock induces the variation of temperature, which results in the changes of stress field. In return, the rock matrix shrinks, causing the aperture of the surrounded fractures to increase. In this paper, we present a novel analytical solution to the temperature and displacement (and stress) field of a rectangular thermoelastic material that is cooling down.

The analytical solution is of great importance to the thermal engineering community, since it enables engineers to fast estimate the stress condition of the material without numerically solving a coupled multiphysical problem, which could be rather computationally expensive. Moreover, the proposed analytical solution can be combined with existing THM simulators (Kim, Sonnenthal and Rutqvist, 2012; Rutqvist *et al.*, 2013) to improve the accuracy of the simulation.

The conceptual model of this problem is shown in Figure 1, where the color refers to the original temperature field and the arrows indicate the traction stress induced by the thermal unloading process.



**Figure 1** A conceptual model demonstrating matrix-fracture interaction in the dual-porosity system (Wang et al., 2016).

## 2. GOVERNING EQUATIONS

This coupled process is governed by the following partial differential equations. Firstly, the thermal governing equation of the material is as below

$$\rho_r C_r \frac{\partial T}{\partial t} = K_r \nabla^2 T \quad 1$$

Where  $\rho_r$ ,  $C_r$  and  $K_r$  is the density, heat capacity and heat conductivity of the material, respectively.  $T$  refers to the temperature and  $t$  is the time term. If we set  $\kappa = K_r / \rho_r C_r$ , then we have

$$\frac{\partial T}{\partial t} = \kappa \nabla^2 T \quad 2$$

Secondly, the stress governing equations ignoring the body force are the extended Beltrami-Michell equation, as follows

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2}{\partial x^2} \left[ \frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left( \sigma_{xx} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) = 0 \quad 3$$

$$\frac{\partial^2 h}{\partial y^2} + \frac{\partial^2}{\partial y^2} \left[ \frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left( \sigma_{yy} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) = 0 \quad 4$$

$$\frac{\partial^2 h}{\partial z^2} + \frac{\partial^2}{\partial z^2} \left[ \frac{3}{2(1+\nu)} (\sigma_m - h) \right] + \frac{1}{2} \nabla^2 \left( \sigma_{zz} - h - \frac{3\nu}{1+\nu} (\sigma_m - h) \right) = 0 \quad 5$$

Where  $\nu$  is the Poisson's ratio.  $G$  is the shear modulus.  $\sigma_{ii}, i = x, y, z$  are the displacement components.  $\sigma_m$  is the mean stress, defined as  $\sigma_m = (\sigma_{xx} + \sigma_{yy} + \sigma_{zz}) / 3$

The thermal-hydraulic term  $h$  is defined as  $h = \alpha P + 3\beta K T$ , where  $\beta$  and  $K$  is the thermal expansion coefficient and the bulk modulus of the material, respectively. As shown in our previous work (Wang et al., 2016; Lei Wang et al., 2017), the thermal effect is much more significant than the hydraulic effect. Therefore, in this work the hydraulic term  $\alpha P$  is ignored.

By rearranging Equation 3 to 5, we have the mean stress equation as follows

$$\nabla^2 \sigma_m = \frac{2(1-2\nu)}{(1-\nu)} \beta K \nabla^2 T \quad 6$$

If we set

$$\alpha = \frac{2(1-2\nu)}{(1-\nu)} \beta K \quad 7$$

Then we have the following mean stress equation

$$\nabla^2 \sigma_m = \alpha \nabla^2 T \quad 8$$

It should be noticed that the stress equations are based on the assumption of instantaneous equilibrium, which provides us great flexibility in decoupling the system. The strategy of the propose workflow is as such. We first analytically solve the thermal governing equation. We then substitute the solved transient temperature field into the mean stress equation, which then becomes an inhomogeneous Poisson's equation and can be solved by separation of variables. The solved mean stress field is then substituted into the three stress governing equation, which also become independent inhomogeneous Poisson's equation and can be solved using a similar approach to the solution of the mean stress equation.

### 3. MATHEMATICAL DERIVATION

In this section, we investigate the thermal-mechanical behavior of the matrix rock, surrounded by fractures with fixed stress boundary, by analytically solving the governing equations described in the previous section. On the fixed stress boundary, the three principal stress components on the boundary are given. We aim to solve the variation of temperature as well as stress within the body of the matrix rock. The fracture temperature is assumed to decrease rapidly to the injection temperature, as follows

$$T_{x=0} = T_{x=L_x} = T_{y=0} = T_{y=L_y} = T_{z=0} = T_{z=L_z} = T_{inj} \quad 9$$

The initial temperature of the matrix rock is  $T_0$ . The transient temperature field can be easily solved from Equation **Error! Reference source not found.** as below. The solution can be found in many PDE textbooks.

$$T(x, y, t) = T_{inj} + \frac{8}{L_x L_y L_z} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} F(p, q, r) \sin \frac{p\pi x}{L_x} \sin \frac{q\pi y}{L_y} \sin \frac{r\pi z}{L_z} \exp \left[ - \left( \frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2} \right) \kappa t \right] \quad 10$$

where

$$F(p, q, r) = \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} T_0 \sin \frac{p\pi x}{L_x} \sin \frac{q\pi y}{L_y} \sin \frac{r\pi z}{L_z} dx dy dz = \frac{T_0 L_x L_y L_z}{\pi^3 p q r} [1 - (-1)^p] [1 - (-1)^q] [1 - (-1)^r] \quad 11$$

By substituting equation 11 into equation 10, we get the temperature filed within the rock matrix as

$$T(x, y, t) = T_{inj} + \frac{8T_0}{\pi^3} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \sum_{r=1}^{\infty} \exp \left[ - \left( \frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2} \right) \kappa t \right] \sin \frac{p\pi x}{L_x} \sin \frac{q\pi y}{L_y} \sin \frac{r\pi z}{L_z} \frac{[1 - (-1)^p] [1 - (-1)^q] [1 - (-1)^r]}{p q r} \quad 12$$

Based on the above equation, The average temperature of the rock matrix can be obtained as

$$\bar{T} = \frac{8T_0}{\pi^6} \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \sum_{r=1}^{\infty} \exp \left[ - \left( \frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^p]^2 [1 - (-1)^q]^2 [1 - (-1)^r]^2}{p^2 q^2 r^2} \quad 13$$

We use the solution of temperature to get the thermal-mechanical response of the matrix rock. As mentioned in the previous section, we firstly solve the mean stress equation. To solve the mean stress equation, we start with the homogeneous equation  $\nabla^2 \sigma_m' = 0$

We use separation of variables by assuming  $\sigma_m'(x, y, z) = X(x)Y(y)Z(z)$

By substituting the separated variables into the homogeneous mean stress equation, we get the following two sets of equations as

$$X'' - \mu X = 0 \quad 14$$

$$Y'' + \mu Y = 0 \quad 15$$

With boundary conditions

$$X(0) = 0 \quad 16$$

$$X(M) = 0$$

17

$$Y(0) = 0 \quad 18$$

$$Y(M) = 0 \quad 19$$

The characteristic solutions of the above equations are

$$X_m(x) = \sin \frac{(m+1)\pi x}{L} \quad 20$$

$$Y_n(y) = \sin \frac{(n+1)\pi y}{M} \quad 21$$

Then for the inhomogeneous system, we have

$$\begin{aligned} T_{xx} + T_{yy} + T_{zz} &= \sigma_{m,xx} + \sigma_{m,yy} + \sigma_{m,zz} \\ &= \frac{8T_0}{\pi^3} \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} \sum_{l=0}^{\infty} \left[ -B_{mnl} \left( \frac{(m+1)^2 \pi^2}{L_x^2} + \frac{(n+1)^2 \pi^2}{L_y^2} + \frac{(l+1)^2 \pi^2}{L_z^2} \right) \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{\pi^3 mnl} \\ &\quad \sin \frac{(m+1)\pi x}{L_x} \sin \frac{(n+1)\pi y}{L_y} \sin \frac{(l+1)\pi z}{L_z} \end{aligned} \quad 22$$

Where

$$\begin{aligned} &-B_{mnl} \left( \frac{(m+1)^2 \pi^2}{L_x^2} + \frac{(n+1)^2 \pi^2}{L_y^2} + \frac{(l+1)^2 \pi^2}{L_z^2} \right) \\ &= \frac{8}{L_x L_y L_z} \int_0^{L_x} \int_0^{L_y} \int_0^{L_z} (T_{xx} + T_{yy} + T_{zz}) \sin \frac{(m+1)\pi x}{L_x} \sin \frac{(n+1)\pi y}{L_y} \sin \frac{(l+1)\pi z}{L_z} dx dy dz \end{aligned} \quad 23$$

We also have

$$\begin{aligned} T_{xx} + T_{yy} + T_{zz} &= \\ &= \frac{8T_0}{\pi^3} \sum_{p=0}^{\infty} \sum_{q=0}^{\infty} \sum_{r=0}^{\infty} \left[ \frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2} \right] \exp \left[ - \left( \frac{p^2 \pi^2}{L_x^2} + \frac{q^2 \pi^2}{L_y^2} + \frac{r^2 \pi^2}{L_z^2} \right) \kappa t \right] \sin \frac{p\pi x}{L_x} \sin \frac{q\pi y}{L_y} \sin \frac{r\pi z}{L_z} \\ &\quad \frac{[1 - (-1)^p][1 - (-1)^q][1 - (-1)^r]}{pqr} \end{aligned} \quad 24$$

Then the mean stress can be solved as

$$\begin{aligned} \sigma_m(x, y, z) &= \sigma_{m,0} + \alpha \frac{8T_0}{\pi^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \frac{\exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right]}{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \end{aligned} \quad 25$$

where  $\sigma_{m,0} = \frac{1}{3}(\sigma_{xx,0} + \sigma_{yy,0} + \sigma_{zz,0})$  is the initial (boundary) stress condition.

For the solution of the stress components, we need to first calculate the partial derivative of  $\sigma_m$  and T with respect to  $x y z$ . We take the partial derivative with respect to  $x$  as an example

$$\frac{\partial \sigma_m}{\partial x} = \alpha \frac{8T_0}{\pi^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{mnl} \frac{m\pi}{L_x} \cos \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \quad 26$$

$$\frac{\partial^2 \sigma_m}{\partial x^2} = -\alpha \frac{8T_0}{\pi^3} \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} \exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{mnl} \frac{m^2 \pi^2}{L_x^2} \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \quad 27$$

Using a similar manner by first solving the homogeneous system then solving the inhomogeneous system, we can get  $\sigma_{xx}$ ,  $\sigma_{yy}$  and  $\sigma_{zz}$  in Equation 28,30 and 32 respectively.

$$\sigma_{xx}(x, y, z) = \sigma_{xx,0} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} C_{mnl}^1 \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \quad 28$$

$$C_{mnl}^1 = - \frac{8}{L_x L_y L_z} \frac{\int_0^{L_x} \int_0^{L_y} \int_0^{L_z} (T_{yy} + T_{zz}) \sin \frac{(m+1)\pi x}{L_x} \sin \frac{(n+1)\pi y}{L_y} \sin \frac{(l+1)\pi z}{L_z} dx dy dz}{\left( \frac{(m+1)^2 \pi^2}{L_x^2} + \frac{(n+1)^2 \pi^2}{L_y^2} + \frac{(l+1)^2 \pi^2}{L_z^2} \right)} \quad 29$$

$$= \frac{12\alpha T_0}{\pi^3} \frac{\frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2}}{\frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2}} \exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{mnl}$$

$$\sigma_{yy}(x, y, z) = \sigma_{yy,0} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} C_{mnl}^2 \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \quad 30$$

where

$$C_{mnl}^2 = \frac{12\alpha T_0}{\pi^3} \frac{\frac{m^2 \pi^2}{L_x^2} + \frac{l^2 \pi^2}{L_z^2}}{\frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2}} \exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{mnl} \quad 31$$

$$\sigma_{zz}(x, y, z) = \sigma_{zz,0} + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{l=1}^{\infty} C_{mnl}^3 \sin \frac{m\pi x}{L_x} \sin \frac{n\pi y}{L_y} \sin \frac{l\pi z}{L_z} \quad 32$$

where

$$C_{mnl}^3 = \frac{12\alpha T_0}{\pi^3} \frac{\frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2}}{\frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2}} \exp \left[ - \left( \frac{m^2 \pi^2}{L_x^2} + \frac{n^2 \pi^2}{L_y^2} + \frac{l^2 \pi^2}{L_z^2} \right) \kappa t \right] \frac{[1 - (-1)^m][1 - (-1)^n][1 - (-1)^l]}{mnl} \quad 33$$

#### 4. SUMMARY AND CONCLUSIONS

In this work, we brought out a novel derivation of the thermal-hydraulic-mechanical model of the matrix-fracture interaction, in order to capture the transient behavior of the fracture system in the thermal unloading process. We analytically solve the temperature field inside a matrix rock that is surrounded by fractures, using Fourier series and separation of variables. The temperature field is used to calculate the stress as well as the displacement field in the matrix rock during the thermal unloading process. Based on the analytical solution of the displacement field, we are able to obtain the deformation of the matrix rock, and therefore obtain the change in the aperture/permeability of the fracture system. Our analytical model has been successfully implemented in an in-house simulator. The proposed model can also be used to fast estimate the permeability variation during real-time injection monitoring in real practices.

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