Coupling Crustal Seismicity to Crustal Permeability –
Power-Law Spatial Correlation for EGS-Induced and Hydrothermal Seismicity

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ABSTRACT

Direct association of induced seismicity with crustal fluid flow properties has long been assumed, particularly for active crustal fault systems, but for a variety of reasons it has been difficult to describe the relationship in terms of subsurface properties. We present microseismicity evidence that, at least in the absence of active faulting, this relationship can be understood in terms of the empirics of power-law scaling porosity-related crustal properties in a critically-strained brittle crust. In summary, the observed power-law scaling two-point spatial-correlation property of microearthquake locations – e.g., induced-seismicity “clouds” – can be directly related to fluctuations in crustal permeability controlled by power-law scaling spatial fluctuations in crustal porosity.

Evidence for such a relation comes from seismicity observed at two geothermal developments: (1) fluid injection at 6 km depth at a Finnish EGS site, and (2) natural seismicity at 3 km depth in an Indonesian geothermal field. Microearthquake locations from both sites show power-law scaling two-point correlation distributions in event separation distance \( r \), \( \Gamma_{\text{mod}}(r) \sim 1/r^n \), \( n \sim \frac{1}{2} \). The observed microseismicity spatial correlation systematics follow from a trio of facts expressed in terms of spatial frequency \( k \) and crustal porosity \( \phi \) and permeability \( \kappa \):

1. Crustal porosity fluctuation power scales inversely as a power-law in spatial frequency \( k \), \( P_\phi(k) \sim 1/k \);
2. Crustal permeability is closely associated with crustal porosity, \( \kappa \sim \exp(\alpha \phi) \), where \( \alpha \approx 3-4 \);
3. In stationary random systems, spatial correlation distributions \( \Gamma(k) \) relate to spectral fluctuation power distributions via Fourier transformation, \( P(k) \sim \int \exp(ikr) \Gamma(r) dr \) (Wiener-Khinchin theorem).

Numerical simulations of permeability \( \kappa \) for a range of porosity spatial correlations \( P_\phi(k) \sim 1/k^m \), \( 0 < m < 2 \), generate a range of two-point correlation functions \( \Gamma_n(k) \sim 1/k^n \). The simulations show that the permeability correlation function \( \Gamma_n(k) \sim 1/k^{n/2} \) arises for the porosity the spatial-correlation scaling distribution \( P_\phi(k) \sim 1/k^l \) that is universally seen in well log data. As it is long recognised that fluid pressure effects promote seismic activity, and as fluid effects are likely to be strongest where permeability is greatest, it is logical to interpret the observed \( \Gamma_{\text{mod}}(r) \sim 1/r^{1/2} \) scaling for seismicity at the EGS and hydrothermal sites as evidence for a fundamental rock-fluid interaction that closely links crustal porosity \( \phi \), permeability \( \kappa \), and microseismicity.

Interpreting the observed spatial correlation \( \Gamma_{\text{mod}}(r) \sim 1/r^{1/2} \) for EGS and hydrothermal system seismicity as statistical evidence that discrete slip events associate with ability of fluid to flow within a crustal volume, we note that close spatial association of natural fluid flow pathways with low-level energy seismic release is extensively observed in shale formations undergoing frac-stimulation. The present data extend evidence for the close spatial association in shales to discrete slip events in magnitude range -1 < \( M \) < 1 in the absence of active tectonic faults and/or faulting. The mechanics of EGS permeability stimulation in crustal heat exchange volumes is revealed by the two-point correlation character of its induced seismicity. In parallel with ability to seismically map complex flow-connectivity structures in shale formations, seismic flow-structure imaging of convective geothermal flow systems can establish suitable production-well drilling targets.

1. INTRODUCTION

Systematic spatial correlation of crustal fluid-flow properties over a many decades of scale length is a signature feature of rock-fluid interactions across a wide range of geological settings. This paper extends the ambit of fluid-rock interaction spatial correlations to microseismic slip displacements induced by extensive wellbore fluid injection into 6km-deep basement rock. Relating induced seismicity to spatial correlation properties of crustal fluid flow mechanics has two practical applications. First, spatial correlations observed in crustal stimulation mechanics fuel clear negative implications for standard scenarios of Enhanced/Engineered Geothermal System (EGS) permeability stimulation of basement rock. Second, closely associating induced seismicity with crustal fluid flow structures supports the use of multi-channel ambient seismic noise data to image large-scale flow structures in convective geothermal systems as a means of guiding production-well drilling.
In the following discussion, §2 links spatial correlations for induced microseismicity to spatial correlations of crustal fluid-rock interactions in basement rock. In §2.1, recently observed induced microseismicity at 6km depth in Fennoscandian basement are contrasted with a standard view of EGS crustal fracture stimulation that ignores spatial correlations. §2.2 attributes the induced seismic event distribution to power-law scaling spatial correlation empirics of crustal fluid-rock interactions seen at all wellbore-accessible depths. §2.3 then uses numerical realisations of §2.2 crustal rock-fluid interaction empirics to derive the observed spatial correlations of induced microseismicity in terms of reactivated fossilised crustal rock-fluid interaction fabric. Speaking mathematically, our numerical modelling ‘predicts’ the observed induced-microseismicity 2-point spatial correlation function \( \Gamma_{\text{mod}}(r) \sim 1/r^{1/2} \) from the combination of attested well-log neutron porosity spatial fluctuation power-law spectral scaling \( P_E(k) \sim 1/k^4 \) with attested well-core relations between spatially varying porosity \( \phi \) and permeability \( \kappa \), \( \kappa(r) \sim \exp(\alpha \phi(r)) \), \( \alpha \sim 3-4 \).

The practical implications of §2 induced-microseismicity spatial correlation have for standard EGS stimulation scenarios and for production well drilling for convective geothermal resources are reviewed in §§3-4. On present evidence, it is not clear how naturally occurring wellbore-centric Peclet number \( P_e \sim 10 \) heat advection can be up-scaled by hypothetical EGS fracture stimulation scenarios to achieve \( P_e \sim 100 \) levels of heat extraction needed to drive turbines. On the other hand, building on naturally occurring Peclet number \( P_e \sim 10 \) heat advection offers means to stimulate \( P_e \sim 10 \) heat advection in long-reach wellbore-centric flow systems for direct use applications. §4 then interprets the spatial correlation of ambient microseismicity in hydrogeothermal systems to support prospects for remote-sensing of major convective fluid flow structures as targets for production well drilling. Significantly increasing production well success rates at hydrogeothermal sites addresses long-standing investor reluctance to sustain and develop hydrogeothermal resource heat extraction worldwide.

2. SPATIALLY-CORRELATED CRUSTAL ROCK-FLUID INTERACTIONS & BASEMENT CRUST EGS PERMEABILITY STIMULATION

Fig 1 synopses our discussion of spatial correlation phenomenology in crustal rock-fluid interactions. The left-hand image illustrates a standard concept of EGS wellbore-centric fluid pressure stimulation of crustal rock [1], while the right-hand image details the distribution of microseismicity recently induced by injecting 18000m\(^3\) of water into 6km-deep Fennoscandian basement rock along a 1km length of open wellbore [2].

![Figure 1](image.png)

**Figure 1**: (Left) Standard hypothetical rock-fluid interaction during wellbore-centric injection of fluid into an EGS crustal heat exchange volume. Blue tints denote cold injected fluid flow that absorbs crustal heat on traversing the heat exchange volume to exit as red-tinted hot fluids at the outtake well. Numerical simulations of the left-hand fluid-rock interaction typically assume that fracture properties such as position, length, width, orientation and aperture are can be represented by uniformly distributed independent random numbers. By the Central Limit Theorem, such numerical constructs lead inevitably to spatially-un correlated normally distributed flow property distributions. Any seismicity putatively associated with this hypothetical rock-fluid interaction will have a constant two-point correlation function \( \Gamma_{\text{hyp}}(r) \sim 1/r^0 \sim \text{const} \). (Right) In contrast to the constant statistical correlation function characteristic of standard hypothetical rock-fluid interactions, seismicity induced by wellbore-centric fluid injection is spatially correlated, \( \Gamma_{\text{mod}}(r) \sim 1/r^{1/2} \), over a decadal range of correlation offsets \( 30m < r < 500m \). The power-law nature of the observed 2-point correlation function indicates that the correlation function is scale-independent. Spatially-correlated scale-independent clustering leads to lognormal flow-property distributions as is observed in crustal flow systems worldwide.
We distinguish the Fig 1 hypothetical from the observed fracture distributions via two-point statistical spatial correlation values $\Gamma(r)$ as a function of inter-event spatial offsets $r$ within the crustal stimulation volumes [3; Appendix]. The hypothetical distribution of random crustal stimulation fracture events at Fig 1 left is typically devoid of spatial correlations [4; cf. §3]. With no correlation, the associated microseismicity two-point correlation function is constant, $\Gamma_{hyp}(r) \propto 1/r^0 \sim const$. In contrast, the two-point spatial correlation function or autocorrelation function for microseismic slip events induced by wellbore-fluid injection in basement rock at Fig 1 right is seen to be power-law scaling, $\Gamma_{meq}(r) \propto 1/r^{1/2}$, over a decadal range of inter-event offsets $30m < r < 500m$.

The two-point correlation statistic $\Gamma_{meq}(r)$ for Fig 1 induced microseismicity measures the average number of events within a given range, $r$ to $r + dr$, from any given sample event [Appendix]. If there is no spatial correlation within a collection of events, then the average number of events at offset interval $r$ to $r + dr$ is independent of offset value $r$, giving $\Gamma_{hyp}(r) \propto 1/r^0 \sim const$. If there is a tendency for fracture-related events to cluster, then the two-point spatial correlation function varies with offset range $r$ in a manner characteristic of the clustering process. The induced seismicity distribution of Fig 1 (right) shows that stimulation-induced microseismicity slip events in deep basement rock cluster in power-law scaling fashion, $\Gamma_{meq}(r) \propto 1/r^{1/2}$. In the observed induced seismicity event spatial distribution, sample events systematically tend to be associated with nearer events (smaller $r$) rather than further events (larger $r$), and the observed clustering is independent of the inter-event offset -- i.e., event clustering occurs without reference to an embedded or structural scale length. §2.1 shows that the induced seismicity is independent of which section of the stimulation wellbore injects stimulation fluids, giving evidence that the ‘stimulated’ flow paths are not freshly created at each crustal section but are instead pre-existing fossil flow structures. §2.2 reviews the widely attested pre-existing power-law scaling nature of well-log porosity fluctuation and its close spatial association with well-core crustal permeability fluctuations at all scale lengths. §2.3 discusses how the observed microseismic spatial correlation exponent value $\frac{1}{2}$ is controlled by the pre-existing crustal porosity well-log fluctuation scaling and its associated permeability.

2.1 Spatial-correlation phenomenology of Fennoscandian basement rock induced seismicity

Over a period of 50 days in June-July 2018, a total of 18000 cubic meters of fresh water were injected at 6km depth into Finnish basement rock at a drill site in the town of Espoo 15km west of Helsinki. Fluid injection was staged over five nominal 200-meter intervals along a 45-degree deviated open wellbore as sketched in Fig 2 [2].

![Figure 2: Sketch of wellbore fluid-injection stimulation project for 6km-deep Finnish basement crustal volume enclosing a 1km-long 45° deviated open section of wellbore OTN3.](image)

Seismic monitoring of fluid injection permitted operator control of induced earthquakes during the five-stage sequence of crustal stimulation [2]. Seismic monitoring was performed with a 24-station 3C-sensor network (12-station areal near-surface sensor array + 12-station vertical array of sensors at 2-3km depth). Information on induced-earthquake rates, locations, magnitudes, and evolution of seismic and hydraulic energy controlled fluid injection; at any stimulation stage, pumping was either stopped or varied between 60-90 MPa wellhead-pressures and 400-800 liter/min flow rates. Over 8000 monitor events were processed within a maximum delay of 5 minutes.
Of these, some 6,150 earthquakes with larger signal-to-noise ratios in magnitude range $-1 < M < 1.9$ formed the catalogue for monitoring and evaluating the five-stage fluid injection process.

Fig 1 (right) shows the cumulative wellbore-centric distribution of induced seismicity for magnitude range $-1 < M < 1.9$ in tight Finnish basement rock. Injection occurred over a 1km open-hole section of wellbore OTN3 at 6km depth. Figs 3-4 expand the observed induced seismicity distributions for two time-steps during the 50-day stimulation programme [2]. The five 200m-long fluid injection segments of open OTN3 wellbore in Fig 1 are marked by different colours – from deepest upward, red/pale green/light green/dark green/purple. Figs 3-4 show similar induced seismicity distributions over periods of 2 days and 30 days respectively:

- Event distribution is wellbore-centric in depth and azimuth, with no evidence of systematic stress alignment or large-scale fracture fabric sections intersecting the wellbore;
- Event distribution is irregular along the 1km section of open wellbore, possibly related to fracture fabric variation along the wellbore;
- Events notably cluster above and particularly below the 1km section of open wellbore, indicating that stimulation fluids access crustal volumes not immediately adjacent to wellbore injection intervals;
- Event distributions are largely the same regardless of which of the five wellbore intervals is the source of the injected fluid.

From Figs 3-4, we see that the wellbore-fluid induced stimulation event distributions in one sense agree with the Fig 1 hypothetical induced seismicity image, while in another sense the rock stimulation event distributions strongly disagree with the details of the hypothetical fracture structure.

The Fig 1 hypothetical fracture distribution without spatial correlations is generically understood to imply that stimulation fluids generate or reactivate a fracture network that is more or less the same without regard to the precise location of the injection wellbore within the crustal stimulation volume [1; §3]. When crustal rock is assumed to be spatial-correlation free, all that matters is a mean fracture density, which by hypothesis does not vary significantly within the crustal volume. Hence there is an implicit expectation that an induced hypothetical fracture network will be more or less the same, regardless of where the stimulation wellbore is located. In this sense, the Fig 1 hypothetical fracture construct can be said to be consistent with the Fig 1 observed stimulation event distributions, which are largely the same regardless of which stimulation interval injects the fluid.

On the other hand, Figs 3-4 show a clear discrepancy between the Fig 1 hypothetical and observed stimulation fracture configurations. While it doesn’t overly matter where the OTN3 open hole stimulation occurs in the accessed crustal volume, the spatial distribution of induced events is materially different from that expected for the hypothetical stimulation.

The seeming paradox of simultaneous similarity and dissimilarity between Fig 1 hypothesis and observation is easily resolved by noting that in both cases we can presume that the crustal fracture fabric is pre-existing, with fluid injections merely activating, or reactivating, a pre-existing fracture fabric. In both hypothetical and observation cases, the specific pre-existing crustal fracture fabric in the stimulation volume is expected to be revealed by stimulation microseismicity.

In light of Figs 1 and 3-4, our question becomes, what is the pre-existing fracture fabric of crustal rock? Is the pre-existing fracture fabric spatially uncorrelated as in Fig 1 (left) or spatially-correlated as in Fig 1 (right)?
The answer is abundantly clear: vast arrays of well-log, well-core, and well-productivity data show that the inherent pre-existing fracture fabric of crustal rock is that of Fig 1 (right) spatial correlation, not that of Fig 1 (left) spatial non-correlation. In essence, Fig 1 induced microseismicity data show that the standard hypothetical fracture fabric and its numerical realisations by means of uncorrelated random numbers are simply wrong. Examining crustal spatial correlation evidence in greater detail establishes that the power-law scaling spatial correlation empirics for crustal fluid-rock interaction yield the observed Fig 1 power-law scaling induced seismicity spatial correlation function $\Gamma_{\text{uc}}(r) \sim 1/r^{3/2}$.

Figure 4: Coloured dots locate induced seismicity events across the 30-day interval 09 June to 09 July 2018. In this period, fluid was injected into the crustal stimulation volume from the first four of the five 200m-long open well injection intervals (red, pale green, light green, dark green). As in Fig 3, induced seismicity occurs essentially in the same stimulation/cluster pattern regardless of where in the 1km-long open wellbore the fluid is injected. A majority of induced seismicity occurs below the toe of the open wellbore for all four open wellbore stimulation intervals, with a sizeable cluster of stimulation events located above the open wellbore stimulation interval. Most particularly, there is no evidence that induced seismicity is proximate to the injection interval; all four injection intervals are seen to stimulate seismic activity in the same set of crustal volume clusters.

2.2 Generic crustal permeability spatial correlation generated by steady-state tectonic crustal deformation

Evidence for a ‘correct’ spatial correlation formulation for crustal fluid-rock interaction, and the resultant pre-existing fracture fabric that imposes spatial correlations on induced seismicity, can be summarised as:

I. Crustal porosity fluctuation power scales inversely as a power-law in spatial frequency $k$, $P_s(k) \sim 1/k$;

II. Spatial variations in crustal permeability $\kappa(\cdot)$ closely associate with crustal porosity $\phi(\cdot)$, $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$, with parameter product for mean porosity having empirical value $\alpha_0 \sim 3-4$;

III. In stationary random systems, spatial correlation distributions $\Gamma(r)$ relate to spectral fluctuation power distributions via Fourier transformation, $P(k) \sim \exp(\alpha_0 k) \Gamma(r) dr$ (Wiener-Khinchin theorem).

KTB deep wellbore data in Figs 5-6 exemplify the well-log spectral observational constraint (I) and well-core poro-perm relation between porosity and permeability (II) for crystalline basement rock to 6km depths [5].

The overarching crustal context for Figs 5-6 is steady-state tectonic deformation acting on fluid-saturated porous rock comprised of tough mineral grains bonded by weak cements [6]. Steady-state residual tectonic deformation at strain rates of order $10^{-5}/\text{yr} < \dot{\varepsilon} < 10^{-7}/\text{yr}$ [7] over $10^4$-$10^5$-year intervals generate crustal strains of order $10^{-4} < \dot{\varepsilon} < 10^{-2}$. Tectonic strains of this order introduce irreversible finite strain damage at weak cements bonding strong mineral grains [8]. The essentially universal well-log spectral systematics of Fig 5 show that ongoing tectonic deformation creates in the brittle-fracture crust a wide-sense stationary random grain-scale fracture connectivity that is spatially correlated at across 6 decades of scale length, mm < $\ell$ < km. In particular, as indicated in Fig 5, crustal well-logs of neutron porosity in basement rock record spatial fluctuations that persist to 6km depth with Fourier spectral power scaling inversely with spatial frequency, $S_\delta(k) \sim 1/k$, over scale range 1/km < $k < 1$/m.

The essentially universal presence of $1/k$-spectral-scaling of empiric (I) comprehensively refutes the standard assumption that crustal porosity in geological formations is effectively determined throughout the formation by a mean value and standard deviation that can be determined by a handful of small-scale samples [9]. Further, as seen in Fig 6, rather than formations having a mild quasi-uniform porosity and permeability in a formation, porosity fluctuations at all scale lengths throughout geological formations generate strongly fluctuating spatial connectivity between pores at all scales as per crustal empiric (II). The associated empirical value $\alpha_0 \sim 3-4$ guarantees lognormal distributions of well-productivity, as observed worldwide [10,11].
Figure 5: (Left) Log-log plots of KTB well-log Fourier power-spectral scaling $P(k) \sim 1/k^\beta$ for spatial fluctuation in six physical variables measured between 4-6 km depths in crystalline basement (LLD = lateral log induction; DTCO = P-wave sonic velocity; RHOB = mass density; PEF = photoelectric absorption; NPHI = neutron scattering ≡ porosity; VPEF = weighted photoelectric absorption); plot title numbers give value of power-law scaling exponent $\beta$ fit to fluctuation power-spectra; well-log ensemble mean exponent is $\beta \sim 1.12 \pm 0.1$ over spatial frequency range $1$cycle/km $< k < 300$cycles/km. (Right) Amplitude of P-wave sonic and neutron porosity spatial fluctuations as a function of depth in KTB main well; well-log fluctuation amplitudes of spectral to left do not significantly reduce with depth in crustal rock.

Figure 6: Open circles give well-core poroperm data for 31 KTB core between 4 and 6 km depths. Red lines fit to plotted poroperm relation log($\kappa$) $\propto$ $\alpha$ $\varphi$. Different plots are for different core axis directions relative to wellbore axis and local rock fabric axis (s = perpendicular to fabric; p = parallel to fabric; a = along wellbore axis; r = perpendicular to wellbore axis). Aside from a single well-core poroperm outlier, poroperm data conform to a linear relation log($\kappa$) $\propto$ $\alpha$ $\varphi$ with proportionality constant having values of order $\alpha \sim 300$ with mean value $\alpha$ $\varphi$ $\sim 3$ for mean basement porosity $\varphi$ $\sim 1\%$. The mean values for 3000 well-core samples from five hydrocarbon-bearing reservoir formations are $3 < \alpha$ $\varphi < 4$ for porosity values of order $\varphi$ $\sim 10-30\%$ [10]. For observed poroperm proportionality constants $3 < \alpha$ $\varphi < 4$, integrating relation log($\kappa$) $\propto$ $\alpha$ $\varphi$ over reservoir-scale poroperm sequences gives observational constraint (ii) expression $\kappa(\cdot) \sim \exp(\alpha \varphi(\cdot))$. For normal distributions of porosity in range $0 < \varphi < 1$, observed values of $3 < \alpha$ $\varphi < 4$ guarantee that reservoir-scale crustal permeability $\kappa(\cdot)$ fluctuations are lognormally distributed, as is observed worldwide for well-productivity data of groundwater, hydrocarbon fluids, geothermal fluids, and for fossilised fluid system mineral distributions [11].

Fig 6 illustrates that the connection between porosity and permeability observed in reservoir rock worldwide [10] extends to crystalline rock at 4-6 depth. Generic irreversible grain-scale finite strain damage is naturally greatest where porosity is greatest. Spatially-correlated grain-scale fracture connectivity thus constitutes a generic rock-fluid interaction that generates the spatially erratic fluid percolation fluid pathways observed at all scales in the drillable crust. In particular, Fig 6 illustrates well-core poroperm data for porosities $< 1\%$, showing that crustal permeability $\kappa(\cdot)$ in basement rock at depth is described by spatially-correlated porosity $\varphi(\cdot)$, $\kappa(\cdot) \sim \exp(\alpha \varphi(\cdot))$, for $\alpha$ an empirical parameter. The observed values of $\alpha$ are such that the exponent product term $\alpha \varphi$ has a fixed mean value, $\alpha$ $\varphi$ $\sim 3-4$ [10]. For all observed ranges of parameter values $\alpha$, crustal permeability $\kappa(\cdot) \sim \exp(\alpha \varphi(\cdot))$ is formally a lognormal distribution, as observed for crustal well-flow productivity distributions worldwide [11].

The multi-decadal duality of spatial frequency and spatial range empirics (I)-(II) gives rise to a third spatial correlation empiric (III), by which correlated random fluctuations in space $\Gamma(r)$ relate to spectral fluctuation power distributions in spatial frequency $P(k)$ as a Fourier transformation duality, $P(k) \sim \exp(\text{ikr}) \Gamma(r) dr$ with its inverse transform $\Gamma(r) \sim \exp(-\text{ikr}) P(k) dk$ [12]. Insofar as the random distribution in a physical system is stationary in a wide sense -- i.e., the physical system mean-values and their fluctuation bounds do not change during physically relevant times -- then the Fourier power-spectrum $P(k)$ of a physical fluctuation sequence is equivalent to the spatial correlation function $\Gamma(r)$ of that fluctuation sequence.
Empirics (I)-(II) enable us to numerically simulate crustal poroperm fields $\phi(\cdot)$ and $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$ in 2 or 3 dimensions for a range of spectral power-law scaling exponents $0 < m < 2$, $S^n_o(k) \sim 1/k^m$. Number field representations with $m \sim 0$ correspond to essentially non-correlated spatial fluctuations in porosity (as is routinely assumed for hydrocarbon reservoir characterisation [9]). At the other spatial correlation extreme, number field representations with $m \sim 2$ correspond to essentially highly correlated spatial fluctuations in porosity such as layered media (layers have step-function structure for which the Fourier power spectra scale inversely with the square of the spatial frequency, $S(k) \sim 1/k^2$ [12]). Accordingly, we can use such numerical representations of porosity fields $\phi(\cdot)$ and associated permeability fields $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$ to compute the resulting spatial correlation function $\Gamma^n(r)$ as a function of exponent $m$ used to generate a range of spatially correlated porosity fields $\phi(\cdot)$. As we can expect that power-law scaling spatial correlation fields $S^n_o(k) \sim 1/k^m$ generate a power-law spatial correlation functions, we can numerically evaluate the correlation function exponent $n$ for spatial correlation form $\Gamma^n(r) \propto 1/r^n$ in the associated permeability number fields.

To make contact with induced seismicity data, we can expect that spatial domains of higher/lower porosity $\phi(\cdot)$ and associated permeability $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$ correspond to spatial domains with higher/lower likelihood of seismic slip. To test this expectation, we next check if the spatial correlation property of Fig 1 observed induced microseismicity has the same spatial correlation function as our numerical constructs for the highest values of porosity $\phi(\cdot)$ and associated permeability $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$.

2.3 Two-point spatial correlation for induced seismicity in crust with spatially-correlated rock-fluid interactions

As we cannot log crustal permeability in the same way we can log crustal porosity, we look to induced seismicity to measure the spatial correlation property of crustal permeability. To this end, consider 3D numerical distributions of crustal permeability with generalised poroperm spectral correlation properties based on crustal rock-fluid interaction empirics (I)-(II). The generalised 3D numerical permeability fields are controlled by porosity spectral scaling according to $S_o(k) \sim 1/k^m$ for spectral scaling exponent $0 < m < 2$. Within this range of generalised 3D permeability fields, we find the consequent range of two-point spatial correlation functions of form $\Gamma^n(r) \propto 1/r^n$, where exponent $n$ is to be found by inspection of the numerical correlation distribution.

Fig 7 shows representative result of such a numerical exercise. For a set of numerical parameters with normally distributed 3D porosity field $0.1 < \phi(\cdot) < 0.3$ distributed across a data cube of 300 nodes on a side, the 3D permeability field is computed for proportionality constant in $\log(\kappa) \propto \alpha \phi$ set to $\alpha = 20$ with a mean product parameter $\omega = 5$. The mean values and deviations of the corresponding 2-point spatial correlation exponents $m(n)$ given by $\Gamma^n(r) \propto 1/r^m$ are determined for incremental steps 0.2 for power-law spatial correlation exponent $n$ between 0.1 and 1.8, $m = 0.1:0.2:1.8$ over 10 layers of $Z = 30$ units each the 3D data volume.

![Fig 7](image)

Fig 7 -- Representative numerical relationship between degree of power-law scaling spatial correlation of 3D porosity field given by exponential parameter $0 < m < 2$, $S_o(k) \sim 1/k^m$, and degree of power-law scaling 2-pt spatial correlation function exponent $n$, $\Gamma^n(r) \propto 1/r^n$, determined for permeability field $\kappa(\cdot) \sim \exp(\alpha \phi(\cdot))$. If it is assumed that the locations of highest porosity and permeability correspond to the mostly likely locations of induced seismicity slip events, then the numerical constructions ‘predict’ that for observed porosity fluctuation power scaling exponent $m \sim 1$, the associated induced seismicity has 2-point correlation function scaling exponent, $\Gamma^{m-1}(r) \propto 1/r^{n-1/2}$, in agreement with the observation of Fig 1 (right).

Fig 8 compares an observed spatial distribution of induced seismicity events (left) with a representative numerical realisation of Fig 7 permeability distributions for correlation exponents $m \sim 1$ and $n \sim 0.64$ (right). The numerical realisation has no errors associated with observed microseismicity data and thus better exhibits the fine-scale spatial correlations that location errors in induced seismic event data do not have. Both the observed numerical simulation event distributions have power-law scaling dependency on interevent range $r$, and the scaling exponent for interevent range is the same $n \sim 0.5-0.6$ for both observation and numerical models.
We conclude from Figs 7-8 numerical constructions that the Fig 1 observed induced seismicity spatial correlation $\Gamma(r) \propto 1/r^{n/2}$ responds to naturally occurring spatial correlation embodied in crustal rock-fluid empirics (I)-(III).

![Image](https://via.placeholder.com/150)

**Figure 8:** (Left) Representative spatial distribution and resultant 2-point correlation function for Fig 1 (right) induced seismicity at 6km depth in crystalline basement. (Right) Representative spatial distribution and resultant 2-point correlation function for numerical simulation of empirical crustal porosity and permeability distributions described in text. There is a strong likelihood that the different degrees of fine-scale local clustering for field data (left) and simulation data (right) is due to limited spatial resolution of field data.

3. EGS STIMULATION FOR AD HOC FRACTURE-FLOW MODELS WITHOUT SPATIAL CORRELATION PHENOMENOLOGY

§2 connects the spatial correlation property of observed EGS stimulation induced seismicity distributions with crustal permeability heterogeneity expressed by the correlation empirics (I)-(II) for well-log and well-core spatial fluctuations. The dual Fourier transform relation (III) between spectral power-law scaling of permeability distributions and spatial power-law scaling for induced microseismicity distributions formalises the link. If Fig 1 right-hand spatially-correlated microseismicity distributions accurately reflect EGS wellbore-centric crustal stimulation reality grounded in spatial correlation properties of crustal rock-fluid interactions, what are we to make of the standard spatially-uncorrelated hypothetical EGS stimulation scenarios represented by Fig 1 left?

Standard EGS fracture-borne fluid flow stimulation scenarios that are designed to achieve heat extraction needed to produce electrical power from the drillable crust heat store draw heavily, even exclusively, on the spatial averaging practices of hydrocarbon reservoir management. Mathematically, spatial averaging of reservoir flow heterogeneity produces physically realistic models only if the random fluctuations in reservoir flow processes are statistically independent – i.e., even if lognormal well production distributions mean a majority of wells produce a minority of hydrocarbons while a minority of wells produce a majority of hydrocarbons, low producing wells are nonetheless profitable; and well management issues centered on the immediate vicinity of the wellbore -- i.e., the ‘skin effect’.

While the spatial non-correlation concept of reservoir heterogeneity is implausible in light of abundant well-log and well-core fluctuation empirics (I)-(III), at least four features of hydrocarbon reservoir management appear to collectively override that inherent implausibility in interests of hydrocarbon production practice:

1. large numbers of wells over which to perform spatial averages;
2. low cost of wells relative to the price of produced fluid -- i.e., drilling more wells is a safe option for unreliable statistical predictions of well production distributions;
3. high probability that wells pay for themselves over a production life -- i.e., even if lognormal well production distributions mean a majority of wells produce a minority of hydrocarbons while a minority of wells produce a majority of hydrocarbons, low producing wells are nonetheless profitable;
4. well management issues centered on the immediate vicinity of the wellbore -- i.e., the ‘skin effect’.

Applying hydrocarbon-reservoir management and cost features that define well performance by spatial averaging over the production field does not, however, match geothermal production well cost profile:

1. geothermal resources serviced by order of magnitude fewer wells, significantly reducing the statistical significance of spatial averaged well performance;
2. geothermal wells costing two orders of magnitude more than hydrocarbon wells per unit price of produced fluid -- i.e., geothermal production wells are a much larger operational cost burden than are hydrocarbon production wells;
3. successful geothermal production wells flowing at one to two orders of magnitude higher rate than hydrocarbon wells, greatly reducing the success rate of geothermal wells and increasing the cost burden of geothermal wells;
4. geothermal wells responding to crustal conditions over one or two orders of magnitude larger radii than do hydrocarbon wells; in particular, EGS wellbores need to be systematically stimulated over long reaches of open hole to generate wellbore-to-wellbore flow across an enclosing heat exchange crustal volume, a process that offers no statistical basis for spatial averaging.

Table 1 high-lights the scale of disparity between crustal reservoir formation spatial averaging practice as applied to hydrocarbon production and to geothermal energy production.
Table 1 – Impact ratios for spatial averaging of flow heterogeneity for geothermal over hydrocarbon production wells.

<table>
<thead>
<tr>
<th>Well Property</th>
<th>Hydrocarbon</th>
<th>Geothermal</th>
<th>G/H Impact Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of wells</td>
<td>High – Hundreds per field</td>
<td>Low – Tens per field</td>
<td>10</td>
</tr>
<tr>
<td>Well cost relative to pay</td>
<td>Low for $50-$100 per barrel</td>
<td>High for $0.5-$1 per barrel</td>
<td>100</td>
</tr>
<tr>
<td>Well success</td>
<td>High – even low flow wells pay</td>
<td>Low – only high flow wells pay</td>
<td>10-100</td>
</tr>
<tr>
<td>Well environment</td>
<td>Local (skin effect ~ 1m-3m)</td>
<td>Non-local (&gt; 30m-300m)</td>
<td>30-100</td>
</tr>
</tbody>
</table>

In light of Table 1 impact of reservoir heterogeneity on geothermal wells relative to hydrocarbon wells, we focus how single or EGS doublet geothermal wells actually (rather than hypothetically) interact with the crustal flow heterogeneity empirics (I)-(II). Focusing on single wells reveals the scale of conceptual flaws affecting many or most EGS permeability stimulation scenarios advanced to generate electrical power from drillable crust heat store. Table 1 also hints that hydrocarbon well spatial averaging practice may have discouraged exploration for effective means of locating production well sites in convective hydrogeothermal fields; §4 outlines one such means emerging from the §2 discussion of observed microseismicity spatially correlations.

Conventional EGS stimulation scenarios characterise fractures as essentially uniform discontinuities in an otherwise ‘effectively uniform’ elastic medium. Residual flow property heterogeneity is, by hypothesis, ‘effectively’ eliminated by spatial averaging. Application of spatial averaging to reservoir formations appeared unquestioned in the early flow models, which drew directly on the mathematics of thermal conduction [13]. Attempts to formalise the practice of spatial averaging were made by Hubbert and by Bear [14]. With spatial averaging a prevailing concept, many schemes were advanced to incorporate fractures into spatially-averaged flow scenarios. Fig 9 illustrates two standard approaches, both categories of which ‘fracture-as-discontinuity’ approximations persisting over a 50-year period. At left, fluid velocity in fractures is approximated by Poiseuille-Hagen cubic law flow, \( V[m/s] \sim g[Pa/m] \cdot h[m^3]/12[\mu Pa\cdot s] \), or close variants accounting for fracture roughness [15]. At right Darcy flow through a ‘double porosity’ medium features a high-porosity/high-permeability material that is systematically embedded with uniform blocks of low-porosity/low-permeability material. Double porosity, effectively introduced by Warren & Root in 1963, was incorporated in Tough2, the oldest, most widely used flow simulation code [16].

![Figure 9](image_url)

**Figure 9:** (Left) Illustration of by Poiseuille-Hagen planar flow approximation to fluid flow in fractures; fluid velocity is given as \( V[m/s] \sim g[Pa/m] \cdot h[m^3]/12[\mu Pa\cdot s] \), where the pressure gradient \( g \) is constant along both axes the fracture plane [15]. (Right) Double porosity approximation to a block of heterogeneous crust introduced by Warren and Root, and later incorporated into the widely used Tough2 flow simulation code [16].

True to their origin in the spatial-averaging conceptual framework, Fig 9 ‘fracture as discontinuity’ flow model computations are implemented using uncorrelated random numbers [17]. As such, the Central Limit Theorem mathematically insures that the resultant model flow distributions are normally distributed rather than, say, the lognormal distribution routinely observed worldwide [9,11].

Considering the large number of diverse Fig 9-type flow models appearing as EGS stimulation scenarios, it is convenient to focus our wellbore-centric discussion on a recent computational exercise that dispenses with any pretense at physical accuracy in favour of a thin uniform permeability flow structure [18]. Fig 10 (left) sketches the computational model crustal heat exchanger as rectangular prism. The prism, \( \sim 1 \)km thick and extending \( \sim 4 \)km laterally and \( \sim 3 \)km vertically, is designed to be large enough to sustain 35MW of electrical power production for 30 years. The key model feature, Zone 5, is a 1m-thick planar vertical layer of uniform permeability \( \kappa \sim 10^{-11} \text{m}^2 \) (10 Darcy) serving as a generic flow structure for carrying fluid from injection wells to production wells. Fig 10 (right) gives three examples of Fig-9-type well-fracture intersections that indicate how the Zone 5 fluid transport structure might be conceived in relation to actual crustal fractures.

Computation with the Fig 10 (left) flow structure shows that the heat exchange prism maximises heat extraction efficiency for input and outtake wells separated by 2km flowing at rate \( V \sim 100L/s \). Expressing wellbore thermal heat production as \( Q_h = VT_\rho C \) calculates the generic well-pair flow performance for a given model crustal temperature \( T \) for water with volumetric heat capacity \( \rho C \sim 4\text{MJ/m}^3 \). For model wellbore-to-wellbore flow rate \( V \sim 100L/s \) and bottom hole temperature \( T = 80^\circ C \), the heat produced is \( Q_h \sim 30\text{MW} \).

While the hypothetical Fig 10 \( Q_h \sim 30\text{MW} \) EGS heat exchanger simply ignores wellbore interfaces to the surrounding crust, we learn from the §2 EGS induced seismicity distribution how the crustal fluid-rock interaction fabric actually responds to wellbore fluid injection. Rather than passing smoothly into a uniform high-permeability planar flow structure, actual injection fluids enter crustal rock via reactivated heterogeneous fossil spatially-correlated fracture-connectivity channels. Taking the opposite approach of Fig 10, we focus on...
the mechanics of radial fluid flow $\varphi v_0$ at the wellbore interface by expressing volumetric flow $V$ in terms wellbore radius $r_0$ along a wellbore length $\ell$, $V = 2\pi r_0 \varphi v_0 \ell$. The associated advection heat transfer Peclet number given by $\varphi v_0 \rho$ is $P_e = r_0 \varphi v_0 / D = r_0 \varphi v_0 \rho C / K$, where rock-fluid thermal diffusivity $D = K / \rho C = 2.7 / 862 / 4510 \sim 0.7 \times 10^{-6} \text{m}^2 / \text{s}$.

**Figure 10:** (Left) Generic crustal heat exchange model for producing 35MW of electrical energy [18]; the principal model feature is Zone 5, a 1-meter thick layer of uniform permeability extending 4km laterally and 3km vertically. (Right) Examples of wellbore-fracture intersections that, for computational simplicity, are replaced by the Zone 5 permeability layer [18].

If we treat Fig 10 thermal heat flow as occurring over wellbore length $\ell \sim 200\text{m}$ at crustal temperature $T_0 = 80^\circ \text{C}$, the effective Fig 10 Peclet number is $P_e = Q / 2\pi K T_0 = 30 \times 10^6 / 2\pi / 200 / 80 / 2.7 \sim 100$. For comparison, Peclet numbers observed for fluid transport in actual crust are occasionally $P_e \sim 10$ but generally $P_e < 1$ [5,6].

Building on well-log data for a 2km deep well in Fennoscandian crustal basement associated with Fig 1 EGS stimulation data, Fig 11 evaluates Peclet number $P_e \sim 10$ heat transport flow for naturally occurring crustal fracture structures [5]. The computed wellbore-centric flow distribution for a 20m-cube of crust is shown in Fig 11 (left). Peclet numbers for wellbore-centric radial flow at meter-scale stages along the wellbore axis are shown in the middle plots for analytic fits (red) to the model radial temperature distributions (blue). The resultant model axial temperature field shown as the red trace in Fig 11 (right) is constrained by the observed wellbore axial temperature profile in black. The agreement between model and observed temperature profiles validates the computation performed using the empirical rock-fluid interaction heterogeneity (I)-(II), and indicates that naturally occurring wellbore-centric crustal advective flow systems can have Peclet numbers of order $P_e \sim 10$ but are generally of order $P_e < 1$.

**Figure 11:** (Left) Model of a m-thick planar permeability flow structure of $\kappa \sim 0.02\text{Darcy}$ leaking crustal fluid into a wellbore at hydrostatic pressure. (Mid) Model radial temperature profiles (blue) for four axial levels at 1m intervals along the wellbore from the planar flow structure matched by 2D analytic radial profiles (red) with Peclet number given above each plot. (Right) Resultant axial temperature field (red) fit to observed temperature along the wellbore axis (black). Data/model from [5; cf. 6].

With Fig 11 calibrating actual wellbore-centric advective flow at order $P_e \sim 10$ or less, and §2 microseismicity indicating that injected fluids enter the crust surrounding a stimulation wellbore by activating pre-existing fossil flow fracture-connectivity pathways, we see that standard EGS stimulation measures can probably generate intervals of $P_e \sim 10$ wellbore-centric advective flow. It follows, however, that in order for actual wellbores to achieve Fig 10 hypothetical EGS wellbore advection heat transfer, every meter of a 200-meter wellbore length associated with Fig 10 has to be stimulated to permeability 10 to 100 times the naturally occurring distribution of advective flow permeability.
Further, while Fig 11 refers to naturally occurring fracture structures feeding the wellbore over a 10-meter radius, Fig 12 shows that realising a Fig 10 EGS stimulation scenario requires wellbores to be stimulated to radii of several hundred meters. Fig 12 shows a formal solution to wellbore-centric radial heat transport, and as such gives approximate stimulation radii required to sustain wellbore-to-wellbore heat advection in a crustal heat exchange volume for heat extraction times up to 30 years at heat extraction rates $2 < P_e < 50$ [5,6].

4. AMBIENT MEQ SPATIAL CORRELATION EVIDENCE SUPPORTING PERMEABILITY STRUCTURE IMAGING OF HYDROGEOThermal FLOW

Given the §2 association of power-law-scaling spatial correlation of EGS basement rock induced seismicity with pre-existing crustal permeability distributions due to power-law-scaling spatially-correlation porosity distributions, it is of interest to look at ambient microseismicity distributions at convective hydrogeothermal sites. Available well-log and well-core data, though sparse, show that convective geothermal flow systems are characterised by the same power-law-scaling spatial correlation empirics (I)-(II) as are observed worldwide [19]. It is thus plausible that ambient microseismicity at convective flow sites [20] has the same power-law-scaling spatial correlation property as observed for in §2 EGS basement rock induced seismicity. Connecting ambient microseismicity distribution statistics with the statistics of convective geothermal flow permeability structures opens the door for using ambient seismicity data to accurately locate significant flow structures of a convective geothermal flow system as targets for production well drilling [21].

Figs 13-14 summarise the ambient microseismicity recorded at a geothermal field in west-central Java, Indonesia [20]. A total of 877 microseismicity event locations occur in three groups. We look for spatial correlation evidence in two tight clusters, with respectively 337 and 365 events, associated with steam field production. The remaining 175 red partition events are either associated with a deep lying structure fault or are diffusely scattered at the near surface. Fig 13 shows planar projections of the tight event clusters. Fig 14 plots the resulting spatial correlation function in log-log format with a power-law fit to the statistical clustering estimates for event-event offsets between 200 and 1000 meters. For offsets > 1km, the event populations become too sparse to return robust estimates. For the 200-1000m offset range, the two microseismicity clusters give power-law-scaling spatial correlation functions consistent with the $\Gamma_{meq}(r) \sim 1/r^{1/2}$ function observed in the §2 induced seismicity event population and its interpretation in terms of crustal permeability distributions.

Crustal rock-fluid interaction empirics (I)-(II) combined with the Fourier transform duality (III) can be interpreted as generic properties of crustal flow systems. We therefore note the considerable evidence from hydrofracture stimulations of shale formations conducted to activate otherwise immobile hydrocarbons that many-channel surface seismic data acquired over the reservoir can be processed to systematically map low-level ambient microseismicity generated in reservoir flow structures [21]. Using accurate surveys of shale play seismic velocity structure, it is possible detect and process small but persistent ambient noise signals to map fracture-connectivity flow structures to ~ 25m spatial accuracy. As indicated in §3, much or most current convective geothermal field surveys are subject to low-resolution spatial averaging structure mapping. Spatially averaged flow structure surveys, with, say, spatial resolution ~ 500 meters (comparable to 40-acre gridding of hydrocarbon reservoirs), provide little guidance for drilling production wells. Inadequate flow structure data is particularly acute in the earlier stages of reservoir development, leading to costly “learning curve” drilling of many unproductive wells. Fig 13-14 evidence for extending crustal flow heterogeneity empirics (I)-(II) to convective geothermal systems clearly indicates that ability of multi-channel seismic data to reliably locate major convective flow channels to, say, 50m resolution will greatly improve the economics production well drilling.
Figure 13: Planar distributions of Indonesia geothermal field microseismic event clusters.

Figure 14: Two-point spatial correlation functions (blue) for Fig 13 planar event distributions with power-law scaling fits across event-to-event offset range 200-1000 meters (red). Power-law scaling exponents in plot titles show the geothermal field ambient microseismicity [20] is consistent with the §2 spatial correlation function $\Gamma_{\text{meq}}(r) \sim 1/r^{1/2}$ derived from spatial correlation empirics (I)-(II) of generic basement crust. Following §2 in associating ambient microseismicity with inherent permeability heterogeneity in convective geothermal flow systems supports use of detailed low-level microseismicity maps to locate major flow structures as targets for production well drilling [21].

5. CONCLUSIONS
Well-log and well-core spatial correlation empirics for crustal poroperm properties -- (I) crustal porosity fluctuation power scaling inversely as a power-law in spatial frequency $k$, $P_{\phi}(k) \sim 1/k$, and (II) spatial variations in crustal permeability $\kappa$ associated with crustal porosity $\phi$, $\kappa \sim \exp(\alpha\phi)$ -- lead to high degrees of spatial heterogeneity in crustal flow structures attested over a wide range of geological settings. Observation of spatial correlation systematics for microseismicity event locations, $\Gamma_{\text{meq}}(r) \sim 1/r^{1/2}$, associated with EGS stimulation of deep basement rock (§2) and convective geothermal flow (§4), can be modelled using numerical representations of crustal empirics (I)-(II) and connected to microseismicity distributions by the Wiener-Khinchin theorem (III). Observational evidence that EGS induced seismicity is spatially correlated directly and comprehensively refutes most or all present-day scenarios for electrical power production requiring stimulation of Peclet number $P_e \sim 100$ heat extraction volumes in the deep crust. Rather, present evidence indicates that inherent crustal flow heterogeneity is compatible with stimulation of $P_e \sim 10$ heat extraction volumes for direct use. Observation of microseismicity spatial correlation systematics $\Gamma_{\text{meq}}(r) \sim 1/r^{1/2}$ over convective geothermal systems indicates that flow-structure generation of low-level ambient seismicity validated by shale formation stimulation data processing has the potential to accurately locate high-volume flow structures within convective flow systems. Significantly enhancing production well drilling precision significantly reduces geothermal field development and operating costs.

REFERENCES
Leary et al.


Appendix – Matlab two-point correlation function

% [ corrfun r rw] = twopointcorr(x,y,dr) Developed by Ilya Valmianski  email: ivalmian@ucsd.edu
% Computes the two-point correlation function over a 2D lattice of a fixed width and height
% x - list of x coordinates of points   y - list of y coordinates of points   dr - binning distance for successive circles
% corrfun – two-point correlation function   r – radial values for corrfun   rw - number of particles for particular r value
% Low rw means corrfun is unreliable at that r.
% function [ corrfun r rw] = twopointcorr(x,y,dr)
   x = reshape(squeeze(x),[length(x) 1]); y = reshape(squeeze(y),[length(y) 1]);
   % real height/width; number particles; largest radius
   width = max(x)-min(x); height = max(y)-min(y); totalPart = length(x); maxR = sqrt((width/2)^2 + (height/2)^2);
   % r bins, area bins   default blksize = 1000
   r = dr:maxR; av_dens = totalPart/width/height; rareas = ((2*pi*r*dr)*av_dens); blksize = 1000;
   corrfun = r*0; rw = r*0; % allocate space for corrfun/rw
   numsteps = ceil(totalPart / blksize); % number of steps to be considered
   for j = 1:numsteps         % loop through all particles, compute correlation function
       indi = (j-1)*blksize+1; indf = min(totalPart,j*blksize);
       [corrfunArr rwArr] = arrayfun(@ (xj,yj) onePartCorr(xj,yj,x,y,r,rareas),x(indi:indf),y(indi:indf),UniformOutput,false);
       rw = rw + sum(cell2mat(rwArr),1); corrfun =  corrfun + sum(cell2mat(corrfunArr),1);
   end
   corrfun = corrfun ./rw;corrfun = corrfun(rw~=0); r = r(rw~=0);rw = rw(rw~=0); % truncate values with no contributions
% function [corrfun rw] = onePartCorr(xj,yj,x,y,r,rareas) compute radii in (xj,yj)-centered coordinates
   rho=hypot(x-xj,y-yj);rho=logical(rho); maxRho = min([max(x-xj),min(x),max(y-yj),min(y)]); % compute maximum unbiased rho
   rho=rho(rho<maxRho); % truncate to highest unbiased rho
   rw=r*0;rw(r>rmaxRho)=1; % indicate for which r-values correlation function is computed
   count=hister(rho,[inf r]); count=count(2:end); % compute count with correct binning
   corrfun = count./rareas; % normalize density
end