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The Onset of Convection in Faulted Aquifers

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ABSTRACT

In any geological formation, faults play an important role as they can range from impermeable seals to fluid pathways. At geological time scales, faults can exhibit steady creep and basins have been known to deform quite considerably. The study of fault mechanics has been widely studied and has been recently shown to play a role in affecting the behavior of fluid dynamics, namely the geometry of hydrothermal convection cells. Whilst the onset of hydrothermal convection has been extensively studied for more than a decade, little is known about the effects of mechanical deformation of faults (expressed as shear heating) on hydrothermal convection. With the need to solve for tightly coupled thermo-hydro-mechanics, we investigate these effects using a new open-source numerical multiphysics simulator, REDBACK. This simulator is specifically designed to study material instabilities in a tightly coupled manner. Using a linear stability analysis and the implementation of a pseudo-arclength continuation method, it provides the ability to compute the values of critical parameters that predict the onset of convection. This study presents the effect of shear heating on the onset of convection.

1. INTRODUCTION

Hydrothermal convection is significant role in the geothermal sector, both for heating and cooling purposes. The theory of heat flow through porous media has been well documented since the last century but was limited by then to laboratory experiments and simplified mathematical equations. With the advancement of technology and computational power, we are now able to simulate coupled processes (e.g. THMC: thermal-hydro-mechanical-chemical) and complex geometry to gain better understanding of more realistic problems.

Aquifers are characterized by faults, which may display static (locked), quasi-static (creeping) and dynamic (seismic slip) behavior. During these responses, faults may generate heat due to resistance to motion (e.g. friction) contributing the energy budget of a reservoir, an effect usually referred to as shear heating. Traditionally, the energy feedback of faults in hydrothermal convection has been considered negligible, mainly due to the fact that the study of mechanics and heat transfer in porous media are separate fields. This study aims to understand the effect of the heat generated by mechanical deformation in hydrothermal environments.

2. METHODOLOGY

2.1 Theory and Background

The problem of hydrothermal convection is traditionally characterized by the Rayleigh number, used to predict the onset of convection in porous media. The Rayleigh number comprises coefficients of volume expansion, kinematic viscosity, thermal conductivity, boundary conditions, and the depth of the layer (Rayleigh, 1916). Convection is predicted to occur past a critical Rayleigh number, Ra_c. There are varying definitions of the Rayleigh number specific to the problem at hand; as such one definition of the Rayleigh number for convection in porous medium can be:

$$Ra = \frac{\kappa g \Delta T \rho_0^2 c_f H \alpha}{\mu \lambda} \tag{2.1.1}$$

where κ is the permeability, g the gravitational constant, ΔT the temperature difference of the top and bottom boundaries of the model, ρ_0 the reference fluid density, c_f the specific heat capacity of the fluid, H the thickness of the model, α the thermal expansion coefficient of the pore fluid, μ the viscosity of the fluid, and λ the thermal conductivity of the fluid-saturated medium. Different definitions and values of critical Rayleigh numbers arise from varying initial and boundary conditions of the problem at hand. Nield and Bejan (2013) provide a comprehensive review of various boundary conditions with corresponding Rayleigh numbers. Conventionally, two assumptions applied are the Oberbeck-Boussinesq approximation for density differences, and Darcy's Law. Following an example scenario from Nield and Bejan (2013), an example of the system of equations used to solve the onset of convection in porous media heated from below can be:

$$\nabla \cdot v = 0 \tag{2.1.2}$$

$$c_a\rho_0\frac{\partial v}{\partial t}=-\nabla P-\frac{\mu}{\kappa_\pi}v+\rho_f\boldsymbol{g} \tag{2.1.3}$$

$$(\rho c)_m\frac{\partial T}{\partial t}+\left(\rho c_p\right)_f\mathbf{v}\cdot\nabla\mathrm{T}=k_m\nabla^2T \tag{2.1.4}$$

$$\rho_f=\rho_0[1+\beta(p-p_0)-\lambda(T-T_0)] \tag{2.1.5}$$

where **v** is the seepage velocity, P the pressure, μ the dynamic viscosity, κ_π the permeability, c the specific heat, k_m the overvall thermal conductivity, β the compressibility coefficient (traditionally assumed zero for incompressible fluid), and λ the thermal volume expansion coefficient.

In this study, the formulation of equations differs from the conventional approach described above, but the physical problem is equivalent. The consideration of a deformable porous medium results in some additional terms (e.g. compressibility of the fluid) in the final system of equations.

In this particular case of observing the onset of convection in deformable media, the system of equations solved includes the mass, energy, and momentum balance laws, as well as Fourier's law for heat diffusion. We assume a Representative Elementary Volume (REV) based on a continuum mechanics approach, which consists of a solid skeleton with fully saturated pore space. The system of equations solved is:

$$0=\partial_i\sigma'_{ij}-\partial_i p_f+\bar{\rho}g_i \tag{2.1.6}$$

$$0=\bar{\beta}\frac{\partial p_f}{\partial_t}-\bar{\lambda}\frac{\partial T}{\partial t}+\left[(1-\phi)\beta_s v_k^{(s)}+\phi\beta_f v_k^{(f)}\right]\frac{\partial p_f}{\partial x_k}-\left[(1-\phi)\lambda_s v_k^{(s)}+\phi\lambda_f v_k^{(f)}\right]\frac{\partial T}{\partial x_k}+\frac{\partial\left(\phi\left(v_k^{(f)}-v_k^{(s)}\right)\right)}{\partial x_k}+\frac{\partial v_k^{(s)}}{\partial x_k} \tag{2.1.7}$$

$$0=\left(\rho C_p\right)_m\frac{D^{(m)}T}{Dt}-\kappa\partial_{ii}^2T-\chi\sigma_{ij}\cdot\dot{\epsilon}_{ij}^{vp} \tag{2.1.8}$$

where σ'_{ij} is the effective stress tensor, p_f the pore fluid pressure, g_i the acceleration of gravity, $\bar{\beta}$ the mixture's compressibility coefficient $\bar{\beta}=\left[(1-\phi)\beta_s v_i^{(s)}+\phi\beta_f v_i^{(f)}\right]$, $\bar{\lambda}$ the mixture's thermal expansion coefficient $\bar{\lambda}=\left[(1-\phi)\lambda_s v_i^{(s)}+\phi\lambda_f v_i^{(f)}\right]$, ϕ the porosity, β_s compressibility of the solid, β_f compressibility of the fluid, $v_k^{(s)}$ the velocity of the solid, $v_k^{(f)}$ the velocity of the fluid, λ_s the thermal expansion coefficient of the solid, λ_f the thermal expansion coefficient of the fluid, $\frac{D^{(m)}T}{Dt}$ the material time derivative with respect to the velocity of the mixture $v_i^{(m)}=\left[(1-\phi)\rho_s v_i^{(s)}+\phi\rho_f v_i^{(f)}\right]\bar{\rho}^{-1}$, κ the thermal conductivity, χ the Taylor-Quinney coefficient, and $\dot{\epsilon}_{ij}^{vp}$ the reference strain rate.

In multiphysics modelling, dimensionless formulations ensure that the changes in every parameter can be analyzed regardless of the natural length scale. The normalization of the following parameters is as follows:

$$T^*=\frac{T-T_{ref}}{T_{ref}} \tag{2.1.9}$$

$$x^*=\frac{x}{x_{ref}} \tag{2.1.10}$$

$$t^*=\frac{c_{th}}{x_{ref}^2}t \tag{2.1.11}$$

$$v^*=\frac{x_{ref}}{c_{th}}v \tag{2.1.12}$$

where T^*, x^*, t^*, v^* are normalized variables of temperature, space, time, and velocity respectively, T_{ref}, x_{ref} are the reference temperature and space, and c_{th} the thermal diffusivity. The normalized Darcy's Law is expressed as:

$$\phi\left(V_k^{*(f)}-V_k^{*(s)}\right)=-\frac{\kappa_\pi}{\mu_f}\frac{\sigma_{ref}}{x_{ref}V_{ref}}\left(\frac{\partial p^*}{\partial x_k^*}-\frac{x_{ref}}{\sigma_{ref}}\rho_f g_k\right) \tag{2.1.13}$$

The final system of dimensionless equations, in this case of convection in porous deformable media are:

$$0=\partial_i\sigma'_{ij}-\partial_i p_f+\bar{\rho}g_i \tag{2.1.14}$$

$$0=\partial_t p_f+v_i^{(p)}\partial_i p_f-v_i^{(T)}\partial_i T-\partial_i\left[\frac{1}{Le}\left(\partial_i p_f-\rho_f g_k\right)\right]-\Lambda\partial_t T+\frac{\dot{\epsilon_V}}{\beta^*} \tag{2.1.15}$$

$$0 = \partial_t T + \bar{v}_i \partial_i T - \partial_{ii}^2 T - Gr\sigma_{ij}\dot{\epsilon}_{ij}^{vp} \tag{2.1.16}$$

where Le is the Lewis number, defined as $Le = \frac{c_{th}}{c_{hy}} = \frac{\mu_f c_{th} \beta_m^*}{\kappa_\pi \sigma_{ref}}$, μ_f the fluid viscosity, c_{th} the thermal diffusivity of the solid-fluid mixture, β_m^* the normalized compressibility of the medium, and κ_π the permeability. Λ is the thermal pressurization coefficient $\frac{\bar{\lambda}}{\beta}\frac{T_{ref}}{\sigma T_{ref}}$, and Gr the Gruntfest number $\frac{\chi \sigma_{ref} \epsilon_{ref} \dot{x}_{ref}^2}{\alpha T_{ref}}$ where σ is the thermal conductivity. For the normalized velocities (stars are dropped for convenience), $v_i^{(p)} = (1-\phi)\frac{\beta_s}{\bar{\beta}} v_i^s + \phi \frac{\beta_f}{\bar{\beta}} v_i^f$, and $v_i^{(T)} = (1-\phi)\frac{\bar{\lambda}_s}{\bar{\beta}} v_i^s + \phi \frac{\lambda_f}{\bar{\beta}} v_i^f$, $\lambda_\alpha T_{ref} = \lambda_\alpha^*$, for $\alpha = s, f$, and $\beta_\alpha T_{ref} = \beta_\alpha^*$, for $\alpha = s, f, m$, where s, f, m are the solid, fluid, and mixture respectively.

2.2 Numerical Implementation

The numerical tool used for this study is REDBACK, an open source, strongly coupled geomechanics simulator. REDBACK is based on MOOSE, a highly scalable finite-element multiphysics framework (Gaston et al., 2009). REDBACK has the capability to solve full mechanics and has been used to investigate pore collapse (Poulet and Veveakis, 2016) and the oscillatory behavior of faults (Poulet et al., 2016).

The onset of convection in porous incompressible media is typically determined by the Rayleigh number (Eq. 2.1.1) with permeability being the governing parameter, as it can vary up to several orders of magnitude in a system. With the consideration of a deformable porous medium however, the permeability parameter is given in the definition of the Lewis number, which expresses the ratio of thermal to mass diffusivities. The relationship of the Lewis and Rayleigh numbers is then inversely proportional to each other:

$$Le \propto \frac{1}{Ra} \tag{2.2.1}$$

Hence, a system which is predicted to convect above a critical Rayleigh number will correspond to a value below a critical Lewis number. The final system of equations solved in this case of the onset of convection in non-deformable porous media are:

$$0 = \partial_t p_f + v_i^p \partial_i p_f - v_i^T \partial_i T - \partial_i \left[\frac{1}{Le}(\partial_i p_f - \rho_f g_i)\right] \tag{2.2.2}$$

$$0 = \partial_t T + \bar{v}_i \partial_i T - \partial_{ii}^2 T \tag{2.2.3}$$

REDBACK is capable of handling full mechanical deformation, as shown in Poulet et al. (2016). In the case of a creeping fault in an aquifer, the energy produced from the creep is expected to impact the overall heat transfer equilibrium. The energy can be conveyed by the Gruntfest number, which is the ratio of the heat generated by mechanical work to the heat diffused in the system. Hence, the final system of equations solved to investigate the impact of shear heating on the onset of convection are:

$$0 = \partial_t p_f + v_i^p \partial_i p_f - v_i^T \partial_i T - \partial_i \left[\frac{1}{Le}(\partial_i p_f - \rho_f g_i)\right] \tag{2.2.4}$$

$$0 = \partial_t T + \bar{v}_i \partial_i T - \partial_{ii}^2 T - Gr\sigma_{ij}\dot{\epsilon}_{ij}^{vp} \tag{2.2.5}$$

Although the capabilities of numerical modelling are vast, results about the onset of convection can be inconclusive especially in systems that are close to criticality. Elder (1967) observed that it takes an extended amount of time to observe the onset of convection when a system is close to criticality. This was recently acknowledged by Reid (2013) that running a sensitivity analysis of a system close to criticality results in expensive computational time and unsubstantial conclusions. Hence, a linear stability analysis is better suited to determine if convection does or does not occur in a system. Early studies performed by Bories (1987) show the evolution of the stability curve for the case of $Ra_c = 4\pi^2$. For stability analyses, the horizontal and vertical axes can represent any parameter. Commonly for convection, the vertical and horizontal axes contain the Nusselt and Rayleigh number respectively. In the case of this study, a pseudo-arclength continuation method is used (Keller, 1977), and the Nusselt number is plotted on the vertical axis while the Lewis number is plotted on the horizontal axis. The Nusselt number expresses the ratio of convection to diffusion and the definition can vary depending on the problem at hand, however in this case it is defined to be:

$$Nu = 1 + \frac{max\|\nabla T_{conv}\|_\Omega}{max\|\nabla T_{diff}\|_\Omega} \tag{2.2.6}$$

where $max\|\nabla T_{conv}\|_\Omega$ is the maximum of the norm of convection over the entire domain, and $max\|\nabla T_{diff}\|_\Omega$ is the maximum of the norm of diffusion over the entire domain. When *Nu*=1 conduction prevails, whereas when *Nu* > 1 convection kicks in. This definition ensures that even in the case of purely diffusive processes with the addition of shear heating, the proper ratio of convection to diffusion can be obtained.

3. RESULTS

3.1 Convection in Porous Media

In this contribution we detail the response of the system of equations (2.2.4) and (2.2.5), where the critical number encompassing permeability is the Lewis number, *Le*. A second dimensionless parameter, the Gruntfest number *Gr*, characterizes the effect of mechanical deformation in terms of shear heating, with the case $Gr = 0$ being the classical hydro-thermal convection in porous media.

We showcase the response of the system equations in a faulted reservoir. The geometry used consists of a 2D block with a dipping layer, representing a fault cross-cutting country rock. The material properties for all blocks are equivalent, with the only differing parameter being the Lewis number. The Lewis numbers for the units above and below the fault are equal and have values past the critical Lewis number (i.e. convecting), whilst the fault has a non-convective Lewis value. A transient simulation was run to identify the critical Lewis number at which convection occurs; particularly $Le = 4.8 \times 10^{-8}$. In this case, the boundary conditions for pressure are a zero-flux (Neumann BC) on the bottom of the model, and fixed (Dirichlet BC) on the top of the model. The temperature boundary conditions are fixed (Dirichlet BC) on both the top and bottom of the model. This simulation is not subject to any mechanical deformation ($Gr = 0$). Figure 3.1.1 shows the result of steady state convection obtained from a transient simulation for this case.

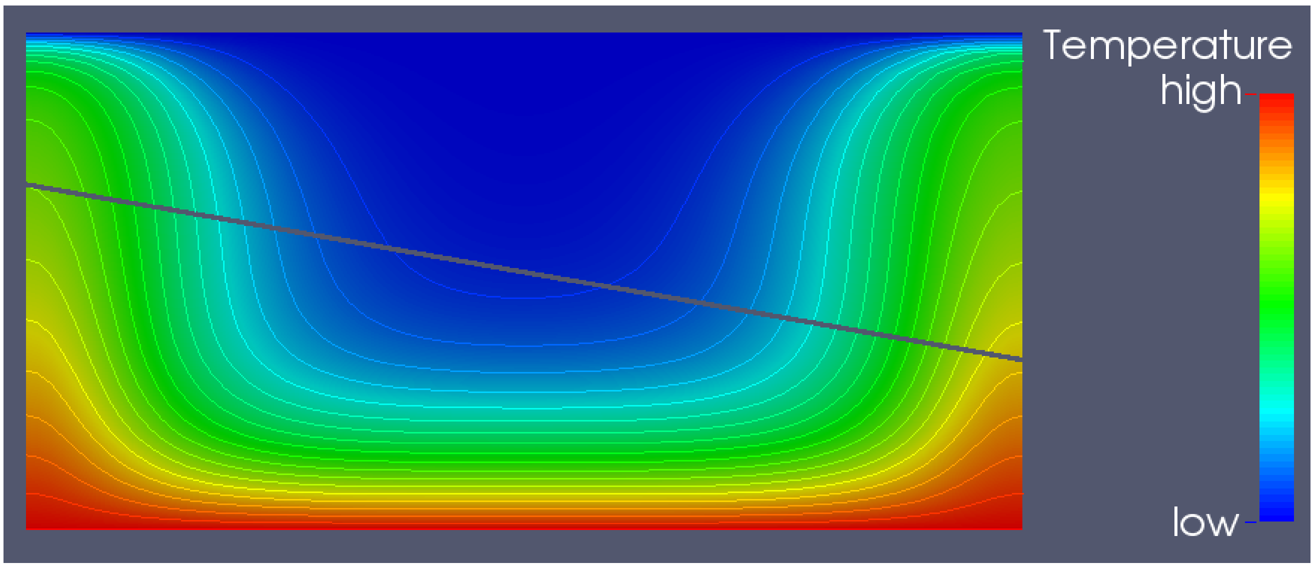

Figure 3.1.1: Steady state convection observed from a transient simulation.

3.2 Convection in Porous Media with Mechanics

This section describes the parameters used to investigate the effect of shear heating on the onset of convection. Based on the previous simulation of convection in non-deformable media, this case was run with the same parameters with the addition of the shear heating term, particularly $Gr > 0$.

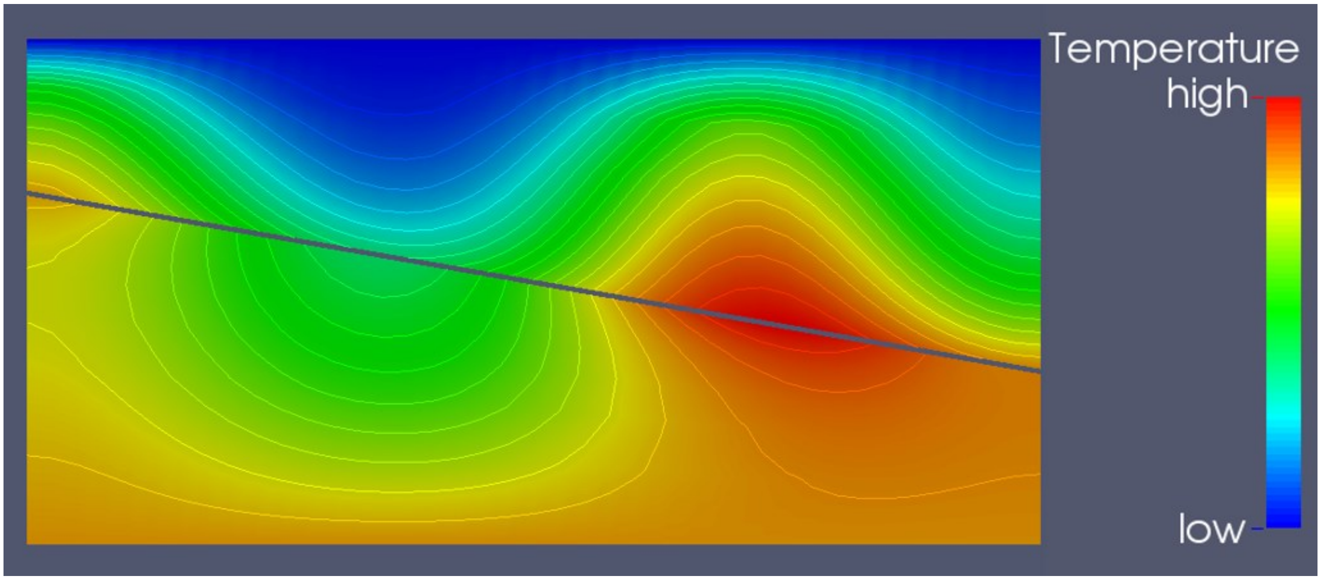

Figure 3.2.1: The effect of shear heating on the onset of convection, observed from a transient simulation.

It can be observed in Figure 3.2.1 that even with the same material properties and boundary conditions for both temperature and pressure, there is a significant change in the convection patterns observed from a transient simulation. The additional heat produced from the creeping fault affects the convection patterns; instead of having uniformly convecting cells, there are two peaks at irregular spacing observed on the upper block, and temperature does not diffuse uniformly on the lower block. The point of interest is that even though the permeability of the fault is lower than the critical value required for convection, convection occurs due to the extra heat generated from the steady creep.

3.3 Bifurcation Analysis

To evaluate and quantify the role of shear heating in the onset of convection, we perform a numerical bifurcation analysis based on an arc-length continuation method (Keller, 1977) of the steady state of our system of equations. The bifurcation analysis is performed with respect to the Lewis number, for various values of Gruntfest.

A preliminary deformation simulation was initially performed in a 3D block experiencing constant shear along a fault. After a given time in the transient simulation, the stresses in the fault become constant which correspond to a constant Gruntfest number. As such, the displacement is not solved; rather the heat generated from the deformation is taken into account. The stability analysis is run using a steady state simulation solving the following system of equations:

$$0 = v_i^p \partial_i p_f - v_i^T \partial_i T - \partial_i \left[\frac{1}{Le} (\partial_i p_f - \rho_f g_i) \right] \tag{3.3.1}$$

$$0 = \overline{v_i} \partial_i T - \partial_{ii}^2 T - Gr \sigma_{ij} \dot{\epsilon}_{ij}^{vp} \tag{3.3.2}$$

In Figure 3.3.1, it is observed that as Gruntfest departs from zero, the shear heating contributes to the temperature equation and the critical value of the Lewis number (respectively Rayleigh number) increases (respectively decreases). In such a scenario, convection is promoted even for lower values of permeability than traditionally calculated, because of the additional heat contribution of the fault with shear heating. This means that faulted settings which are considered stable and non-convective in the traditional sense, may be in reality convecting settings due to the presence of active faults. This result is of major importance in real geological settings, in which faults are more often than not in abundance.

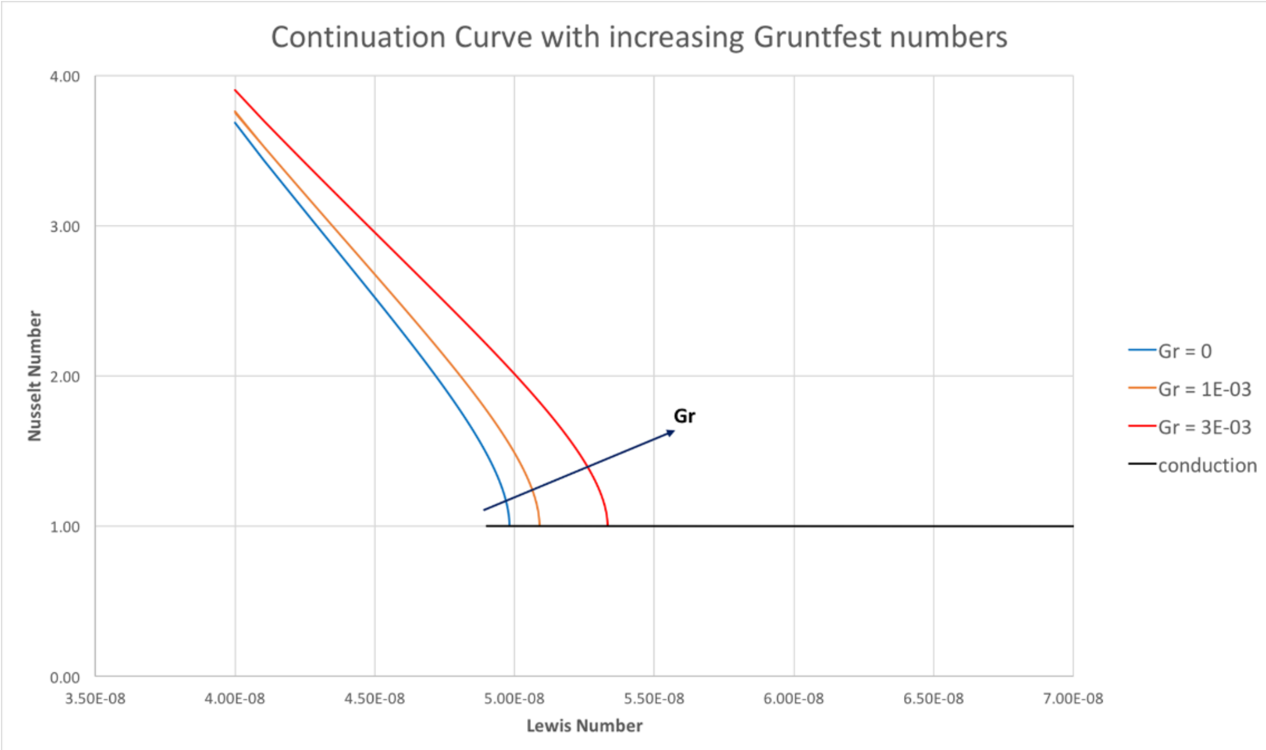

Figure 3.3.1: Continuation curves increasing only Gruntfest numbers in identical simulations. The critical value at which convection is predicted to occur decreases as more heat (higher values for shear heating) is present in the system.

DISCUSSION AND CONCLUSION

The onset of convection in porous media has been intensively studied, yet there is little understanding of the effect of creeping faults in aquifers. Faults are present in any geological reservoir, and may exhibit constant creep in geological timescales. The energy generated from this creep is also known as shear heating, and is expected to impact the overall heat-transfer equilibrium of the system. This study aims to investigate the effect of shear heating on the onset of convection, using REDBACK, a geomechanics simulator.

The mathematical formulation of REDBACK is mechanics-based, so the derivation of equations are different to conventional methods. In this case of predicting the onset of convection, the Lewis number is used instead of the traditional Rayleigh number, and convection is predicted to occur below a critical Lewis number, Le_c.

The most important observation of this study was that due to the effects of shear heating in deforming media, the permeability predicted for convection to occur is less than traditional values where non-deformable media are considered. This may be significant in areas of constant seismic activity where convection may not be expected to occur, due to faults either being considered an impermeable barrier, or having a permeability lower than the predicted critical value.

It was also observed that an actively creeping fault may change convection patterns; the heat generated from the fault causes convection cells to move along the fault. This finding could have significant implications for industrial applications; compared to simulations for non-deformable media, the location of the temperature peak may differ when shear heating is considered.

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