A General Scaling Method for Spontaneous Imbibition
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Abstract
Scaling the experimental data of spontaneous imbibition without serious limitations has been difficult. To this end, a general approach was developed to scale the experimental data of spontaneous imbibition for most systems (gas-liquid-rock and oil-water-rock systems) in both cocurrent and countercurrent cases. We defined a dimensionless time with almost all the parameters considered. These include porosity, permeability, size, shape, boundary conditions, wetting and nonwetting phase relative permeabilities, interfacial tension, wettability, and gravity. The definition of the dimensionless time was not empirical, instead, was based on theoretical analysis of the fluid flow mechanisms that govern spontaneous imbibition. The general scaling method was confirmed against the experimental data from spontaneous water imbibition conducted at different interfacial tensions in oil-saturated rocks with different sizes and permeabilities. A general analytical solution to the relationship between recovery and imbibition time for linear spontaneous imbibition was derived. The analytical solution predicts a linear correlation between the imbibition rate and the reciprocal of the recovery by spontaneous imbibition in most fluid-fluid-rock systems.

Introduction
An important fluid flow phenomenon during water injection or aquifer invasion into reservoirs is spontaneous water imbibition. Scaling the experimental data of spontaneous water imbibition in different fluid-fluid-rock systems is of essential importance to designing the water injection projects and predicting the reservoir production performances. Ignoring the effects of relative permeability, capillary pressure, and gravity in the dimensionless time might be the reason that the existing scaling methods do not always function successfully. It is known that these parameters influence the spontaneous imbibition in porous media significantly. For that reason, these parameters should be honored properly in the scaling.

Many papers have been published to characterize and scale spontaneous water imbibition in both oil-water-rock systems\(^1\)-\(^6\) and gas-liquid-rock systems\(^21\)-\(^24\). However, few have included the effects of capillary pressure, relative permeability (both wetting and nonwetting phases), wettability, and gravity simultaneously. This is important because all the parameters may play an important role in many cases and may not be ignored. For example, a lot of enhanced/improved oil recovery processes relate to low interfacial tension (IFT). In these cases, capillary pressure as a driving force may be small and gravity may not be neglected. In some cases, gravity may also be a driving force as pointed out by Schechter et al.\(^25\).

One of the frequently-used dimensionless time groups in the past to scale spontaneous imbibition data is defined as follows (Ma et al.\(^15\)):

\[
t_D = \frac{k \sigma}{\phi \mu m L_a^2 t}
\]  

where \(t_D\) is the dimensionless time, \(k\) is the rock permeability, \(\phi\) is the porosity, \(\sigma\) is the interfacial tension between the wetting and nonwetting phases, \(t\) is the imbibition time, \(\mu_m\) is the geometric mean of the viscosities of the two phases and \(L_a\) the characteristic length.

The dimensionless time defined in Eq. 1 is suitable for oil-water-rock systems under specific conditions. These include: (1) wettability must be the same, (2) relative permeability functions must be identical, (3) capillary pressure functions must be identically proportional to interfacial tension, (4) initial fluid distributions must be duplicated, and (5) gravity must be neglected. On the other hand, Eq. 1 implies that higher IFT systems have higher imbibition rate, which is not true in many cases. For example, Schechter et al.\(^25\) observed experimentally that the imbibition rate in oil-water-rock systems with low IFT was greater than with high IFT in high permeability core samples. Al-Lawati and Saley\(^18\) reported a similar phenomenon.

Zhang et al.\(^17\) pointed out that Eq. 1 could not scale the experimental data in gas-liquid-rock systems. To relax the
limitations, we developed a method\textsuperscript{22} to scale the cocurrent spontaneous water imbibition into gas-saturated rocks, which was based on a spontaneous imbibition model\textsuperscript{21}. The effects of relative permeability, surface tension, wettability, and gravity were considered in this method for gas-liquid-rock systems. The dimensionless time is expressed as follows:

\[ t_{d1} = c^2 \frac{kk_{ro}^* P_e^* S_{of} - S_{wi}}{\mu_e \phi L_a^2} - t \]  \hspace{1cm} (2)

where \( c \) is the ratio of the gravity force to the capillary force, \( \mu_e \) is the viscosity of the liquid (wetting) phase, \( S_{of} \) is the water (or the wetting phase) saturation behind the imbibition front, \( S_{wi} \) is the initial water saturation in the core sample, \( k_{ro}^* \) is the water phase relative permeability at \( S_{of} \), \( P_e^* \) is the capillary pressure at \( S_{of} \), and \( t_{d1} \) is the dimensionless time appropriate to gas-liquid-rock systems.

However this method can not be applied to other systems such as oil-water-rock systems because the mobility of the nonwetting phase was assumed infinite. To solve this problem, we developed another scaling method for oil-water-rock systems in which relative permeabilities of both wetting and nonwetting phases were considered. The dimensionless time in this case is expressed as follows:

\[ t_{d2} = \frac{kk_{ro}^* P_e^* S_{of} - S_{wi}}{\mu_e \phi L_a^2} - t \]  \hspace{1cm} (3)

where \( k_{ro}^* \) is the relative permeability pseudofunction associated with \( k_{ro}^* \) (the oil phase relative permeability at \( S_{of} \)) and \( k_{ro}^* \), \( \mu_e \) is the effective viscosity of oil and water phases, and \( t_{d2} \) is the dimensionless time suitable for oil-water-rock systems in which gravity is negligible.

As seen in Eq. 3, gravity was neglected in this method. However gravity may play an important role in some cases, as stated previously. Such cases include low interfacial tension between oil and water\textsuperscript{18, 25}, weak water wetness, and high rock permeability. This will be discussed later in more detail.

In this paper, we focused on the development of a general scaling approach for spontaneous imbibition by relaxing all the limitations of Eqs. 1-3 and honoring almost all the parameters involved.

**Theory**

A general scaling approach for spontaneous imbibition was derived as shown in Appendix A in which a dimensionless time and a normalized recovery were defined. The dimensionless time is expressed as follows:

\[ t_d = c^2 \frac{kk_{ro}^* P_e^* S_{of} - S_{wi}}{\phi \mu_e \mu_c L_a^2} - t \]  \hspace{1cm} (4)

where \( t_d \) is the dimensionless time, \( kk_{ro}^* \) is the effective mobility of the two phases, \( M_e^* \), which can be calculated from the experimental data of spontaneous imbibition.

The dimensionless time defined in Eq. 4 is similar to that defined in Eq. 2. Note that the relative permeability of the nonwetting phase is included in Eq. 4 but not in Eq. 2.

We can see from Eq. 4 that almost all the parameters that govern spontaneous imbibition in porous media are honored in the definition of the dimensionless time.

The normalized recovery used in the general scaling approach is defined as follows:

\[ R' = cR \]  \hspace{1cm} (5)

Here, \( R' \) is the normalized recovery and \( R \) is the recovery in the units of pore volume.

Also shown in Appendix A is a linear correlation between the imbibition rate and the reciprocal of the recovery by spontaneous imbibition in most fluid-fluid-rock systems:

\[ q_w = a_0 \frac{1}{R} - b_0 \]  \hspace{1cm} (6)

where \( q_w \) is the volumetric imbibition rate, \( a_0 \) and \( b_0 \) are two constants associated with capillary and gravity forces respectively. Eq. 6 is similar to the linear model developed by Li and Horne\textsuperscript{21} for gas-liquid-rock systems but the expressions for the two constants here are more complicated.

The relationship between the normalized recovery and dimensionless time can be obtained to solve Eq. A-16, which is expressed as follows:

\[ (1 - R^*)e^{R^*} = e^{-td} \]  \hspace{1cm} (7)

We can see from Eq. 7 that \( R' \) is only a function of the newly defined dimensionless time (see Eq. 4). This feature shows that experimental data from spontaneous imbibition in rocks with different size, porosity, permeability, initial fluid saturation, relative permeability, interfacial tension, and wettability can be scaled to a single curve of \( R' \) vs. \( t_d \). We will verify the theoretical prediction later using the data from experiments of spontaneous imbibition in oil-water-rock systems conducted by Schechter \textit{et al}.'s\textsuperscript{25}.

Theoretically Eqs. 4 and 6 are suitable for both cocurrent and countercurrent spontaneous imbibition. The difference between the two is the calculation of the effective mobility as
well as the characteristic length. Zhang et al.\textsuperscript{17} discussed the calculation of the characteristic length in detail. We only discuss the calculation of the effective mobility here.

For cocurrent spontaneous imbibition, the effective mobility is calculated as follows:

\[ M'_e = \frac{k k'_{ce}}{\mu_e} = \frac{M'_w M'_{nw}}{M'_w + M'_{nw}} \tag{8} \]

where \( M'_w \) and \( M'_{nw} \) are the wetting and nonwetting phase mobilities at \( S_{nw} \).

For countercurrent spontaneous imbibition, the expression of the effective mobility is represented as follows:

\[ M'_e = \frac{M'_w M'_{nw}}{M'_w + M'_{nw}} \tag{9} \]

The procedure to scale the experimental data of the spontaneous imbibition using the new method is described briefly in the following. The calculated imbibition rate is first plotted vs. the reciprocal of the recovery (the amount of the wetting phase imbibed into the core in terms of pore volume). A straight line is expected from which the values of the two constants, \( a_0 \) and \( b_0 \), could be obtained from a linear regression analysis. The effective mobility, \( M'_e \), and the capillary pressure, \( P'_c \), at \( S_{nw} \) could also be calculated once the values of \( a_0 \) and \( b_0 \) are available. Therefore the dimensionless time defined in Eq. 4 and the normalized recovery defined in Eq. 5 could be computed. Finally, the normalized recovery is plotted vs. the new dimensionless time. According to Eq. 7, the experimental data of spontaneous imbibition in different rocks with different specific properties is expected to correlate in the form of the normalized recovery vs. the new dimensionless time.

In gas-liquid-rock systems, the mobility of the nonwetting phase (gas) is usually much greater than that of the wetting phase. Hence \( M'_e \) is almost equal to \( M'_w \). In this case, the general scaling model (Eqs. 4 and 5) developed in this study could be reduced to the model that we developed\textsuperscript{22} previously. All the other equations are also the same as in gas-liquid-rock systems.

In some oil-water-rock systems in which gravity is negligible, the value of \( b_0 \) is equal to zero. In this case, the general scaling model represented by Eq. 4 could be reduced to Eq. 3\textsuperscript{20}. The derivation to achieve this has been presented by Li and Horne\textsuperscript{20}.

The general dimensionless time defined in Eq. 4 could also reduce to the definition by Mattax and Kyte\textsuperscript{10} if the effects of relative permeability, wettability, initial fluid saturation, and gravity are neglected (only permeability is considered). This could be derived by assuming that capillary pressure is directly proportional to interfacial tension and inversely proportional to \( \sqrt{k / \phi} \).

**Results**

The general scaling approach (Eqs. 4 and 5) and the analytical solution (Eq. 6) proposed in this study can be reduced in cases in which gravity is negligible or one phase is gas, as stated previously. The simplified forms of the general scaling approach in the two cases were verified previously by Li and Horne\textsuperscript{20, 22}. To confirm the models in oil-water-rock systems in which gravity may not be negligible, the experimental data of spontaneous water imbibition (with equilibrated fluids) by Schechter et al.\textsuperscript{25} were used and scaled using different approaches. Schechter et al.\textsuperscript{25} conducted spontaneous water imbibition in rocks with different permeability. In each core sample, experiments were conducted at three different interfacial tensions ranging from 0.10 to 38.01 mN/m. Gravity may play an important role in cases of low IFT and high permeability. Therefore gravity may not be neglected.

**Scaling in a 15 md Indiana Limestone Core**

The experimental data\textsuperscript{25} of spontaneous water imbibition conducted at three different interfacial tensions in an Indiana limestone core with a permeability of 15 md are depicted in Fig. 1. The water imbibition rate increased with the interfacial tension in a certain period of time, which is consistent with Eq. 1 in the low permeability core sample.

Fig. 2 shows the scaling results obtained by using the existing, frequently-used dimensionless time defined in Eq. 1 for the spontaneous water imbibition data at three different interfacial tensions (see Fig. 1). The units of the oil recovery in Fig. 2 (the same for the rest of the paper) are oil originally in place (OOIP). It is supposed that all the experimental data points obtained at different IFT should sit close to a single curve if the dimensionless time is appropriate for scaling. However Fig. 2 demonstrates that the experimental data points scatter significantly, which shows that the existing dimensionless time defined in Eq. 1 failed to scale the experimental data obtained at the three different interfacial tensions.

Note that the experimental data measured at 1.07 and 38.1 mN/m could be scaled using the dimensionless time defined in Eq. 1 but those measured at 0.10 mN/m could not, as shown in Fig. 2. The reason is stated in the following. The further decrease in IFT from 1.07 to 0.10 mN/m might result in significant increase in relative permeability of oil and water. However the effect of relative permeability on spontaneous water imbibition is not considered in Eq. 1. On the other hand, gravity might become an important factor because of the significant decrease in capillary pressure caused by the further lowering of IFT. Gravity is not included either in the dimensionless time defined in Eq. 1.

Capillary pressure is expected to further decrease in rock with higher permeabilities. Considering this and the explanation above, we foresee that the experimental data in core samples with a permeability much higher than 15 md will
not be scaled with the existing dimensionless time (see Eq. 1) for $IFT=1.0$ and $38.1 \, \text{mN/m}$. The prediction will be examined using the experimental data in 100 and 500 md core samples respectively, which will be discussed in the next section.

To use the general scaling approach proposed in this study, it is necessary to plot the imbibition rate vs. the reciprocal of the oil recovery. The values of $a_0$ and $b_0$ required to calculate the new dimensionless time and the normalized oil recovery can be estimated from such plots. We plotted the experimental data for each IFT because of the very different scale of the imbibition rates at different IFT. Figs. 3a-3c show that the relationships between the imbibition rate and the reciprocal of the oil recovery by spontaneous water imbibition are linear, as foreseen by Eq. 6.

One interesting feature in Figs. 3a-3c is that the straight lines do not go through the origin point, as suggested by Eq. 6. This feature demonstrates the significant effect of gravity on spontaneous water imbibition. If there is no effect of gravity, the value of $b_0$ is equal to zero.

Using the scaling approach developed in this study (see Eqs. 4 and 5) and the parameters obtained from Fig. 3, the same experimental data presented in Fig. 2 are plotted as the normalized oil recovery versus the new dimensionless time and the results are shown in Fig. 4. All the experimental data of the spontaneous water imbibition at three different interfacial tensions sit near a single curve. We can see from Fig. 4 that the proposed scaling approach works successfully for the spontaneous water imbibition in the low permeability limestone core sample at the three different interfacial tensions.

Scaling in a 100 md Berea Sandstone Core

Using the same fluids, Schechter et al.\textsuperscript{25} conducted spontaneous water imbibition in another core sample with a higher permeability (100 md). The experimental data at three different interfacial tensions are plotted in Fig. 5. As pointed out by Schechter et al.\textsuperscript{25}, the rate behavior shown in Fig. 5 differs sharply from that implied by Eq. 1. In this case, the high IFT brine imbibed more slowly than either the intermediate or the low IFT brine. The observed phenomenon may be explained using the dimensionless time defined in Eq. 4. The decrease in IFT may result in the increase in relative permeabilities. For example, Li and Firoozabadi\textsuperscript{26} demonstrated theoretically that both the wetting and nonwetting phase relative permeabilities increase with the decrease in IFT. Henderson et al.\textsuperscript{27} confirmed the mechanism experimentally. According to Eq. 9, the effective mobility during the process of countercurrent spontaneous water imbibition may increase when the relative permeabilities of the two phases increase. Therefore, the high IFT brine may imbibe more slowly or faster than the low IFT brine in the same rock based on Eq. 4, which depends on the comparison of the relative increment of the effective mobility to the reduction of the capillary pressure.

The scaling results obtained using the existing dimensionless time defined in Eq. 1 are shown in Fig. 6. The experimental data are not scaled at all in this case for all the three different IFT values, as foreseen in the last section. Fig. 6 shows that the existing dimensionless time defined in Eq. 1 is not suitable for such oil-water-rock systems.

The parameters required to conduct scaling with the approach developed in this study were obtained from the relationships between the imbibition rate and the reciprocal of the oil recovery at three different interfacial tensions. The scaling results are shown in Fig. 7. Compared to the results in Fig. 6, we can see that the proposed scaling approach works better than the existing method for the systems studied.

Scaling in a 500 md Berea Sandstone Core

Fig. 8 shows the experimental data\textsuperscript{25} at three different interfacial tensions in a Berea sandstone core with a permeability of about 500 md. The rate behavior shown in Fig. 8 is similar to that in Fig. 5. The high IFT brine imbibed more slowly than the intermediate IFT brine and had the smallest oil recovery.

The values of the dimensionless time defined in Eq. 1 were calculated and the scaling results are shown in Fig. 9. As before, the experimental data can not be scaled for the oil-water-rock systems at all the three different interfacial tensions as foreseen. Gravity might play a greater role in the high permeability core sample than in the low permeability core sample.

The parameters required to calculate the dimensionless time defined in Eq. 4 were estimated from the relationships between the imbibition rate and the reciprocal of the oil recovery at three different interfacial tensions. Then the values of the new dimensionless time and normalized oil recovery were computed. Fig. 10 shows the scaling results. We can see from Fig. 10 that the scaling approach proposed in this study works well for the spontaneous water imbibition in the high permeability Berea sandstone core sample at different interfacial tensions.

Fig. 11 shows the relationships between the oil recovery in the units of OOIP and the dimensionless time proposed in this study (see Eq. 4). We can see that the experimental data can not be scaled when the oil recovery instead of the normalized oil recovery is used. The same phenomenon was observed for the experimental data shown in Figs. 1 and 5.

Considering the results in Figs. 10 and 11, we concluded that it would be necessary to normalize the recovery by spontaneous imbibition to scale the experimental data. Note that the traditional scaling methods only focus on the normalization of the imbibition time.

Scaling in Different Rocks and at Different Interfacial Tensions

The experimental results discussed previously were measured in three core samples with different porosity (15.8%, 18.7%, and 21.3%), different permeability (15, 100, and 500 md), and different size in length (61, 57, and 57 cm). The three core samples were also different in rock type. One was limestone and the other two were Berea sandstone. It will be interesting to scale the experimental data from all the three core samples for all the three different IFT.
All the scaling results shown in Figs. 4, 7, and 10 except for the data at 0.1 mN/m in the 100 md core sample are plotted in Fig. 12. We can see that all the experimental data measured at different interfacial tensions in rocks with different permeabilities and different sizes could be scaled satisfactorily using the approach proposed in this paper. Fig. 12 further proves the validity of the general scaling approach.

Discussion
Theoretically the general scaling approach proposed in this study may be suitable for almost all the fluid-fluid-rock systems in both cocurrent and countercurrent spontaneous imbibition cases. However the approach was only tested in oil-water-rock systems with limited experimental data in countercurrent cases. It would be useful to test the general scaling approach using more experimental data in different cases such as cocurrent spontaneous water imbibition in rocks with initial water saturation.

One of the important features of the proposed scaling approach is that the variables, as a group, required to calculate the dimensionless time can be determined from the spontaneous imbibition tests. It may be interesting to calculate the proposed dimensionless time using relative permeability and capillary pressure data measured using methods other than spontaneous imbibition techniques. That noted we showed previously that the values of capillary pressure at \( S_{wf} \) extracted from the experimental data of spontaneous water imbibition into gas-saturated rock were equal to that measured using a different technique\(^7\). The end-point relative permeability values extracted from spontaneous imbibition tests were equal to those measured using a steady-state method\(^21, 28\).

The general model (Eq. 7) to characterize spontaneous water imbibition may be used to predict the oil production by water injection in fractured reservoirs once rock properties, fluid properties, relative permeability, and capillary pressure data are available. The linear model (Eq. 6) may be deployed to extract permeability, relative permeability and capillary pressure data once oil production history is known.

Conclusions
The following conclusions may be drawn according to our current study:

1. A general scaling approach was developed for cocurrent and countercurrent spontaneous imbibition in almost all fluid-fluid-rock systems.
2. A dimensionless time and a normalized recovery were defined by honoring almost all the parameters that govern spontaneous imbibition to complete the general scaling approach.
3. The proposed scaling approach functions satisfactorily for the spontaneous imbibition in oil-water-rock systems with different permeability and interfacial tension while the frequently-used model failed.
4. The dimensionless time derived in this study could explain why the imbibition rate in systems with high interfacial tension was smaller than that in systems with low interfacial tension.
5. A general analytical solution to spontaneous imbibition was derived. The solution predicts a linear correlation between the imbibition rate and the reciprocal of the recovery in most cases, which was verified in oil-water-rock systems.

Nomenclature

- \( a_0 \) = coefficient associated with capillary forces, m/t
- \( A \) = cross-section area of the core, L\(^2\)
- \( b_0 \) = coefficient associated with the core, L\(^2\)
- \( c \) = constant, the ratio of the gravity force to the capillary force.
- \( g \) = gravity constant, L/t\(^2\)
- \( k \) = absolute permeability, L\(^2\)
- \( k_e \) = effective permeability of the two phases (but considered as one phase), L\(^2\)
- \( k_{ew} \) = effective permeability of the two phases at \( S_{wf} \), L\(^2\)
- \( k_{rw} \) = effective permeability of the nonwetting phase, L\(^2\)
- \( k_{re} \) = relative permeability pseudofunction of the two phases at \( S_{wf} \)
- \( k_{rw}^* \) = relative permeability of the wetting phase at \( S_{wf} \)
- \( k_w \) = effective permeability of the water or the wetting phase, L\(^2\)
- \( k_{rw}^* \) = relative permeability of the water or the wetting phase at \( S_{wf} \)
- \( L \) = core length, L
- \( L_a \) = characteristic length, L
- \( M_e \) = effective mobility of the two phases, mL/t
- \( M_{ew} \) = mobility of the nonwetting phase, mL/t
- \( M_{rw} \) = mobility of the nonwetting phase at \( S_{wf} \), mL/t
- \( M_w \) = mobility of the water or the wetting phase, mL/t
- \( M_{w}^* \) = mobility of the water or the wetting phase at \( S_{wf} \), mL/t
- \( N_{wt} \) = volume of water (or the wetting phase) imbibed into the core, L\(^3\)
- \( P_c \) = capillary pressure, m/Lt\(^2\)
- \( P_c^* \) = capillary pressure at \( S_{wf} \), m/Lt\(^2\)
- \( p_{nw} \) = pressure of the nonwetting phase, m/Lt\(^2\)
- \( p_w \) = pressure of the wetting phase, m/Lt\(^2\)
- \( q_w \) = imbibition rate of the wetting phase, L\(^3\)/t
- \( R \) = recovery by spontaneous imbibition in the units of pore volume.
- \( R^* \) = normalized oil (or the nonwetting phase) recovery
- \( S_{wf} \) = water (or the wetting phase) saturation behind imbibition front
- \( S_{wi} \) = initial water (or the wetting phase) saturation
- \( t \) = imbibition time, t
- \( t_d \) = dimensionless time derived in this study.
- \( t_{dl} \) = dimensionless time appropriate to gas-liquid-rock systems.
\[ t_{ls} = \text{dimensionless time suitable for oil-water-rock systems in which gravity is negligible.} \]
\[ t_p = \text{dimensionless time defined by Eq. 1.} \]
\[ V_p = \text{pore volume of the core sample, L}^3 \]
\[ v_{nw} = \text{the volumetric flux of the nonwetting phase, L}/t \]
\[ v_w = \text{the volumetric flux of the water phase, L}/t \]
\[ \mu_c = \text{the effective viscosity of the two phases (but considered as one phase), m/Lt} \]
\[ \mu_m = \text{geometric mean of the wetting and nonwetting phase viscosities, m/Lt} \]
\[ \mu_{nw} = \text{viscosity of the nonwetting phase, m/Lt} \]
\[ \mu_w = \text{viscosity of water (or the wetting phase), m/Lt} \]
\[ \phi = \text{porosity} \]
\[ \rho_{nw} = \text{density of the nonwetting phases, m/L}^3 \]
\[ \rho_w = \text{density of the wetting phases, m/L}^3 \]
\[ \Delta \rho = \text{density difference between the wetting and nonwetting phases, m/L}^3 \]

**Acknowledgements**

This research was conducted with financial support to the Stanford Geothermal Program from the Geothermal and Wind division of the US Department of Energy under grant DE-FG07-99ID13763, the contribution of which is gratefully acknowledged.

**References**

Appendix A: Scaling Analysis

The theoretical analysis was conducted to derive the general model to characterize spontaneous imbibition without neglecting gravity, initial fluid saturation, capillary pressure, and relative permeability of both the wetting and nonwetting phases. Also derived was a dimensionless time with almost all the parameters (those involved in spontaneous imbibition) included.

Assuming Darcy’s Law during the process of spontaneous imbibition that occurs vertically upward in a core with a specific value of initial saturation of the wetting phase ($S_{wi}$), including the zero $S_{wi}$, the volumetric fluxes of the wetting and the nonwetting phases in the core sample are expressed as follows:

\[ v_w = -\frac{k_w}{\mu_w} \left( \frac{\partial p_w}{\partial x} + \rho_w g \right) \]  
\[ v_{nw} = -\frac{k_{nw}}{\mu_{nw}} \left( \frac{\partial p_{nw}}{\partial x} + \rho_{nw} g \right) \]

where $v_w$ and $v_{nw}$ are the volumetric fluxes of the wetting and nonwetting phases; $k_w$ and $k_{nw}$ are the effective permeabilities of the wetting and nonwetting phases; $\mu_w$ and $\mu_{nw}$ are the viscosities of the wetting and nonwetting phases; $\rho_w$ and $\rho_{nw}$ the densities of the wetting and nonwetting phases; $p_w$ and $p_{nw}$ are the pressures of the wetting and nonwetting phases at the position $x$. From the definition of capillary pressure, the pressure of the wetting phase can be calculated:

\[ p_w = p_{nw} - P_c \]  

where $P_c$ is the capillary pressure.

Substituting Eq. A-2 into Eq. A-1a:

\[ v_w = M_w \left( \frac{\partial P_c}{\partial x} - \frac{\partial p_{nw}}{\partial x} - \rho_w g \right) \]  

where

\[ M_w = \frac{k_w}{\mu_w} \]  

Eq. A-1b could be written as follows:

\[ \frac{\partial p_{nw}}{\partial x} = -\left( \frac{v_{nw}}{M_{nw}} + \rho_{nw} g \right) \]  

where

\[ M_{nw} = \frac{k_{nw}}{\mu_{nw}} \]  

In cocurrent spontaneous imbibition flow, the following equation applies:

\[ v_w = v_{nw} \]  

while in countercurrent spontaneous imbibition flow, the following equation applies:

\[ v_w + v_{nw} = 0 \]
\[
\frac{\partial P_w}{\partial x} = -\left(\frac{v_w}{M_{nw}} + \rho_w g\right)
\]  
(A-9)

Substituting Eq. A-9 into Eq. A-3:

\[
v_w = \frac{M_e M_{nw}}{M_{nw} - M_w} \left(\frac{\partial P_e}{\partial x} - \Delta \rho g\right)
\]  
(A-10)

where \(\Delta \rho\) is the density difference between the wetting phase and the nonwetting phase \((\rho_w - \rho_{nw})\).

We define that:

\[
M_e = \frac{k_e}{\mu_e} = \frac{M_{nw} M_{nw}}{M_{nw} - M_w}
\]  
(A-11)

where \(M_e\) is a coefficient referred as the effective mobility, representing the combined effect of the mobilities of both the wetting and the nonwetting phases on the spontaneous imbibition. \(k_e\) and \(\mu_e\) are the effective permeability and the effective viscosity of the two phases respectively.

It is assumed in this study that the following equation holds:

\[
\frac{\partial P_e}{\partial x} = \frac{P_e}{x}
\]  
(A-12)

One of the cases in which Eq. A-12 holds is piston-like spontaneous imbibition. The validity of Eq. A-12 has been discussed in detail by Li and Horne\(^{20}\).

Substituting Eqs. A-11 and 12 into Eq. A-10:

\[
v_w = M_e \frac{P_e}{x} - \Delta \rho g
\]  
(A-13)

Assuming that the distribution of \(S_{wf}\) in the porous medium is homogeneous, the cumulative volume of the wetting phase imbibed into the core with \(S_{wf}\) can be calculated as follows:

\[
N_{wt} = Ax \phi (S_{wf} - S_{wi})
\]  
(A-14)

where \(N_{wt}\) is the cumulative volume of the wetting phase imbibed into the core and \(A\) is the cross-section area of the core.

The imbibition rate of the wetting phase \(q_w\) is equal to \(Av_w\) in cocurrent spontaneous imbibition. Therefore, Eq. A-13 can be expressed as follows:

\[
q_w = AM_e \left(\frac{P_e}{x} - \Delta \rho g\right)
\]  
(A-15)

In Eq. A-15, \(P_e\) and \(M_e\) are the capillary pressure and the effective mobility at \(S_{wf}\). We define \(P_e^* = P_e (S_{wf})\) and \(M_e^* = M_e (S_{wf})\) to distinguish the capillary pressure and the effective mobility at any other specific water saturation. Considering this and substituting Eq. A-14 into Eq. A-15, the following equation can be obtained:

\[
q_w = \frac{A^2 \phi M_e^* p_e^* (S_{wf} - S_{wi})}{N_{wt}} - A M_e^* \Delta \rho g
\]  
(A-16)

Define that:

\[
a_0 = \frac{A M_e^* (S_{wf} - S_{wi})}{L} P_e^*
\]  
(A-17a)

\[
b_0 = A M_e^* \Delta \rho g
\]  
(A-17b)

\[
R = \frac{N_{wt}}{V_p}
\]  
(A-17c)

here \(R\) is the recovery in the units of pore volume.

Eq. 6 in the text can be obtained by substituting Eqs. A-17a, A-17b, and A-17c into Eq. A-16.

Define that:

\[
c = \frac{b_0}{a_0}
\]  
(A-18a)

\[
R^* = cR
\]  
(A-18b)

and

\[
t_d = c^2 \frac{k_e^*}{\phi} \frac{p_e^*}{\mu_e} \frac{S_{wf} - S_{wi}}{L_a^2}
\]  
(A-18c)

here \(c\) is the ratio of the gravity force to the capillary force, \(t_d\) is the dimensionless time with gravity and capillary forces included. \(R^*\) is the normalized recovery. In the cocurrent spontaneous imbibition case, \(L_a\) is equal to the core length. In Eq. A-18c, \(k_e^* = M_e^*\).
Substituting Eqs. A-18a, A-18b, and A-18c into Eq. A-16, the following equation is obtained:

\[
\frac{R^* dR^*}{dt_d} = 1 - R^*
\]  

(A-19)

In cases of \( R^* \) less than 1.0, the solution of Eq. A-19 can be obtained, as expressed in Eq. 7. Note that \( R^* \) is only a function of the newly defined dimensionless time. Eq. 4 can be obtained from Eq. A-18c in terms of relative permeability.

![Fig. 1: Spontaneous water imbibition behavior in a 15 md Indiana limestone core.](image1)

![Fig. 2: Scaling results using the existing dimensionless time for a 15 md Indiana limestone core.](image2)

![Fig. 3: Relationship between imbibition rate and the reciprocal of oil recovery at different IFT in a 15 md Indiana limestone core.](image3)
Fig. 4: Scaling results using the new scaling model for a 15 md Indiana limestone core at different IFT.

Fig. 5: Spontaneous water imbibition behavior in a 100 md Berea sandstone core.

Fig. 6: Scaling results using the existing dimensionless time for a 100 md Berea sandstone core.

Fig. 7: Scaling results using the new scaling model for a 100 md Berea sandstone core at different IFT.

Fig. 8: Spontaneous water imbibition behavior in a 500 md Berea sandstone core.

Fig. 9: Scaling results using the existing dimensionless time for a 500 md Berea sandstone core.
Fig. 10: Scaling results using the new scaling model for a 500 md Berea sandstone core at different IFT.

Fig. 11: Relationships between oil recovery and the new dimensionless time in a 500 md Berea sandstone core at different IFT.

Fig. 12: Scaling results using the new scaling model for different rocks at different IFT.