Experimental Verification of Methods to Calculate Relative Permeability Using Capillary Pressure Data
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Abstract
The Brooks and Corey model has been accepted widely to calculate relative permeability using capillary pressure data. However, the Purcell model was found to be the best fit to the experimental data of the wetting phase relative permeability for the cases studied, as long as the measured capillary pressure curve had the same residual saturation as the relative permeability curve. The differences between the experimental and the Purcell model data were almost negligible. A physical model was developed to explain the insignificance of the effect of tortuosity on the wetting-phase. For nonwetting phase relative permeability, the model results were very close to the experimental values in drainage except for the Purcell model. However, calculated data in imbibition were different than the experimental data. This study showed that relative permeability could be calculated satisfactorily by choosing a suitable capillary pressure technique, especially in drainage processes. In the reverse procedure, capillary pressure could also be computed once relative permeability data are available.

Introduction
Relative permeability is of central importance to reservoir engineering but difficult to measure in some cases. Such cases include extremely low permeability rocks and special fluid systems in which there are phase transformation and mass transfer between the two phases as pressure changes. Steam-water flow is an example of such a system. Steam-water relative permeability plays an important role in controlling reservoir performance for steam injection into oil reservoirs and water injection into geothermal reservoirs. At the same time, Li and Horne\textsuperscript{1} found significant differences between steam-water and air-water capillary pressures, and Horne \textit{et al.}\textsuperscript{2} found differences between steam-water and air-water relative permeabilities. According to these studies, steam-water flow properties may not be replaced simply by air-water or nitrogen-water flow properties. It would be helpful for reservoir engineers to be able to calculate steam-water relative permeability once steam-water capillary pressure data are available.

There are many papers\textsuperscript{3-13} related to techniques for the calculation of relative permeabilities from capillary pressure data. Purcell\textsuperscript{3} developed a method to calculate the permeability using pore size distribution derived from mercury-injection capillary pressure curves. This method was used to calculate the multiphase relative permeabilities, as reported by Gates and Leits\textsuperscript{4}. Later, Burdine\textsuperscript{5} introduced a tortuosity factor in the model. Corey\textsuperscript{6} and Brooks and Corey\textsuperscript{7} summarized the previous works\textsuperscript{3-5} and modified the method by representing capillary pressure curve as a power law function of the wetting phase saturation. Honarpour \textit{et al.}\textsuperscript{14} reviewed the literature in this field. The published literature and experimental data for relative permeability and capillary pressure were not sufficient to conclude which method should be the standard one.

Unlike for oil-gas and oil-water flow properties, there are few studies for the calculation of steam-water relative permeabilities by the capillary pressure technique. Historically, the capillary pressure techniques were developed for drainage situations and were useful to obtain gas-liquid (oil or water) relative permeability when fluid flow tests were not practical.

In this study, we calculated the gas-liquid (including gas-oil, air-water, and steam-water) and oil-water relative permeabilities using experimental data of capillary pressure from different rocks by different methods such as the Purcell, Burdine, Corey, and Brooks-Corey methods. The calculated results were compared to the relative permeability data measured in the same core sample. The purpose of this study was to verify which capillary pressure technique would achieve the best fit to the experimental data of relative permeability.

Mathematical Background
We chose four representative models developed by various authors (Purcell\textsuperscript{3}; Burdine\textsuperscript{5}; Corey\textsuperscript{6}; Brooks and Corey\textsuperscript{7}) to
calculate relative permeabilities using the capillary pressure
techniques in different fluid-rock systems. The mathematical
expressions of the four models are described briefly in this
section.

**Purcell Model.** Purcell developed an equation to compute
rock permeability by using capillary pressure data. This
equation can be extended readily to the calculation of
multiphase relative permeability. In two-phase flow, the
relative permeability of the wetting phase can be calculated as
follows:

\[ k_{rw} = \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \]  

where \( k_{rw} \) and \( S_w \) are the relative permeability and saturation of
the wetting phase; \( P_c \) is the capillary pressure as a function of
\( S_w \).

Similarly, the relative permeability of the nonwetting
phase can be calculated as follows:

\[ k_{rnw} = \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \]

where \( k_{rnw} \) is the relative permeability of the nonwetting phase.
It can be seen from Eqs. 1 and 2 that the sum of the wetting
and nonwetting phase relative permeabilities at a specific
saturation is equal to one. This may not be true in most porous
media. In the next section, the relative permeabilities
calculated using this method are compared to the experimental
data. The comparison shows that Eq. 1 is close to experimental
values of the wetting phase relative permeability but Eq. 2 for
the nonwetting phase is far from the experimental results.

**Burdine Model.** Burdine developed equations similar to
Purcell’s method by introducing a tortuosity factor as a
function of wetting phase saturation. The relative permeability
of the wetting phase can be computed as follows:

\[ k_{rw} = \left( \lambda_{rw} \right)^2 \frac{\int_0^{S_w} dS_w / (P_c)^2}{\int_0^1 dS_w / (P_c)^2} \]  

where \( \lambda_{rw} \) is the tortuosity ratio of the wetting phase.
According to Burdine, \( \lambda_{rw} \) could be calculated as follows:

\[ \lambda_{rw} = \frac{\tau_w(1.0)}{\tau_w(S_w)} = \frac{S_w - S_m}{1 - S_m} \]

where \( S_m \) is the minimum wetting phase saturation from the
capillary pressure curve; \( \tau_w \) (1.0) and \( \tau_w(S_w) \) are the
tortuosities of the wetting phase when the wetting phase
saturation is equal to 100% and \( S_m \) respectively.

In the same way, relative permeabilities of the nonwetting
phase can be calculated by introducing a nonwetting phase
tortuosity ratio. The equation can be expressed as follows:

\[ k_{rnw} = \left( \lambda_{rnw} \right)^2 \frac{\int_0^{S_n} dS_n / (P_c)^2}{\int_0^1 dS_n / (P_c)^2} \]  

where \( \lambda_{rnw} \) is the tortuosity ratio of the nonwetting phase,
which can be calculated as follows:

\[ \lambda_{rnw} = \frac{\tau_{rnw}(1.0)}{\tau_{rnw}(S_n)} = \frac{1 - S_n - S_e}{1 - S_m - S_e} \]

Here \( S_e \) is the equilibrium saturation of the nonwetting phase;
\( \tau_{rnw} \) is the tortuosity of the nonwetting phase.

Honarpour et al. pointed out that the expression for the
wetting phase relative permeability (Eq. 3) fits the
experimental data much better than the expression for the
nonwetting phase (Eq. 5).

**Corey Model.** According to the Purcell and Burdine models,
an analytical expression for the wetting and nonwetting phase
relative permeabilities can be obtained if capillary pressure
curves can be represented by a simple mathematical function.
Corey found that oil-gas capillary pressure curves could be
expressed approximately using the following linear relation:

\[ 1 / P_c^2 = CS_w^* \]  

where \( C \) is a constant and \( S_w^* \) is the normalized wetting phase
saturation, which could be expressed as follows for the
drainage case:

\[ S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \]

where \( S_{wr} \) is the residual saturation of the wetting phase. In
Corey’s case, \( S_{wr} \) is the residual oil saturation.

Although originally the Corey model was not developed
for the imbibition case, in this study it was used to calculate
the imbibition relative permeabilities by defining the
normalized wetting phase saturation as follows:
where \( S_{wr} \) is the residual saturation of the nonwetting phase.

Substituting Eq. 7 into Eqs. 3 and 5 with the assumption that \( S_e = 0 \) and \( S_w = S_{wr} \), Corey6 obtained the following equations to calculate the wetting (oil) and nonwetting (gas) phase relative permeabilities for drainage cases:

\[
k_{wr} = (S_{wr}^*)^4
\]

(10)

\[
k_{rw} = (1 - S_{wr}^*)^2[1 - (S_{wr}^*)^2]
\]

(11)

A constraint to the use of Corey's Model (Eqs. 10 and 11) is that the capillary pressure curve should be represented by Eq. 7.

**Brooks-Corey Model.** Because of the limitation of Corey's model, Brooks and Corey7 modified the representation of capillary pressure function to a more general form as follows:

\[
P_c = p_e(S_w^*)^{-1/\lambda}
\]

(12)

where \( p_e \) is the entry capillary pressure and \( \lambda \) is the pore size distribution index.

Substituting Eq. 12 into the Burdine model (Eqs. 3 and 5) with the assumption that \( S_e = 0 \), Brooks and Corey7 derived equations to calculate the wetting and nonwetting phase relative permeabilities as follows:

\[
k_{rw} = (S_{wr}^*)^{2+3\lambda}/\lambda
\]

(13)

\[
k_{nrw} = (1 - S_{wr}^*)^2[1 - (S_{wr}^*)^{2+\lambda}/\lambda]
\]

(14)

When \( \lambda \) is equal to 2, the Brooks-Corey model reduces to the Corey model.

**Results**

The experimental data of capillary pressure from our previous study15 and the literature were used in this paper. The calculation and comparison in steam-water, nitrogen-water, oil-water, and oil-gas flow are presented and discussed in this section.

**Steam-water flow.** The data of both drainage and imbibition steam-water capillary pressure from Li and Horne15 were used to calculate the corresponding steam-water relative permeability. Note that the capillary pressure data were represented using Eq. 12 in all the calculations by the Purcell model. The calculated results were compared to the experimental data of steam-water relative permeability measured by Mahiya16. During the process of the fluid flooding tests, the water saturation in the core sample was first decreased from 100% to the residual water saturation, about 28%, representing a drainage process. The water saturation was then increased, representing an imbibition.

Fig. 1 shows the experimental data of the steam-water relative permeability16 and capillary pressure15 in drainage. All these data were measured at a temperature of about 120°C in the same Berea core sample. The permeability and porosity of this core were 1400 md and 24.8%; the length and diameter were 43.2 cm and 5.04 cm, respectively. Because the relative permeability and the capillary pressure were measured simultaneously, the two curves had the same residual water saturations. This feature is important and will be discussed later in more detail. Note that the steam relative permeability data shown in Fig. 1 have been calibrated under the consideration of gas slip effect (Klinkenberg Effect17) in two-phase flow by Li and Horne18.

The drainage steam-water relative permeabilities were calculated using the experimental data of the drainage steam-water capillary pressure shown in Fig. 1 and plotted versus the normalized water saturation that is defined in Eq. 8. The calculated results and the comparison to the corresponding experimental data are shown in Fig. 2. The relative permeabilities in Fig. 2 were normalized to conduct the comparison. The method to do so is to divide the experimental relative permeabilities by the corresponding end point values. The same normalization was applied to the experimental relative permeabilities shown in figures used to compare results in the rest of this paper.

We can see from Fig. 2 that the water relative permeabilities calculated using the Purcell model are the best fit to the experimental data. This implies that it may not be necessary to adjust the calculation of the wetting phase relative permeabilities by introducing the concept of the tortuosity factor in such a case. The water phase relative permeabilities calculated by all the other models are less than the experimental values. It can be seen from Fig. 2 that the steam phase (nonwetting phase) relative permeabilities calculated by all the models but the Purcell model are almost the same and consistent with the experimental data for the drainage case. The steam phase relative permeabilities calculated by the Purcell model are not shown in Fig. 2 and all the figures following in this section because the curve is concave downwards, which is unexpected and far from the experimental values.

The experimental data of the imbibition steam-water relative permeability from Mahiya16 and the imbibition capillary pressure from Li and Horne15 are shown in Fig. 3. These data were also measured simultaneously in the same Berea core sample at a temperature of about 120°C. The steam relative permeability data shown in Fig. 3 have also been...
calibrated under the consideration of gas slip effect in two-phase flow. The imbibition steam-water relative permeabilities were then calculated using the measured data of the imbibition steam-water capillary pressure shown in Fig. 3 and also plotted versus the normalized water saturation. Fig. 4 shows the calculated results and the comparison to the experimental values. The water relative permeabilities from the Purcell model are also the best fit to the experimental data, the same as in drainage. The results from the Corey model are a good fit too. The water phase relative permeabilities calculated by the Burdine and the Brooks-Corey models are less than the experimental values. Actually the results calculated using the two models are the same if the capillary pressure data in the Burdine model are represented using Eq. 12. The steam phase relative permeabilities calculated by all the models except the Purcell model are not significantly different from each other but are less than the experimental data in the imbibition case.

Nitrogen-water flow. In the following section, we will discuss the calculated results and the comparison in nitrogen-water systems to further confirm the phenomena that we observed. Li and Horne measured the nitrogen-water relative permeabilities in a fired Berea core sample similar to that used in the measurement of steam-water relative permeabilities by Mahiya. In this study, we drilled a plug from another part of the same fired Berea sandstone that was used by Li and Horne. The length and diameter of the plug sample were 5.029 cm and 2.559 cm respectively; the porosity was 24.37%. The drainage nitrogen-water capillary pressure of the plug was measured by using the semipermeable porous-plate method. The measured data of the drainage nitrogen-water capillary pressure along with the relative permeabilities from Li and Horne are plotted in Fig. 5. Although the nitrogen-water capillary pressure and relative permeability curves were not measured simultaneously, the residual water saturations were almost the same for both.

The results calculated using the capillary pressure models for the nitrogen-water flow (drainage) and the comparison to the experimental data are shown in Fig. 6. The experimental data of water relative permeability are located between the Purcell model and the Corey model. The two models provide a good approximation to the experimental data in this case. The features of gas phase relative permeability curve calculated by these models are similar to those of steam-water flow (see Fig. 4) except that the calculated results are greater than the measured data.

Oil-water flow. Kleppe and Morse reported the experimental data of imbibition oil-water relative permeability and capillary pressure in Berea sandstone with a permeability of 290 md and a porosity of 22.5%. The three curves are shown in Fig. 7. The calculated results of oil and water relative permeability and the comparison to the experimental data are plotted in Fig. 8. In oil-water flow, the best fit to the wetting phase (water phase in this case) relative permeability is also from the Purcell model. The water phase relative permeabilities calculated using other models are not notably different from each other but are much less than the experimental data in this case. For the nonwetting phase (oil phase in this case) relative permeability, all the models except the Purcell model give good fit to the experimental data.

Beckner et al. reported imbibition oil-water relative permeability and capillary pressure data which were representative of actual field data (see Fig. 9). The capillary pressure data were also used to calculate oil-water relative permeability with various methods. The results and the comparison are shown in Fig. 10. The Purcell model produced the best fit to the water phase relative permeability, the same as observed previously. The water phase relative permeabilities calculated using other models are less than the relative permeability data from Beckner et al.

Oil-gas flow. We made the same calculation and comparison using the data of oil-gas relative permeability and capillary pressure measured in Berea sandstone by Richardson et al. The permeability and porosity of the core were 107 md and 17.7%; the length and diameter were 30.7 cm and 6.85 cm, respectively. The oil phase was kerosene and the gas phase was helium. The experimental data of the drainage oil-gas relative permeability and the capillary pressure are shown in Fig. 11. The calculated results of relative permeability and the comparison to the experimental values are demonstrated in Fig. 12. We also observed that the best fit to the wetting phase relative permeability in oil-gas flow was from the Purcell model.

All the relative permeability and capillary pressure curves we used in the previous sections have a common feature: the residual saturation from the capillary pressure curve is equal to that from the relative permeability curve. Gates and Lietz reported oil-gas relative permeability and capillary pressure curves without such a feature. The experimental data of drainage oil-gas relative permeability and capillary pressure, taken from Fig. 4 in the paper by Gates and Lietz, were used in this study and are depicted in Fig. 13. These data were measured in a Pyrex core with a permeability of 1370 md and a porosity of 37.4%. The oil phase was kerosene and the gas phase was air. The residual oil saturation was about 30% according to the oil phase relative permeability curve but was about 12% according to the capillary pressure and the gas phase relative permeability curves (see Fig. 13). The reason might be the evaporation of oil caused by continuous gas injection even after the residual oil saturation by displacement was reached.

The oil and gas relative permeabilities calculated using various capillary pressure techniques were compared to the experimental data measured by Gates and Lietz and the results are demonstrated in Fig. 14. We observed that all the models except the Purcell model yielded the best fit to both the wetting and nonwetting phase relative permeabilities.

In summarizing all the calculations that we have made, including some not presented here, the Purcell model was the best fit to the wetting phase relative permeability if the
measured capillary pressure curve had the same residual saturation as the relative permeability curve.

**Calculation of capillary pressure using relative permeability data.** In some cases, relative permeability data are available but capillary pressure data are not. A method to calculate capillary pressure function using relative permeability is proposed in this section. As observed previously, the Purcell model may be the best fit to the experimental data of the wetting phase relative permeability. Substituting $P_c$ using Eq. 12, the Purcell model can be expressed as follows:

$$k_{rw} = (S_w^*)^{2+\lambda}$$

(15)

Therefore we can fit the experimental data of the wetting phase relative permeability using Eq. 15 to obtain the value of the pore size distribution index $\lambda$. According to Eq. 12, the corresponding capillary pressure function can be determined once the value of the pore size distribution index $\lambda$ is available. The entry capillary pressure may be measured readily or can be evaluated using other methods.

**Discussion**

The techniques using capillary pressure to calculate relative permeability were developed in the late forties. Burdine\(^5\) pointed out that the calculated relative permeabilities are more consistent and probably contain less maximum error than the measured data because the error in measurement is unknown. This may be true in some cases. However, the differences between different capillary pressure models are obvious, especially for the wetting phase. Therefore, one important question is which model is most appropriate for practical use. The calculations in this study showed that the Purcell model was the best fit to the wetting phase relative permeability. This seems surprising because the concept of the tortuosity factor as a function of wetting phase saturation is not introduced for the calculation of the wetting phase relative permeability in such a case. A physical model was developed to demonstrate the insignificant effect of the tortuosity factor on the wetting phase, as shown in Fig. 15. $L$ is the direct distance between the ends of a single capillary tube and $L_t$ is the length of the tortuous capillary tube.

Burdine\(^5\) obtained an empirical expression of the effective tortuosity factor as a function of wetting phase saturation (see Eq. 4). $\lambda_{rw}$ is actually the ratio of the tortuosity at 100% wetting phase saturation to the tortuosity at a wetting phase saturation of $S_w$. According to Eq. 4, the tortuosity of wetting phase is infinite at the minimum wetting phase saturation that is equal to residual water saturation $S_{wr}$ here. This may not be true for the wetting phase because the wetting phase may exist on the rock surface in the form of continuous film, as shown in Fig. 15b. In this case, $\tau_w (S_w = S_{wr})$ may be close to $\tau$ (1.0) (see Fig. 15a), which demonstrates that there is little effect of the wetting phase saturation on the tortuosity of the wetting phase. Similarly, based on Eq. 6, the tortuosity of the nonwetting phase is infinite when the wetting phase saturation is equal to 1-$S_e$. This may be true because the nonwetting phase may exist in the form of discontinuous droplets (see Fig. 15c). In this case, $S_e$ is equal to $S_{gr}$.

It can be seen from the analysis here that the tortuosity of wetting and nonwetting phases would behave differently as a function of wetting phase saturation. This may be why it is necessary to introduce the tortuosity for the nonwetting phase but not for the wetting phase.

As stated previously, capillary pressure techniques were developed originally in cases in which it is difficult to measure relative permeability. Actually these techniques may also be useful even in cases in which both relative permeability and capillary pressure data are available. In these cases, we can still calculate relative permeability using the appropriate models with the capillary pressure data and compare the results to the experimental values. If the calculated results are consistent with the experimental data, we may have more confidence on the experimental measurements. This idea may also be applied to numerical simulation. For example, it may be helpful to check the relative permeability curves obtained from upscaling using the capillary pressure techniques.

**Conclusions**

Based on the present study, the following conclusions may be drawn:

1. The calculated results in gas-liquid and oil-water flow indicate that the Purcell model may be the best fit to the experimental data of the wetting phase relative permeability for both drainage and imbibition processes but is not a good fit for the nonwetting phase.
2. It may not be necessary to introduce the tortuosity factor in calculating the wetting phase relative permeability as long as the measured capillary pressure curve had the same residual saturation as the relative permeability curve.
3. Except for the Purcell model, the results of the nonwetting phase relative permeability calculated using the models for the drainage case were almost the same and very close to the experimental values. However, those for the imbibition cases were different from the measured data.
4. The capillary pressure techniques would be valuable not only in cases in which it is difficult to measure relative permeability curves but also in cases in which both relative permeability and capillary pressure data are available.
5. In general, the Purcell model is proposed to calculate the wetting phase relative permeability and the Brooks-Corey model is proposed to calculate the nonwetting phase relative permeability once reliable capillary pressure data are available.

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**Nomenclature**

- $C$ = constant
- $k_{nw}$ = relative permeability of nonwetting phase
- $k_w$ = relative permeability of wetting phase
- $L = $ direct distance between the ends of a single capillary tube
- $L_a = $ the length of the tortuous capillary tube
- $P_c = $ capillary pressure
- $p_e = $ entry capillary pressure
- $S_e = $ equilibrium saturation of wetting phase
- $S_m = $ minimum wetting phase saturation
- $S_w = $ wetting phase saturation
- $S_w^* = $ normalized wetting phase saturation
- $S_{nwr} = $ residual saturation of nonwetting phase
- $S_{wr} = $ residual wetting phase saturation
- $\lambda = $ pore size distribution index
- $\lambda_{nw} = $ tortuosity ratio of nonwetting phase
- $\lambda_w = $ tortuosity ratio of wetting phase
- $\tau_w = $ tortuosity of wetting phase

**References**

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Fig. 2: Calculated steam-water relative permeability and the comparison to the experimental data from Mahiya in drainage.

Fig. 3: Experimental data of imbibition steam-water relative permeability and capillary pressure.

Fig. 4: Calculated steam-water relative permeability and the comparison to the experimental data in imbibition.

Fig. 5: Experimental data of drainage nitrogen-water relative permeability and capillary pressure.
Fig. 6: Calculated nitrogen-water relative permeability and the comparison to the experimental data in drainage.

Fig. 7: Imbibition oil-water relative permeability and capillary pressure from Kleppe and Morse\textsuperscript{19}.

Fig. 8: Calculated oil-water relative permeability and the comparison to the experimental data from Kleppe and Morse\textsuperscript{19}.

Fig. 9: Imbibition oil-water relative permeability and capillary pressure from Beckner et al.\textsuperscript{20}.
Fig. 10: Calculated oil-water relative permeability and the comparison to the data from Beckner et al.\textsuperscript{20}

Fig. 11: Drainage oil-gas relative permeability and capillary pressure from Richardson et al.\textsuperscript{21}

Fig. 12: Calculated oil-gas relative permeability and the comparison to the experimental data from Richardson et al.\textsuperscript{21}

Fig. 13: Drainage oil-gas relative permeability and capillary pressure from Gates and Lietz\textsuperscript{4}.
Fig. 14: Calculated oil-gas relative permeability and the comparison to the experimental data from Gates and Lietz⁴.

Fig. 15: Tortuosity in a single capillary tube.