Fractal Characterization of The Geysers Rock

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Abstract
Fractures play an important role in steam production from geothermal reservoirs. It has been a challenge for a long time to characterize materials with a high density of fractures such as The Geysers rock. Experimental data showed that the capillary pressure curves of The Geysers rock are very different from that of Berea sandstone. Methods to characterize the difference between the two have been few. It was also found that the frequently-used Brooks-Corey model could not be used to represent the capillary pressure curves of The Geysers rock samples studied. To this end, a fractal technique was proposed to model the features of the capillary pressure curves and to characterize the difference between The Geysers rock and Berea sandstone. The calculated values of the fractal dimension of all the core samples studied (six from The Geysers and one Berea sandstone) were in the range from 2 to 3. The results demonstrated that The Geysers rock with a high density of fractures had a greater fractal dimension than Berea sandstone that is almost without fractures. This shows that The Geysers rock has greater heterogeneity, as expected. The significance of fractal dimension inferred from capillary pressure data is consistent with the traditional qualitative observation from the frequency graph of pore size distribution. The proposed fractal technique may be extended from core scales to reservoir scales to characterize fracture density in geothermal reservoirs.

Introduction
Reserves of geothermal reservoirs depend mainly on the porosity of the rock matrix while steam and water production rates primarily depend on fractures. It is relatively easy to measure the values of porosity in the rock matrix. On the other hand, it is difficult to measure or characterize the fracture systems quantitatively as well as the entire rock with both matrix and fractures. Recently attention has been paid to the fractal characterization of the fracture surface (Babadagli and Develi, 2000) and the space distribution of fractures in geothermal reservoirs (Sammis et al., 1992; Tateno et al., 1995; Tsuchiya and Nakatsuka, 1995).

Tateno et al. (1995) demonstrated that the distribution of open fracture width in a geothermal reservoir was fractal. Sammis et al. (1992) studied the fracture pattern and the distribution of fractures in two dimensions at The Geysers geothermal field. Sammis et al. (1992) found that the values of the fractal dimension measured at different scales, the outcrop, roadcut, and regional scales, were almost the same.

However publications on the characterization of the entire rock with both matrix and fractures have been few. Studies on the features of capillary pressure curves of The Geysers rock and the
corresponding pore size distribution have also been few in the literature. In this study, the capillary pressure curves of The Geysers rock were measured using a mercury-injection technique. A fractal approach was proposed to characterize the rock heterogeneity quantitatively using the data from the capillary pressure curves. The values of the fractal dimension inferred from capillary pressure curves were used to represent rock heterogeneity as well as the differences between The Geysers rock and the more uniform, unfractured Berea sandstone.

**Methodology**

There have been many methods to characterize heterogeneity in nature. Of all the approaches, fractal geometry has been utilized widely in many areas. Fractal geometry is a branch of mathematics and is used mainly to characterize extremely disordered or heterogeneous systems that appear similar in some way when observed at different scales. This feature is known as self-similarity or scale invariance. The Geysers rock is highly heterogeneous and extremely disordered and may be fractal in nature. In this study, we chose fractal geometry as a tool to characterize the heterogeneity of The Geysers rock with both matrix and fractures.

According to the basic concept of fractal geometry, the following expression applies to a fractal object:

$$N(r) \propto r^{-D_f}$$

where $r$ is the radius (or characteristic length) of a unit chosen to fill the fractal object, $N(r)$ is the number of the units (with a radius of $r$) required to fill the entire fractal object, and $D_f$ is the so-called fractal dimension. The fractal dimension is a representation of the heterogeneity of the fractal object. The greater the fractal dimension, the more heterogeneous the fractal object.

Capillary pressure curves measured by a mercury-injection technique are often used to infer the pore size distribution of rock samples. In making this inference, rock with solid skeleton and pores is represented by using a capillary tube model. $N(r)$ can be calculated easily once capillary pressure curves measured using a mercury-injection technique are available. The unit chosen in this study was a cylindrical capillary tube with a radius of $r$ and a length of $l$. So the volume of the unit is equal to $\pi r^2 l$ and $N(r)$ at a given radius of $r$ is then calculated easily.

Once $N(r)$ is known, the value of fractal dimension, $D_f$, can be determined from the relationship between $N(r)$ and $r$. The relationship between $N(r)$ and $r$ should be linear on a log-log plot if the pore system of the rock is fractal.

**Experimental Measurements**

Six core samples from different wells at The Geysers geothermal field were used in this study. Unfortunately the core samples were irregular and too small to drill a plug for permeability measurements. The measured porosity of the core samples ranged from 0.1 to 4.0%. One core sample of Berea sandstone was also used as a comparison. The porosity of the Berea sample was about 23.0% and the air permeability was about 804 md.
The surface tension of air/mercury is 480 mN/m and the contact angle through the mercury phase is 140° (Purcell, 1949).

Results

Capillary pressure curves of the six core samples from The Geysers geothermal field and one core sample of Berea sandstone were measured using the mercury-injection technique. The results are shown in Fig. 1. There are substantial differences between The Geysers rock and Berea sandstone. The normalized wetting-phase saturation is calculated as follows:

\[
S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}}
\]

where \(S_w^*\) is the normalized wetting-phase saturation (the wetting-phase in this study is air), \(S_w\) is the wetting-phase saturation, and \(S_{wr}\) is the residual saturation of the wetting-phase.

![Capillary pressure curves of The Geysers rock and Berea sandstone.](image)

The capillary pressure curve of the Berea sandstone core sample is linear on a log-log plot, which implies that the capillary pressure curve can be represented using the frequently-used Brooks-Corey model (1964). However all the capillary pressure curves of The Geysers rock samples are nonlinear, which implies that the capillary pressure curves may not be represented using the Brooks-Corey model. Qualitatively one can see that The Geysers rock samples are more heterogeneous than Berea sandstone, as expected.

The pore size (pore throat radius) distributions inferred from capillary pressure measurements are shown in Fig. 2. Note that the capillary tube model was used as usual. Fig. 2 shows that the pore sizes in The Geysers rock are much smaller than in the Berea sandstone.
Figure 2:  
*Pore size distributions of The Geysers rock and Berea sandstone.*

To characterize the difference in heterogeneity between The Geysers rock and Berea sandstone, the capillary pressure curves shown in Fig. 1 were transferred to relationships between $N(r)$ and $r$ according to Eq. 1. The results are shown in Fig. 3. We can see that all the curves of $N(r)$ vs. $r$ are linear on a log-log plot, which implies that the pore systems of The Geysers rock and Berea sandstone are fractal and can be characterized using the theories of fractal geometry. Note that about three data points in the beginning of mercury intrusion were off the main trend and were removed to conduct the regression analysis.

Figure 3:  
*Relationships between $N(r)$ and $r$ of The Geysers rock and Berea sandstone.*

One can see from Fig. 1 that both Berea and The Geysers rock are fractals. However the values of the fractal dimension are different and can be used to characterize the differences between the two different rocks.
The values of the fractal dimension were calculated according to Eq. 1 for all the core samples using the data shown in Fig. 3 and the results are listed in Table 1.

Table 1: Fractal Dimensions of The Geysers rock and Berea sandstone

<table>
<thead>
<tr>
<th>Sample</th>
<th>SB15D_1</th>
<th>MLM_3</th>
<th>Pc 92</th>
<th>SB15D_2</th>
<th>PRATI_5</th>
<th>CA1862_4</th>
<th>Berea</th>
</tr>
</thead>
<tbody>
<tr>
<td>φ (%)</td>
<td>4.0</td>
<td>2.5</td>
<td>0.4</td>
<td>0.1</td>
<td>1.1</td>
<td>0.8</td>
<td>23.0</td>
</tr>
<tr>
<td>$D_f$</td>
<td>2.2510</td>
<td>2.3407</td>
<td>2.4983</td>
<td>2.4356</td>
<td>2.3070</td>
<td>2.3645</td>
<td>2.0582</td>
</tr>
</tbody>
</table>

Table 1 shows that all the values of fractal dimension range from 2 to 3. The fractal dimension of Berea sandstone is smaller than that of all the rock samples from The Geysers geothermal field. The observation implies that The Geysers rock is more heterogeneous than Berea sandstone, which is obviously true.

Rock heterogeneity can also be observed qualitatively from the frequency graph of pore size distribution. Figs. 4 and 5 show the pore size distributions of Berea sandstone and The Geysers rock (PRATI_5) respectively.

![Pore size distribution of Berea sandstone.](image)

The fraction of pores (Y-axis in Figs. 4 and 5) was calculated with the volume of injected mercury divided by the pore volume of the core sample. One can see that the pore size distribution of Berea sandstone is a normal distribution with a relatively narrow peak. However, the pore size distribution of The Geysers rock (PRATI_5) is not a normal distribution and has many peaks. Comparing Fig. 4 to Fig. 5, it is obvious that The Geysers rock (PRATI_5) has greater heterogeneity than Berea sandstone. This qualitative observation is verified by the values of fractal dimension inferred from capillary pressure curves (see Table 1).
The previous description also demonstrates that the significance of fractal dimension determined using Eq. 1 from capillary pressure data is consistent with the traditional qualitative observation from the frequency graph of pore size distribution.

Figure 5: Pore size distribution of The Geysers rock (PRATI_5).

The results discussed in this section demonstrated that the fractal approach can be used to characterize the heterogeneity of The Geysers rock satisfactorily.

Discussion

In petroleum industry, heterogeneity is a significant problem for oil production. Usually oil recovery in water drive reservoirs decreases with increase in heterogeneity. However this may not be the case in geothermal industry. Fractures are beneficial to the production of steam and water. Steam production rate may increase with fracture density. The greater fracture density implies greater heterogeneity, which can be characterized using fractal dimension. It would be helpful for reservoir engineers if a relationship between fractal dimension and steam production from geothermal reservoirs.

For water injection into geothermal reservoirs, the injection rate should not be greater than a specific value and should be optimized as Li and Horne (2000) pointed out. In this case, the injected water will not breakthrough fast at production wells and the fractures are beneficial. However the fractures may not be beneficial if the water injection rate is very high and greater than a critical value.

Shen and Li (1994, 1995) found a relationship between the fractal dimension of core samples and the oil recovery by water flooding. The core samples with greater values of fractal dimension had smaller values of water drive oil recovery. The experimental observation is theoretically reasonable. This is because fractal dimension is a representation of rock heterogeneity. The greater fractal dimension implies greater heterogeneity. It is known that the oil recovery by water flooding is inversely proportional to the heterogeneity.
The relationship between the fractal dimension and the oil recovery by water flooding observed in laboratory was even true for oil recovery in many reservoirs developed by water injection, as reported by Shen and Li (1995). This is also understandable according to the features of fractal objects. Self-similarity is an important feature of a fractal. This feature implies that fractal dimension at a core scale may be equal to that at a reservoir scale. As reported by Sammis et al. (1992), the values of the fractal dimension measured at The Geysers geothermal oil field at different scales, the outcrop, roadcut, and regional scales, were almost the same, at least in two dimensions.

Based on the previous studies, we speculated that there may be a relationship between the fractal dimension and the steam production rate in geothermal reservoirs. However this speculation is yet to be studied and verified using production data.

**Conclusions**

Based on the present work, the following conclusions may be drawn:

1. The results demonstrated that the pore systems of the core samples from The Geysers geothermal field are fractals.
2. The fractal technique proposed in this article to characterize the capillary pressure curve of The Geysers rock works satisfactorily. The values of the fractal dimension can be used to represent the heterogeneity of different rock samples quantitatively.
3. The fractal dimension of The Geysers rock was greater than that of Berea sandstone, which implies that The Geysers rock is more heterogeneous than Berea sandstone.
4. The Geysers rock samples studied have similar features in capillary pressure curves. However the features of The Geysers rock are different from those of Berea sandstone.
5. Unlike Berea sandstone, the capillary pressure curves of The Geysers rock could not be represented using the frequently-used Brooks-Corey model.

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**Nomenclature**

- $D_f$: fractal dimension
- $N(r)$: number of the unit needed to fill the fractal object
- $r$: radius of the unit
- $S_w$: wetting-phase saturation
- $S_w^*$: normalized wetting-phase saturation
- $S_{nr}$: residual saturation of the wetting-phase

**References**


