53. $(\mathfrak{o}\mathfrak{s}')_\nu = (\mathfrak{o}_\mathfrak{x})_\nu \cdot (\mathfrak{s}'_\mathfrak{x})_\nu + (\mathfrak{o}_\mathfrak{y})_\nu \cdot (\mathfrak{s}'_\mathfrak{y})_\nu + (\mathfrak{o}_\mathfrak{z})_\nu \cdot (\mathfrak{s}'_\mathfrak{z})_\nu$

54. $\mathfrak{u}_{\nu+1} = \mathfrak{r}_\nu \cdot (\mathfrak{o}\mathfrak{s}')_\nu + \mathfrak{k}_{\nu+1} \cdot (\mathfrak{s}'_\mathfrak{z})_\nu$

55. $(\mathfrak{o}\mathfrak{s})^2{}_{\nu+1} = \dfrac{\mathfrak{u}_{\nu+1} \cdot \mathfrak{u}_{\nu+1} + 1 \cdot \mathfrak{v}_{\nu+1} + 2\,\mathfrak{k}_{\nu+1} \cdot \mathfrak{z}_\nu}{\mathfrak{r}_{\nu+1}{}^2}$

56. $(\mathfrak{o}\mathfrak{s})_{\nu+1} = + \sqrt{(\mathfrak{o}\mathfrak{s})_{\nu+1}{}^2}$

57. $\tilde{\mathfrak{d}}'_\nu = 1 \cdot \mathfrak{u}_{\nu+1} - \mathfrak{r}_{\nu+1} \cdot (\mathfrak{o}\mathfrak{s})_{\nu+1}$

58. $\mathfrak{x}_{\nu+1} = \tilde{\mathfrak{d}}'_\nu \cdot (\mathfrak{s}'_\mathfrak{x})_\nu + \mathfrak{x}_\nu$

59. $\mathfrak{y}_{\nu+1} = \tilde{\mathfrak{d}}'_\nu \cdot (\mathfrak{s}'_\mathfrak{y})_\nu + \mathfrak{y}_\nu$

60. $\mathfrak{z}_{\nu+1} = \tilde{\mathfrak{d}}'_\nu \cdot (\mathfrak{s}'_\mathfrak{z})_\nu + \mathfrak{z}_\nu - \mathfrak{k}_{\nu+1}$

61. $(\mathfrak{s}_\mathfrak{x})_{\nu+1} = (\mathfrak{s}'_\mathfrak{x})_\nu$

62. $(\mathfrak{s}_\mathfrak{y})_{\nu+1} = (\mathfrak{s}'_\mathfrak{y})_\nu$

63. $(\mathfrak{s}_\mathfrak{z})_{\nu+1} = (\mathfrak{s}'_\mathfrak{z})_\nu$

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Beam Displacement at Total Reflection: The Goos-Hänchen Effect, IV*

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Abstract

The paper is divided into four parts. Part I comprises the Introduction and the two chapters entitled "Reflection and Refraction of a Beam of Light" and "Total Reflection of an E-Polarized Beam". Part II treats the *Goos-Hänchen* effect in classical optics. The different descriptions are discussed and their results are compared with *Wolter's* measurements Part III deals with the *Goos-Hänchen* effect in other branches of physics such as acoustics, quantum mechanics, plasma physics and nonlinear optics. The Schoch effect is introduced; furthermore the total reflection of diverging and converging waves is investigated. The final Part IV is devoted to several applications of the *Goos-Hänchen* effect, including the case of absorbing media. In addition, it contains the Summary and Conclusion, and the extensive list of references.

Inhalt

Strahlversetzung bei der Totalreflexion. Die vorliegende Arbeit ist in vier Teile unterteilt. Teil I umfaßt die Einleitung und die beiden Kapitel „Reflexion und Brechung eines Lichtstrahles“ und „Totalreflexion eines E-polarisierten Lichtstrahles“. Teil II behandelt den *Goos-Hänchen*-Effekt in der klassischen Optik. Die verschiedenen Beschreibungen werden besprochen und ihre Ergebnisse mit den Messungen von *Wolter* verglichen. Teil III befaßt sich mit dem *Goos-Hänchen*-Effekt in anderen Zweigen der Physik, nämlich der Akustik, der Quanten-Mechanik, der Plasma-Physik und der nichtlinearen Optik. Der Schoch-Effekt wird eingeführt; ebenso wird die Totalreflexion divergierender und konvergierender Wellen untersucht. Der abschließende Teil IV ist einigen Anwendungen des *Goos-Hänchen*-Effektes, einschließlich dem Fall absorbierender Medien, gewidmet. Außerdem enthält er die Zusammenfassung und Schlußbetrachtung und das ausführliche Literaturverzeichnis.

6. Application and/or Utilization of the Goos-Hänchen Effect

This chapter is concerned with several cases in which the *Goos-Hänchen* effect is either applied or utilized. The discussion of each case is necessarily brief in order to stay within the scope of the present paper, but the reader will find ample references to pertinent literature. Our point of departure is

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a physical interpretation of Brewster's law in Section 6.1. This interpretation is then utilized in Section 6.2 to discuss the *Goos-Hänchen* effect in the case of absorbing media. Light is shed on the physics involved in the phenomenon of negative penetration at H polarization, noted by *Wolter*. Thereafter two important applications are considered: namely, the total reflection holography in Section 6.3 and the internal reflection spectroscopy in Section 6.4. The final section deals with reflection phenomena at radio frequencies, including the case of a varying dielectric constant in the less-dense medium. It is shown that the well-known phenomenon of long-distance propagation along the ionosphere is, in a sense, nothing but the *Goos-Hänchen* effect modified for the case of a diverging beam.

The application of the *Goos-Hänchen* effect to plasma measurements was mentioned in Section 4.3. That discussion can readily be extended to include the case of dispersive media in general. The critical angle for total reflection determines the real part of the complex index of refraction and the blurring of the reflected image its imaginary part [19]. Furthermore, the *Goos-Hänchen* effect takes place at two distinct angles of incidence[14] if the less-dense medium is represented by an anisotropic and uniaxial crystal [19]. Measurement of the angle difference provides information about the orientation of the crystal.

Up to this point, we have assumed that the less-dense medium is homogeneous and isotropic with either a fixed or a varying (see Section 6.5) dielectric constant. This assumption can be relaxed, and our treatment of the *Goos-Hänchen* effect may be extended to take into account a stratified medium occupying the $z < 0$ half-space in Fig. 3. Unfortunately, such a treatment in general terms becomes rather cumbersome due to the many parameters involved, as may be inferred from the theory of electromagnetic-wave propagation in stratified media (e. g., [30d] and [82]). Total reflection will take place if the stratified medium is, on the average, optically less dense than the first medium in Fig. 3, but the angle range may be interrupted by discrete ranges of partial reflection. This is because a stratified medium reveals the feature of a filter with stop and pass bands ([82c] and [83]). In the regions of total reflection the reflection coefficient can be expressed in the general form of (20a), whose phase shift is determined by the optical properties and the physical dimensions of the multi-phase stratified medium. After having calculated this phase shift, the *Goos-Hänchen* effect is given by either (43) or (46). Particular attention must be given to such cases in which guided modes [38] are excited in some layer of the multi-phase stratified medium. At the angle of incidence for phase-velocity matching with a mode a rather sudden change in phase may occur, leading to a phenomenon[15] analogous to the Schoch effect introduced in Section 4.1. These modes propagate away energy which may return to the denser medium in a manner similar to the illumination-trailing phenomenon discussed in Section 5.3, [78b]. Such a scheme can be used to advantage when coupling energy into modes of a planar dielectric waveguide ([84] and [58]).

Another class of problems deals with the frustrated total reflection which is sometimes called optical tunneling or barrier penetration. If a beam of light is incident on the gap, for example, between two prisms at an angle of incidence greater than the critical angle for total reflection, it is found that some light penetrates the gap when the spacing is small (a few wavelengths). Without the second prism, total reflection would occur. This phenomenon was discovered by *Newton* [Ref. 2, Pages 194 and 205] and was later studied by several researchers (e. g., [31a, b] and [85]). Its investigation with microwaves was carried out by *Schaefer* and *Gross* at 15-cm waves [86], *Culshaw* and *Jones* at 1.25-cm waves [87], and *Brady* et al. at 3-cm waves [88]. Several recent papers [89] deal with different applications of frustrated total reflection to problems of modern optics.

There are conceivable applications of a more practical nature, which can only be mentioned briefly due to the limitation on space. Such applications may, for example, be encountered in schemes for modulating and deflecting narrow laser beams, in the design of optical delay lines, in arrangements to improve the mode quality of a solid-state laser, etc. The latter example involves the reduction of the filamentary laser mechanism [90a] in a marginally unstable resonator as in Reference 90b but without increasing the beam divergence.

6.1 *Brewster's Law*

The denominators in (13a) and (16a) are finite, except when $(\varphi_1 + \varphi_2) = \pi/2$. Then $\tan(\varphi_1 + \varphi_2) = \infty$, and consequently $R_{\parallel} = 0$. In this case the reflected and refracted rays are perpendicular to each other, and it follows from the law of refraction (12b), since now $\sin\varphi_2 = \sin(\pi/2 - \varphi_1) = \cos\varphi_1$, that

$$\tan\varphi_1 = n. \tag{66}$$

The angle of incidence thus defined is called the polarizing or Brewster angle. If light is incident under this angle, the electric vector of the reflected light has no component in the plane of incidence; i.e., the reflected light is E-polarized.

The above result, often called Brewster's law, can also be explained by the following more direct argument [91]: The incident field gives rise to vibrations of electrons in the atoms of the second medium. These vibrations are in the direction of the electric vector of the refracted wave. The vibrating electrons give rise to another wave, the reflected wave, which will be propagated back into the first medium like the reflected light harmonics in nonlinear optics (see, e.g., [57a]). Now a linearly vibrating electron, i.e., an electric dipole, radiates transversely [92] so there is no flux of radiant energy in the direction of vibration. It follows that, when the reflected and refracted rays are at right angles to each other, the reflected ray does not receive any energy for oscillations in the plane of incidence.

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[14] Note the similiarity with the discussion about phase matching in the nonlinear-optics case (Section 4.4).

[15] The optical problem has very recently been indicated in a short note [58a] and the acoustical one involving Lamb's waves has previously been treated ([30a] and [53]).

The above argument reveals that the phase of the reflected wave with H polarization changes abruptly by π if the angle of incidence passes through the Brewster angle. Recalling the interpretation of Section 4.0, a phenomenon analogous to the *Goos-Hänchen* effect may consequently be anticipated. Whether it can be observed in an experiment is doubtful since the amplitude of the reflected H-polarized wave is extremely small about the Brewster angle, [23a].

Attention is directed to an interesting feature of the *Goos-Hänchen* effect in the case of H polarization. The denominators in (37), (40b) and (44b) vanish at the Brewster angle. This singularity is immaterial since the Brewster angle (66) is smaller than the critical angle (18) for total reflection, with the one exception briefly mentioned below. Therefore, the electric field of the reflected wave always oscillates in-phase with the electrons vibrating in the less-dense medium. The situation changes drastically if absorption must be taken into account, as will be shown in the following section.

In the limit, as the index of refraction (12) approaches zero, $\tan\varphi_1 \approx \sin\Phi$; but for $\varphi_1 \leq \Phi$ the incident H-polarized wave, if normal to the interface, cannot excite the inhomogeneous wave (22) which would propagate along the interface. Phase velocity and wavelength of this wave would be infinite for $\sin\varphi_1 = 0$. Hence, the limiting values $R_\parallel = -1$ and $G_\parallel = 0$, obtained from (16) for the case under consideration, are physically consistent with the aforementioned argument.

6.2 *Absorbing Media*

The phenomena of reflection and refraction, treated in Chapter 1, are modified to a striking degree by the presence of an absorption (conductivity σ) in either medium. The law of refraction and Fresnel's formulas, given in Section 1.2, are still valid in a purely formal way but, as in the case of total reflection, the range of the complex angle φ_2 leads to a very different physical interpretation [93]. There is no longer a fixed critical angle for total reflection, but a transition of finite angular width, [112].

Let us suppose that in Fig. 3 the first medium is still a perfect dielectric with the real dielectric constant ε_1 but that the second medium is now absorbing. As in the case of a plasma, considered in Section 4.3, absorption is conveniently described by the complex dielectric constant $\varepsilon_2 = \varepsilon_2' - i4\pi\sigma_2/\omega = (n_2 - i\varkappa_2)^2$, where the former relation is repeated from (53a) and the latter defines the optical constants n_2 and $\varkappa_2$ ([10] and [94]). Hence, the real index of refraction (12), defined by the ratio $\sqrt{\varepsilon_2/\varepsilon_1}$, is now represented by the complex number $n = (n_2 - i\varkappa_2)/n_1$, where the shorthand n_1 stands for $\sqrt{\varepsilon_1}$. On substituting into the law of refraction (12b), the angle of refraction φ_2 becomes complex, and the refracted field is represented by a system of inhomogeneous plane waves as in the case of total reflection. The planes of constant amplitude are parallel to the interface, and the direction of propagation is determined by the normal to the planes of constant phase. The angle Φ_2 made by this wave normal with the negative z-axis is the true angle of refraction. It is defined by the equation

$x \sin\Phi_2 - z \cos\Phi_2 = \text{constant}$ and for $(4\pi\sigma_2/\omega)^2 \gg 1$ given by the approximate relation [93]

$$\sin\Phi_2 \simeq \sqrt{\frac{2\omega\varepsilon_1}{\sigma_2}}\,\sin\varphi_1. \tag{67}$$

As the conductivity σ_2 increases or the angular frequency ω decreases, the planes of constant phase align themselves parallel to the planes of constant amplitude, and the propagation takes place into the conductor in a direction normal to the surface.

The *Goos-Hänchen* effect is conveniently investigated on the basis of *Wolter's* theory outlined in Section 3.2, if the second medium is absorbing. The depth of penetration $Z_{AW\perp}$ is obtained by substituting the Artmann-Wolter expression (44a) into (32a), interpreting the complex result on grounds of (46), i.e.,

$$Z_{AW\perp} = \pm \frac{\lambda_0}{2\pi} \frac{1}{\sqrt{n_1^2 \sin^2\varphi_1 - n_2^2 + \varkappa_2^2 + i2n_2\varkappa_2}}, \tag{68a}$$

where n has been replaced by $(n_2 - i\varkappa_2)/n_1$ and $\lambda_0 = n_1\lambda_1$ is the vacuum wavelength. The sign of $Z_{AW\perp}$ is determined by the condition $\mathrm{Re}\{Z_{AW\perp}\} > 0$ so that the exponential functions in (27) decay. The corresponding expression for H polarization is obtained from (44b) and (32a) after a straightforward calculation, i.e.,

$$Z_{AW\parallel} = Z_{AW\perp} \left[\frac{(n_2^2 - \varkappa_2^2 - i2n_2\varkappa_2)/n_1^2}{\sin^2\varphi_1 - \cos^2\varphi_1\,(n_2^2 - \varkappa_2^2 - i2n_2\varkappa_2)/n_1^2} \right]. \tag{68b}$$

Wolter [16] calculated the reflection of light from a thick layer of silver behind glass ($n_1 = 1.52$, $n_2 = 0.18$ and $\varkappa_2 = 3.31$ at $\lambda_0 = 5500$ Å and $\varphi_1 = 45°$) and found

$$Z_{AW\perp} = \lambda_0\, 0.046\,(1 - i0.05) \quad \text{with} \quad \mathrm{Re}\left\{Z_{AW\perp}\right\} = 2.6 \times 10^{-6}\,\text{cm}$$

and

$$Z_{AW\parallel} = Z_{AW\perp}\,(-1.84 - i0.02) \quad \text{with} \quad \mathrm{Re}\left\{Z_{AW\parallel}\right\} = -4.6 \times 10^{-6}\,\text{cm}. \tag{69}$$

This numerical example reveals two significant facts: Firstly, the minimum ray is hardly blurred since the imaginary parts amount to about 10 percent of the already small real parts. Secondly, the depth of penetration for E polarization is positive and that for H polarization is negative. That is, so to speak, the H-polarized beam is reflected by a virtual surface close to the front of the interface but outside the second medium, as already mentioned in Section 3.2. In both cases the depth of penetration is minute and amounts to less than 10 percent of the wavelength.

The latter phenomenon, noted by *Wolter* [16], is readily understood in view of the physical interpretation for Brewster's law, sketched in Section 6.1. The inhomogeneous wave no longer travels along the interface in the less-dense medium, as indicated in Fig. 4b. On account of absorption the direction of propagation is given by (67), and energy is propagated into the second medium. Consequently, the reflected and refracted rays intersect at an obtuse angle, and the oscillation of the reflected wave is out-of-phase with the vibration of the electrons in the absorbing medium. The situation would be reversed if the second medium exhibits gain. The point of inflection is represented by the Brewster angle in the sense of metal optics.

The general problem of reflection and refraction at a plane interface between absorbing media is rather involved. This is because the superposition of incident and reflected waves does not produce a purely standing wave, even at normal incidence, in the first medium. An interference term is responsible for the energy redistribution between spatial regions in which the resultant field has nodes or antinodes, [95]. A detailed treatment of this problem would exceed the scope of the present paper and the reader is referred to elsewhere [96]. The corresponding problem for converging and diverging waves was briefly reviewed in Section 5.2.

The present discussion has tacitly emphasized homogeneous and isotropic media with strong absorption; the application to weakly absorbing material and films with randomly oriented absorber dipoles is deferred to Section 6.4.

6.3 Total Reflection Holography

Total (internal) reflection holograms are produced by object and reference beams after total reflection of either beam or both, as proposed by *Stetson* [97]. Suppose that the photographic emulsion occupies the first medium in Fig. 3, and that the reference beam is totally reflected at the emulsion/air interface. Then, if the object disturbance impinges from the second medium, the object and its hologram can be spaced very closely without blocking the reference beam. The experimental results demonstrate a remarkable insensitivity with regard to influences degrading conventional holograms.

Conversely *Nassenstein* [98] and *Bryngdahl* [99] proposed to form the holograms with the evanescent wave-fields in the less-dense medium, now representing the photographic emulsion. The latter investigator, in particular, discussed the features of these fields as they apply to the holography under consideration. He pointed out the significance of the depth of penetration, illustrated by Fig. 6, and that of its dependence upon the angle of incidence, the polarization and the magnitude of the critical angle for total reflection. If the object beam is totally reflected, too, the choice of the optimum angle of incidence is not so obvious and is likely to be determined by the experimental setup. This is because all the plane-wave components composing the object beam should be confined to the region of total reflection. Furthermore, the dependence upon polarization will be modified by the electric conductivity (absorption) of the photographic emulsion due to the generation of silver atoms, [100]. This generation, although negligible in conventional photography, is determined not only by the intensity of the object beam, assuming a constant reference beam, but also by the period of time the emulsion is exposed. The H-polarized beam, of course, penetrates most deeply into the less-dense medium if nonabsorbing, but its depth of penetration may change sign if the absorption becomes significant, according to the previous section. This phenomenon, if significant, is apt to limit the ultimate resolution of such a hologram, in particular for unpolarized light.

The phase velocity $c/(\sqrt{\varepsilon}\sin\varphi)$ and wavelength $\lambda/\sin\varphi$ of the evanescent wave are determined by the primary field in the denser medium according to (22); only the depth of penetration depends upon the properties of the less-dense medium. It thus appears possible, by appropriately selecting the angle of incidence, to exceed the classical limit on resolution when illuminating a structure within the less-dense medium with the evanescent wave [101][16]. – A similar argument seems to apply to the case of a diffraction grating with a grating constant smaller than the wavelength.

Nassenstein [102] demonstrated the interference of evanescent waves produced by two impinging waves with slightly different angles of incidence. His recordings substantiate *Wolter's* interpretation of the light penetration into the less-dense medium [11], outlined in Section 2.2.

6.4 Internal Reflection Spectroscopy

Multiple total internal reflections within a parallel slab of an optically denser material [38], as discussed in connection with the historical experiment of *Goos* and *Hänchen* in the Introduction, can be used to advantage in the optical probing of surfaces of optically transparent materials ([37] and [103]). Because the reflectivity is high (about 100%), hundreds of internal reflections can be employed, thus enabling weak absorptions to be amplified and measured. Mechanisms associated with surfaces, both inside and outside the reflecting interface, have been measured using multiple internal reflections. It was found in particular, that because of the light penetration into the less-dense medium, absorption spectra of adsorbed monolayer surface-films on the outside of the interface can be measured [104]. Another application of internal reflection spectroscopy is in the measurement of optical constants of materials [105].

Harrick [106b] recognized that the electric field strength, being the crucial quantity in absorption processes, depends critically upon both the polarization and the angle of incidence, in particular near the critical angle for total

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[16] This is true in principle but may encounter difficulties in its realisation at optical wavelengths. Most conventional photographic plates are manufactured with a thin layer covering the emulsion. The thickness of this layer is comparable with the optical wavelength and thus, except near the critical angle for total reflection (Fig. 6), the evanescent field decays too rapidly to provide sufficient exposure to the photographic emulsion in general. This conclusion is supported by measurements on film blackening after development, as reported in Reference 98b. Special photographic plates without such a layer are now available, too. (The author acknowledges recent discussions on this subject with Prof. *H. Nassenstein*, Farbenfabriken Bayer AG, Leverkusen, Germany).

reflection. In the case of H polarization the E_z-component even reveals a discontinuity because of the requirement that the dielectric displacement be continuous, i.e., in the less-dense medium the electric field strength is magnified by the dielectric constant of the denser medium. – This analysis can be extended to multi-phase stratified media [82b]. – Therefore, the interaction with an absorber (e.g., a dipole) depends upon its orientation relative to the surface as well as upon the angle of incidence. Very strong coupling to dipoles outside of, and oriented normal to, the interface can be obtained because E_z is large; for a film with randomly oriented dipoles the strength of coupling to the H-polarized wave component in the less-dense medium (which depends upon both E_x and E_z) is always greater than the coupling to the E-polarized wave component (i.e., E_y). From such measurements it is possible to determine the orientation of dipoles on the interfaces. In the denser medium, however, the coupling to the two orthogonally polarized plane-wave components is about the same; unlike a propagating wave, where components of the electric field strength are found only perpendicular to the direction of propagation, respective components in all directions are found in front of a totally reflecting interface.

The absorption in the less-dense medium can be expressed in terms of an "effective thickness" due to the magnified electric field strength in the less-dense medium. The effective thickness is defined as the thickness of a film of the same material which would give identical absorption for transmission at normal incidence [106]. A pictorial interpretation stemming from the *Goos-Hänchen* effect was recently proposed by *Hirschfeld* [107]. A still better description may be derived from the path length within the less-dense medium, given by (59).

6.5 *Reflection Phenomena at Radio Frequencies*

The theory of the *Goos-Hänchen* effect is readily applicable to reflection phenomena at radio frequencies since no constraint was imposed on the wavelength. *Wolter* initially developed his theory, outlined in Sections 3.2 and 3.3, to explain a strange reflection phenomenon above seawater in the near-field zone of an antenna at long wavelengths, [16][17]. We assume that the incident wave with a 10-km wavelength impinges from the air upon seawater with electric conductivity $\sigma_2 = 10^{-2}\Omega^{-1}\mathrm{cm}^{-1} = 9 \times 10^9$ el. st. cgs, occupying the second medium in Fig. 3. The complex dielectric constant of seawater is approximately given by $\sqrt{\varepsilon_2} = (1-\mathrm{i})\sqrt{2\sigma_2\lambda/c} = 775\,(1-\mathrm{i})$ according to (53a) since $\varepsilon_2' = 80 \ll 2\sigma_2\lambda/c = 6 \times 10^5$. Neglecting $\sin^2\varphi$ in the Artmann-Wolter expression (44a) yields with (32a)

$$Z_{AW_\perp} = \frac{(1-\mathrm{i})}{4\pi}\sqrt{\frac{c\lambda}{2\sigma_2}} = 98\,(1-\mathrm{i})\ \mathrm{cm}\,. \tag{70a}$$

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[17] Unfortunately, these investigations, originally carried out during World War II, are unpublished, except for the second example in Reference 16.

The Artmann Wolter expression (44b) for H polarization can be simplified because the magnitude of $\csc^2\Phi = 1/n^2$ is negligible in comparison to 1, i.e.,

$$Z_{AW_\parallel} = -Z_{AW_\perp}/\cos^2\varphi\,. \tag{70b}$$

The negative depth of penetration indicates that the H-polarized beam is reflected by a virtual surface above the water. Despite its value it seems not easily accessible to measurement since the blurring of the minimum ray is of roughly the same order of magnitude. This example is readily understood on the basis of the interpretation of Brewster's law outlined in Section 6.1; it represents the other limiting case, namely that of a small angular frequency.

It is well known that radio waves are reflected and guided by the ionosphere, in particular by the Kennelly-Heavyside layer [108]. The ionosphere represents a plasma, as discussed in Section 4.3, having a dielectric constant which decreases with increasing height above the earth, and thus constituting a phenomenon similar to total reflection at a discontinuous change in the index of refraction. The theory of the *Goos-Hänchen* effect is not applicable in a strict sense [24] but, since the characteristics of the plasma change drastically over a distance comparable with the wavelength, it is nevertheless tempting. The ionosphere is separated from the antenna by a relatively small number of wavelengths, and the electromagnetic radiation encounters the ionosphere as a diverging wave. Consequently, v. Schmidt's lateral wave is generated at the total reflection according to Section 5.2. This wave seems responsible for the phenomenon of long-distance propagation along the ionosphere familiar to us from daily experience. The physical connection becomes even more apparent when recalling *Maecker's* interpretation of the path a ray of light traces through the less-dense medium, [17]. He noted that this path can be described by a rapid but continuous change in the index of refraction over the distance of a few wavelengths. At radio frequencies this distance is comparable to the width of the ionosphere. Therefore, the great similarity of our Fig. 17 with Fig. 6 of Reference 108a is not entirely surprising, and the physical relationship of this phenomenon with the *Goos-Hänchen* effect is established.

The electromagnetic-wave propagation in the space between earth and ionosphere can be treated as a waveguide problem [109] in a manner similar to the planar dielectric waveguide at optical wavelength [38]. The criss-crossing waves are reflected totally at the ionosphere and metallically at the earth. These reflections are, however, accompanied by v. Schmidt's lateral waves, due to the diverging nature of the long-wavelength radiation. A similar waveguiding problem seems likely in the evaporation inversion-layer duct above the sea at very low frequencies.

The propagation of electromagnetic waves through a nonuniform medium represents a very important, but extremely difficult problem and has not been solved in general. Several special cases involving nonuniform dielectric constants have been treated in the literature (see, e.g., [30b], [78b], and [110]).

Finally, let us digress from the topic of the present section and have a second glance at the numerical exercise treated above. This problem represents the special case in which the real part of the complex dielectric constant is negligible in comparison with the imaginary part. It also holds in the case of a large conductivity so that the above mentioned phenomenon, discovered by *Wolter* at radio frequencies, should equally well arise at higher frequencies. It is conceivable that this phenomenon must be taken into account in higher-order theories, for example, for the vectorial photoelectric effect, the excitation of surface plasmas, and the generation of light harmonics at reflection. Its significance vanishes in the limit of infinite conductivity in agreement with Maxwell's theory.

Summary and Conclusion

The theory of the *Goos-Hänchen* effect has been treated extensively from different points of view. This phenomenon, representing a lateral beam displacement in the plane of incidence, arises in the total reflection of a parallel beam linearly polarized. It was originally discovered in classical optics but is also encountered in other branches of physics, as discussed in greater detail. Emphasis has been placed on the optical problem for historical reasons but the results, properly interpreted, apply to other branches of physics as well. The mathematical description of reflection and refraction at a plane interface, assuming a beam of light rather than a plane wave, has been treated on grounds of an approximate solution of Maxwell's equations. This theory was originally proposed by *Schaefer* and *Pich*, but their treatment had to be modified by *Lotsch* to comply with the principle of conservation of energy. The reflected and refracted beams obey the laws of reflection and refraction. Their respective amplitudes are related to the amplitude of the incident beam by both Fresnel's formulas and extra quantities due to the limited transverse extent of the beam-fields. The latter expressions lose their significance in the limit as the incident beam becomes so wide that it can be approximated by a plane wave. This plane-wave ansatz is usually presented in text books on optics. For the case of E polarization the total reflection has been treated in greater detail, stating explicitly the electromagnetic fields as well as the Poynting vectors in both media. The reflection is not quite total in the border zones of the beams due to the limited extent of the beam-fields. In the plane of incidence energy is crossing the interface below the "leading" border zone of the incident beam. This energy propagates along the interface within the less-dense medium and afterwards re-emerges into the denser medium below the "trailing" border zone. As a consequence, the line of gravity of the totally reflected beam is shifted into the forward direction, as pointed out by *v. Fragstein*. This phenomenon has been called the *Goos-Hänchen* effect by Professor *H. Wolter* in recognition of the fundamental research of *Goos* and *(Lindberg-)Hänchen*. It can be expressed equivalently either as a beam displacement, as a depth of penetration into the less-dense medium, or as a shift of the reflection center.

The *Goos-Hänchen* effect interpreted as light penetration into the less-dense medium is consistent with *Renard's* viewpoint: It vanishes in the limit of grazing incidence and reduces to the classical formulations near the

critical angle for total reflection. These classical formulations derived by *Artmann* and *Wolter* on grounds of physical optics are fully substantiated by *Wolter's* measurements for those angles of incidence which are of interest in practice. This experimental verification also holds in the transition region from total to partial reflection, described by *Wolter's* extended theory. Various approximations are discussed and compared by actual numerical calculations.

The *Goos-Hänchen* effect arising in acoustics, quantum mechanics, and plasma physics has been treated in detail. In the former case the problem can be rather involved if both longitudinal and shear waves can be excited. The *Goos-Hänchen* effect may take place at more than one angle of incidence due to the critical refraction of these waves at different angles of incidence. A large beam displacement localized at the angle for excitation of the Rayleigh wave has been termed the Schoch effect, originally predicted and since demonstrated by *Schoch*. In plasma physics the propagation of electromagnetic waves is described by a complex dielectric constant. Additional complication arises from the frequency-dependence of the plasma characteristics. The nonlinear-optics experiment, assuming a less-dense nonlinear medium, substantiates the viewpoint of light penetration into the less-dense medium. A simple method to calculate the *Goos-Hänchen* effect of light harmonics on linear grounds has been proposed.

There are a number of phenomena which are related to the *Goos-Hänchen* effect. A perpendicular beam displacement, predicted by *Fedorov*, may occur if the incident beam is not simply E or H polarized. The reflection of a non-parallel beam at the plane interface toward a less-dense medium has been treated comprehensively, assuming either converging or diverging waves. The reflection of a diverging wave is accompanied by v. Schmidt's lateral wave. This wave may be visualized by a conical wave front. It was first noted by *Mintrop* in seismic shock experiments and later explained by *v. Schmidt* in acoustics. *Maecker* demonstrated it in optics and established the physical correlation with the *Goos-Hänchen* effect. He showed that v. Schmidt's lateral wave arises only in the total reflection of a diverging beam, but not in the case of parallel or converging beams. This lateral wave seems responsible for the illumination-trailing phenomenon observed by *Acloque* and *Guillemet*, and the dip in the acoustic reflectivity, at a liquid/solid interface, near the critical angle for total reflection, reported by *Weinstein* and others.

The *Goos-Hänchen* effect is encountered in a variety of applications. Brewster's law has been considered to shed some light on the physical interpretation of the *Goos-Hänchen* effect in the case of an absorbing less-dense medium. The phenomenon of a negative penetration, noted by *Wolter*, is easily understood on this basis. Brief account has been given to the *Goos-Hänchen* effect in total reflection holography and internal reflection spectroscopy. Reflection phenomena at radio frequencies are sketched and related to the *Goos-Hänchen* effect. The well known phenomenon of long-distance propagation along the ionosphere has been explained with the *Goos-Hänchen* effect, suggesting that the anomalous propagation may be caused by v. Schmidt's lateral wave.

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Note added to the galley proof: Horowitz and *Tamir* [113] are currently engaged in a detailed investigation of the Goos-Hänchen effect in the transition region from partial to total reflection. They arrived at the significant conclusion that the finite beam displacement at the critical angle for total reflection depends upon the width and the shape of the incident beam as well as upon the angle of incidence. They also found that the maximum beam displacement occurs at an angle that is slightly larger than the critical angle for total reflection. A similar result was found in Section 4.3 for the case of absorption.

The energy coupling into modes of a planar dielectric waveguide [38] has been the subject of several recent papers over and above the already cited References 58 and 84. The prism-film coupler was considered in [114] and the grating coupler in [115]. The effect of v. Schmidt's lateral wave on the energy coupling has been discounted so far. – It should be pointed out that the scheme of the prism-film coupler was previously used by *Otto* [117] to excite non-radiative surface plasma waves in silver.

It is interesting to note that the Schoch effect was recently utilized to excite and to probe elastic surface waves [116].

The beam displacement perpendicular to the plane of incidence, sketched in Section 5.1, can be interpreted as resulting from a photon-spin interaction according to *Costa de Beauregard* and *Imbert* [118]. Recently, this phenomenon *has been* verified by experiment [118c].

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