Estimating flow properties of porous media with a model for dynamic diffusion

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Summary

We present an approach for estimating effective compressibility and permeability from Differential acoustic resonance spectroscopy (DARS) laboratory measurements. The effective compressibility of fluid-saturated porous medium, located in a harmonic pressure field, is a function of the loading frequency and the fluid in the pore space, as well as the permeability and porosity of the medium. The process is describable by a diffusion process that relates effective compressibility and permeability. DARS is used to measure the effective compressibility. Then the diffusion model is used to estimate the permeability. This method is tested with DARS lab data.

Introduction

Harris et al. (2005) presented a method for estimating acoustic attenuation with DARS. The key component of DARS consists of a resonant fluid-filled cavity. At resonance, the standing wave inside the cavity provides a spatially varying but harmonic pressure field along the major axis of the cavity. The introduction of a porous sample of rock (Figure 1) perturbs the resonant modes of the cavity in a way that allows estimation of an effective attenuation and effective compressibility. If the sample is placed at a pressure anti-node, according to Morse and Ingard (1968), and Harris et al. (2005) the perturbed resonance frequency of the resonator can be expressed as

$$\omega_s^2 - \omega_0^2 = -\frac{\omega_0^2}{\lambda} \left(\frac{V_s}{V_c}\right)^2 \delta \kappa.$$  \hspace{1cm} (1)

In equation (1), $\omega_s$ and $\omega_0$ are the resonance frequencies of the cavity with and without the sample, respectively; $\Phi$ is the acoustic pressure inside the cavity; $\lambda$ is a constant coefficient; $V_s$ is the volume of the sample and $V_c$ is the volume of the cavity. The parameter $\delta \kappa$ is defined by $\delta \kappa = (\kappa_f - \kappa_s)/\kappa_f$, where $\kappa_f$ and $\kappa_s$ are the compressibilities (reciprocal of bulk modulus) of the fluid and the sample, respectively; $\kappa_f$ is defined by $\kappa_f = (\rho c)^{-1}$, in which $\rho$ and $c$ are the density and acoustic velocity of the fluid, respectively, and $\kappa_s$ is given by $\kappa_s = \left[\frac{\kappa}{\rho v_p v_s}\right]$, where $v_p$ and $v_s$ are the P-wave and S-wave velocities of the sample.

Rearranging (1), we get an expression for compressibility:

$$\kappa_s = (1 + A \xi) \kappa_f,$$  \hspace{1cm} (2)

where $\xi = \left(\frac{\omega_s^2 - \omega_0^2}{\omega_0^2}\right) \left(\frac{V_s}{V_c}\right)^2$, $A = -\frac{\lambda}{(\Phi)^2}$.

In equation (2), $\xi$ is the measured frequency perturbation, $\kappa_f$ is the compressibility of the fluid inside the cavity, and the coefficient $A$ can be obtained from calibrations with a reference sample. The calibration factor $A$ is considered not to change for the other unknown samples. The compressibility or bulk modulus of an unknown sample can be calculated from equation (2) with the perturbation it produces in the acoustic resonator.

Figure 1: The key component of DARS is the resonator. In the measurement, the unknown sample is placed at the center of the cavity, where the acoustic pressure is maximum at the first mode.

We used DARS to estimate the compressibilities of both nonporous and porous materials and compared the results with that derived by ultrasound measurements. We found that the compressibilities obtained by two different methods are comparable for nonporous materials (Figure 2) but not always for porous samples. Figure 3 shows the comparison result of eight rock samples. The data points of the samples with extremely low permeability, such as the coal, chalk and the granite, are along the 45-degree line, which indicates that the compressibilities obtained by the two different techniques are equivalent for the three materials. The data for samples with median and high permeability, such as two Berea sandstones and the Boise sandstone, plot away the 45-degree line. The samples with higher permeability illustrate larger deviation from the 45-degree line. Porosity does not show this effect. For instance, the Chalk has high porosity; but plots close to the 45-degree line. Because the ultrasound measurements are on dry rocks, we therefore estimated the equivalent wet frame compressibility by using Gassmann fluid substitution procedure, and then compared the corrected results with that derived by the perturbation observations. However, the data points deviated even farther away from the 45-degree line. This phenomenon indicates that the compressibility derived by DARS measurements is apparently not the compressibility usually quantified by other techniques, e.g., ultrasound method. This observation motivated us to investigate the mechanism of the fluid and solid matrix interaction in the DARS measurements.
Estimating flow properties of porous media with a model for dynamic diffusion

Consider a fluid-saturated porous medium that is subjected to a small amplitude oscillatory pressure gradient; the pressure fluctuation will cause micro fluid flow through the sample to release the differential pressure. This phenomenon can be described by dynamic diffusion. In a cylindrical coordinate system, the pressure diffusion equation in a homogeneous porous medium with circular symmetric geometry has the following form:

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) + \frac{\partial^2 p}{\partial z^2} = \frac{1}{D} \frac{\partial p}{\partial t},
\]

with \( D = k/(\phi \eta \beta) \). Here, \( \phi \) and \( k \) are porosity and permeability of the porous sample, respectively; \( p \) is the acoustic pressure in the fluid, and \( \eta \) is the viscosity of the fluid; \( \beta \) is the compressibility factor involving both the fluid and the solid matrix simultaneously. Detailed derivation of the diffusion equation is presented, e.g., in Barenblatt et al. (1990).

If we ignore the flow in the axial direction, then we can simplify the expression as

\[
\frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial p}{\partial r} \right) = \frac{1}{D} \frac{\partial p}{\partial t}.
\]

Furthermore, if acoustic pressure is a sinusoidal function of time, i.e., \( p(r,t) = P(r)e^{i\omega t} \), we can rewrite equation (4) as

\[
\frac{\partial^2 p}{\partial r^2} + \frac{1}{r} \frac{\partial p}{\partial r} - i\omega \frac{p}{D} = 0.
\]

The solution of this equation is

\[
P(r) = \Delta PK_0\left(\sqrt{i\omega/Dr}\right).
\]

Here, \( K_0 \) is the zero order modified Bessel function of the second kind. \( \Delta P \) is the amplitude of pressure fluctuation at sample surface \( r = r_0 \), and \( \omega \) is the resonant frequency of the acoustic system.

Effective compressibility (EC) of porous materials in a periodic pressure environment can be expressed by the ratio of the net volumetric strain of the material to the corresponding stress on the sample. The net volume change of the sample consists of contributions from the solid matrix and the pore fluid. Therefore, the EC of the porous sample can be written as

\[
\kappa_e = \frac{1}{V_i} \left( \frac{\Delta V_m + \Delta V_f}{\Delta P} \right),
\]

where \( \Delta V_m \) and \( \Delta V_f \) are the net volume change of the rock matrix and the pore fluid inside the pore space, respectively; \( V_i \) is the bulk volume of the sample.

According to the definition of compressibility, the net volume change of the frame, \( \Delta V_m \), can be written as:

\[
\Delta V_m = -\kappa_m V_i (1 - \phi) \Delta P,
\]

where \( \phi \) is the sample porosity and \( \kappa_m \) is the compressibility of the frame. The net volume change of the pore fluid is equal to the amount of fluid flows into and out the pore space driven by the changing pressure. Because of the spatial distribution of the pressure inside the rock, the fluid volume change can be written as

\[
\Delta V_f = -\int_{r_0}^{R} \kappa_f P(r) dr, \quad \text{where} \quad dV_f = 2\pi r dr.
\]

Substituting the expression of \( \Delta V_m \), \( \Delta V_f \) and \( P(r) \) into equation (7), we get the EC expression,

\[
\kappa_e = \kappa_m (1 - \phi) + \frac{2\phi \kappa_f}{r_0} \int_{r_0}^{R} \frac{K_0(\sqrt{i\omega/Dr})}{K_0(\sqrt{i\omega/D_{r_0}})} r dr.
\]

It can be seen from equation (8) that the EC of a fluid-saturated porous material is a function of the frequency of the pressure field, the property of the pore fluid, and the porosity and permeability of the sample.
Estimating flow properties of porous media with a model for dynamic diffusion

Experiment
The key component of DARS is an open-ended cylindrical cavity, which is immersed in a fluid-filled tank. In the measurement, the sample is placed at the center of the cavity, where the acoustic pressure is highest (for the first resonant mode). As equation (1) shows, at the pressure antinode, the smaller compressibility of the samples shifts the frequency higher than the empty cavity resonance response. A typical resonance response of the system with and without the sample is shown in Figure 4.

Figure 4: Power spectrum of the acoustic system. \( \omega_0 \) is the resonant frequency without the sample. Under the interference of a small sample, the resonant frequency shifts to \( \omega_s \).

The measurement results discussed in this paper involved eight rock samples. Their acoustic and flow properties are listed in Table 1. All the samples were fully saturated with the same fluid contained in the cavity. Furthermore, we considered only the first resonance mode.

<table>
<thead>
<tr>
<th>Sample</th>
<th>( v_p ) (km/sec)</th>
<th>( v_s ) (km/sec)</th>
<th>( \rho ) (g/cc)</th>
<th>( \phi ) (%)</th>
<th>( k ) (mDarcy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berea 1</td>
<td>2.656</td>
<td>1.650</td>
<td>2.101</td>
<td>20.85</td>
<td>370</td>
</tr>
<tr>
<td>Berea 2</td>
<td>2.600</td>
<td>1.544</td>
<td>2.142</td>
<td>28</td>
<td>6000</td>
</tr>
<tr>
<td>Boise</td>
<td>2.837</td>
<td>1.658</td>
<td>1.786</td>
<td>34.5</td>
<td>2.1</td>
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<tr>
<td>Chalk</td>
<td>2.045</td>
<td>0.840</td>
<td>1.130</td>
<td>1.9</td>
<td>0.1</td>
</tr>
<tr>
<td>Coal</td>
<td>5.140</td>
<td>2.720</td>
<td>2.630</td>
<td>0.1</td>
<td>0</td>
</tr>
<tr>
<td>Granite</td>
<td>2.336</td>
<td>1.305</td>
<td>1.893</td>
<td>28.3</td>
<td>4200</td>
</tr>
<tr>
<td>Sandstone 1</td>
<td>2.053</td>
<td>1.205</td>
<td>1.982</td>
<td>24.9</td>
<td>1850</td>
</tr>
</tbody>
</table>

Table 1: Acoustic and flow properties of eight rock samples. The density, \( P \)- and \( S \)-wave velocities are measured in the dry state.

Results
The perturbation responses of several of the samples are shown in Figure 5. As Figure 5 illustrates, the resonance frequency of the system always increases under the interference of each sample; however, the magnitude of the increment is different. According to the perturbation theory, equation (1), this phenomenon states that the compressibilities of the relevant samples are different from each other but all are larger than that of the fluid inside the cavity. Figure 5 also shows that the Boise and the Chalk exhibit similar perturbation effect, which indicates that they have equivalent compressibility even though the two samples have dramatically different acoustic and flow properties (Table 1).

We calculated the compressibility of the eight rocks by the perturbation data using equation (2), and also estimated the corresponding theoretical values by the EC model, equation (8). The results derived by two different methods are compared in Figure 6 and Table 2. As Figure 6 shows, the cross plot of the eight rocks is along a 45-degree line. The correlation of the two observations is 0.998 and standard deviation of the data points from the 45-degree line is 0.0042. These features verify that the interaction of the fluid and the solid skeleton in DARS measurements on permeable samples is a dynamic diffusion process, thus we can use the EC concept to interpret the DARS observations. As equation (8) shows, the EC of porous materials contains the information of the loading frequency, the properties of the fluid stored inside the pore space, and most importantly, the permeability and porosity of the media. Therefore, DARS provides the potential to investigate the flow properties of porous media. The fluctuation of the data points around the 45-degree line may attribute to: measurement errors in DARS and ultrasound measurements; the studied materials might be heterogeneous; the flow in the axial direction should also be considered in the derivation of the effective compressibilities.

We finally investigated the feasibility to estimate permeability of porous materials through the combination of DARS observations and the diffusion model. Berea 1 and the two sandstone samples were used with the assumption that the permeability was the only unknown parameter. The procedure was as follows: we first calculated the compressibility of the three samples by using DARS observations. Then we searched the optimal permeability by forcing the theoretically calculated EC to match with that estimated by DASR. The measured
permeability of the three samples by gas-injection are 370, 1850 and 4200 mDarcy respectively. The corresponding estimated permeability of each of the samples are 394, 1770 and 4230 mDarcy separately. The little mismatching between the measured permeability and that estimated by the EC model may attribute to errors within DARS observations and the gas-injection measurement of permeability.

In estimating the EC of the studied samples by equation (8), the involved porosity and skeleton compressibility are obtained by other techniques. We anticipate quantifying theses parameters solely by DARS measurements. The methodology will be addressed in a future study.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$\kappa$-ultrasound (GPa$^{-1}$)</th>
<th>$\kappa$-DARS (GPa$^{-1}$)</th>
<th>$\kappa$-diffusion (GPa$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Berea 1</td>
<td>0.138988</td>
<td>0.233421</td>
<td>0.246050</td>
</tr>
<tr>
<td>Berea 2</td>
<td>0.130360</td>
<td>0.340430</td>
<td>0.336044</td>
</tr>
<tr>
<td>Boise</td>
<td>0.098804</td>
<td>0.116515</td>
<td>0.104023</td>
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<tr>
<td>Chalk</td>
<td>0.099033</td>
<td>0.102417</td>
<td>0.096707</td>
</tr>
<tr>
<td>Granite</td>
<td>0.022967</td>
<td>0.023077</td>
<td>0.022949</td>
</tr>
<tr>
<td>Coal</td>
<td>0.273031</td>
<td>0.276650</td>
<td>0.273221</td>
</tr>
<tr>
<td>Sandstone 1</td>
<td>0.017663</td>
<td>0.331198</td>
<td>0.331083</td>
</tr>
<tr>
<td>Sandstone 2</td>
<td>0.012070</td>
<td>0.326637</td>
<td>0.327851</td>
</tr>
</tbody>
</table>

Table 2: Cross plot the compressibility evaluated by perturbation observations and the analytical results from the dynamic diffusion model for the eight rock samples.

**Conclusion**

The interaction of the fluid and the solid skeleton in DARS measurements on permeable samples is a dynamic diffusion process. The compressibility estimated by the perturbation measurements is a function of permeability and porosity, frequency of the pressure variation, and the properties of the fluid inside the pore space.

The experimental results of eight rock samples show that permeability has considerable effect on the EC of porous media. For samples with very low permeability (~1mD), the compressibility derived from the perturbation observations are comparable to that derived by ultrasound method. Contrarily, for those materials with relatively high permeability, the compressibilities observed by the perturbation measurements are much lower than that obtained by ultrasound method.

We derived an effective compressibility model based upon a dynamic diffusion process, and compared the analytical results for eight rocks with that derived by the perturbation observed. The results agree well, which indicates that we can utilize the dynamic diffusion model to interpret the perturbation measurements, and estimate the flow properties of porous media.

We use three rock samples to validate the feasibility to estimate permeability of porous materials through DARS observations. The estimated permeability of the three rocks is comparable with that measured by gas-injection method.

**References**


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REFERENCES