Differential Acoustic Resonance Spectroscopy: An experimental method for estimating acoustic attenuation in porous media
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Summary
A new acoustic method of estimating attenuation in rocks is presented. This method, called Differential Acoustic Resonance Spectroscopy or DARS, uses the perturbation in the $Q$ and resonant frequency of a fluid-filled cavity resonator caused by the introduction of a small sample of rock. The changes in the resonant frequency and the quality factor are used to characterize the velocity and attenuation properties of the rock sample. The frequency of DARS operation is determined by the size of the fluid-filled cavity, not the size of the rock sample; therefore, DARS can be used to measure an acoustically small sample in the laboratory at low frequencies. This paper describes the DARS concept, theory of operation, and some attenuation results for small porous samples at a frequency near 1 kHz.

Introduction
Two challenges make it difficult to draw solid conclusions regarding attenuation mechanisms: (1) Inherent limitations of current laboratory techniques. Both pulse transmission/reflection and resonance bar techniques give reliable estimates, but pulse techniques operate in the hundreds of kilohertz range and resonance bar techniques usually require large samples or time-consuming sample preparation (Boubie, 1987). (2) The issue of frequency scaling to the band of interest where different attenuation mechanisms may dominate in comparison to the band of the measurement.

We developed DARS to estimate attenuation on small samples of rocks in the laboratory at frequencies applicable to field seismic and logging experiments, e.g., around 1 kHz. Moreover, we expect to use DARS to develop a better understanding of attenuation and the fluid flow properties of porous media.

The acoustic cavity can be spherical, rectangular, cylindrical, or any other shape. The essential concept of DARS is the perturbation caused by the introduction of the sample of rock. We used a fluid-filled cylindrical resonator as illustrated in Figure 1. The first resonant frequency of longitudinal mode in the cylindrical cavity is well known:

$$f = \frac{c_0}{2L},$$  \hspace{1cm} (1)

where $c_0$ is the acoustic velocity of the fluid that fills the cavity and $L$ is the cavity length. The quality factor of the cavity is the resonant frequency $f$ divided by the half-power bandwidth $W$ of the resonance curve (Figure 2):

$$Q = \frac{f}{W}. \hspace{1cm} (2)$$

The resonant frequency and half power bandwidth are found by fitting a Lorentzian curve (Mehl, 1978; Migliori & Sarrao, 1997). When a rock sample is introduced, the resonant frequency either increases or decreases depending primarily on the velocity and density properties of the sample and its location in the cavity. Similarly, the quality factor changes to reflect the attenuation properties of the sample.

Figure 1: A fluid-filled cylindrical resonator with a small sample of rock inside. We measure the resonance frequency and $Q$ with the sample at different locations in the cavity.

Figure 2: DARS resonance curves. $f_1$ is the resonant frequency without the sample and $W_1$ is the half-power bandwidth. When a sample is introduced, the resonant frequency shifts to $f_2$ and the bandwidth changes to $W_2$. 

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Theory

The introduction of the sample perturbs the resonance properties of the otherwise empty cavity. Under the influence of the perturbation, the empty pressure \( p_1 \) in the fluid changes to \( p_2 \), and the resonant frequency shifts from \( \omega_1 \) to \( \omega_2 \). We use the acoustic wave equation to estimate first-order changes in resonance frequency and \( Q \) as follows:

\[
-\kappa_1 \omega_1^2 p_1 = \nabla \cdot \left( \frac{1}{\rho_1} \nabla p_1 \right) \quad (4)
\]

\[
-\kappa_2 \omega_2^2 p_2 = \nabla \cdot \left( \frac{1}{\rho_2} \nabla p_2 \right) \quad (5)
\]

where \( \kappa_1 = 1/(\rho_1 c_1^2) \). The differential equations (4) and (5) are solved with the boundary conditions of \( \nabla p = 0 \) at the boundaries of the cavity and \( p = 0 \) at the open ends of the cavity. Multiplying (1) by \( p_2 \) and (2) by \( p_1 \), and integrating over the cavity volume \( (V_c) \), we have

\[
-\int_{V_c} \kappa_1 \omega_1^2 p_1 p_2 dV = \int_{V_c} \nabla \cdot \left( \frac{1}{\rho_1} \nabla p_1 \right) p_2 dV \quad (6)
\]

and

\[
-\int_{V_c} \kappa_2 \omega_2^2 p_2 p_1 dV = \int_{V_c} \nabla \cdot \left( \frac{1}{\rho_2} \nabla p_2 \right) p_1 dV \quad (7)
\]

With further manipulation of (6) and (7) and application of boundary conditions we get

\[
\omega_2^2 - \omega_1^2 = -\omega_2^2 \frac{\partial \kappa}{V_c} A - \omega_1^2 \frac{\partial \rho}{V_c} B \quad (8)
\]

where \( c_1^2 = 1/\rho_1 k_1 \), \( \partial \rho = (\rho_2 - \rho_1)/\rho_2 \), \( \partial \kappa = (\kappa_2 - \kappa_1)/\kappa_1 \), and

\[
A = \frac{V_c}{V_s} \int_{V_s} p_1 p_2 dV / \int_{V_c} p_1 dV \quad (10)
\]

\[
B = \frac{V_c}{V_s} \int_{V_s} \nabla p_2 \cdot \nabla p_2 dV / \int_{V_c} p_2 dV \quad (11)
\]

In (8), \( V_s \) is the volume of the sample and \( V_c \) is the volume of the cavity, \( \rho_1 \) is the density of the fluid, \( k_1 \) is the compressibility of the fluid, \( p \) is acoustic pressure in the fluid, and \( \nabla p \) is proportional to acoustic particle velocity. If the sample is much smaller than the cavity, i.e., \( V_s \ll V_c \) then \( p_2 \approx p_1 \) and \( \nabla p_2 \approx \nabla p_1 \). The coefficients \( A \) and \( B \) are obtained from calibrations using standard samples. If the sample is placed at a pressure antinode, where \( \nabla p = 0 \), then the calibration constant \( B \) in (8) vanishes. At this location, we can estimate sample compressibility and the quality factor associated with compressibility. If the sample is placed at a velocity antinode, where \( p = 0 \), we can estimate the inertial density and quality factor associated with micro flow.

Attenuation is added by introducing complex frequency \( \omega = \omega_1 + i \omega_i \) and complex modulus \( \kappa = \kappa_1 - i \kappa_i \). Here, we assume that \( \omega_1 \ll \omega_i \). The quality factor \( Q \) of the resonant cavity is defined as

\[
Q = \frac{\omega_1}{2 \omega_i} \quad (12)
\]

After some mathematics and empirical considerations, we find that the quality factor \( Q \) of the sample at a pressure antinode can be estimated from:

\[
\frac{1}{Q_s} = C \frac{V_c}{V_s} \left( \frac{1}{Q_2} - \frac{1}{Q_0} \right) \quad (13)
\]

Here, \( Q_2 \) is the quality factor of the cavity with a sample loaded. It can be calculated with equation (2). \( Q_0 \) is an expected quality factor of the cavity corresponding to an experiment when an imaginary sample with \( Q_2 = \infty \) is loaded in the cavity. \( Q_0 \) and \( C \) together can be found through calibration with two known samples.

The DARS Measurement

A block diagram of the DARS setup is shown in Figure 3. The key component is the cylindrical cavity, which is immersed in a tank filled with fluid. Piezoceramic sources are used to excite the fluid resonance. A high sensitivity hydrophone is embedded in the wall of the cavity to detect acoustic pressure. The sample is moved vertically along the axis of the cavity to test various regimes of pressure and flow. A computer-controlled stepper motor provides accurate and repeatable positioning of the sample. A lock-in amplifier is used to scan the frequency, track and record a selected resonance curve. The typical frequency scan is executed in steps of 0.1 Hz. The system is automated and controlled by a computer. Typical measurement results for resonant frequency and resonance bandwidth of the first mode are shown in Figures 4 and 5 for the sample at different locations in the cavity. At the center of the cavity, the acoustic pressure dominates and the smaller compressibility of the samples shifts the frequency higher than the empty value (Figure 4). At the ends of the cavity,
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the acoustic velocity dominates and the higher density of the sample causes the frequency to shift lower. There are two points within the cavity (for the first mode) where the compressibility and density contributions exactly cancel each other and the resonant frequency is identical to the empty cavity. The bandwidth $W$ (Figure 5) exhibits similar behavior as changing sample positions.

The preliminary test of DARS included four common rock types (Table 1). There were two different types of Berea sandstone, one sample of Boise sandstone, and one sample of Chalk. We also looked at synthetic samples (not shown), made from aluminum, with the purpose of evaluating the attenuation properties with well-controlled flow properties. The pore space and permeability of the synthetic samples range from 8-27% and from 250-1300 mDarcy respectively. Thirty synthetic samples with a variety of permeability and porosity combinations were studied. Both the real rocks and synthetic samples were fully saturated with the same fluid contained in the cavity. Furthermore, we conducted all measurements at temperature of 22±0.2°C and considered only the first resonance mode.

The measured values for $f$ and $W$ for the four rock samples are shown in Figure 6 for changing position of the sample within the cavity. The common point marked “empty cavity” corresponds to the sample far outside the cavity. The point marked “x” corresponds to the sample at the middle of the cavity. The values indicated by the data shown in Figure 6 correspond to slightly varying $Q$ for different sample positions inside the cavity but show distinctly different and easily separated values for the four samples. The slight variations are easily seen in Figure 7, where $Q$ is plotted versus frequency, a surrogate for sample position.

![Figure 3: The DARS setup includes computer controlled sample positioning and swept frequency data acquisition.](image1)

![Figure 4: Variations of resonant frequency with sample location for four porous samples. The baseline corresponds to the empty cavity.](image2)

**Attenuation Results**

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![Figure 5: Variations of resonant bandwidth with sample location for four porous samples. The baseline corresponds to the empty cavity.](image3)

<table>
<thead>
<tr>
<th>Rock type</th>
<th>$\rho$ (g/cc)</th>
<th>$\phi$ (%)</th>
<th>$k$ (mDarcy)</th>
<th>$Q_s$</th>
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<td>1.85</td>
<td>22</td>
<td>368</td>
<td>3.1</td>
</tr>
</tbody>
</table>

Table 1: Properties of the four rock samples used in DARS.

The $Q$-value of the samples can be calculated with equation (13). The results are shown in Table 1. Aluminum ($Q_s=200,000$) and Lucite ($Q_s=23$) are used as standard samples for the calibration. We compared $Q$-values with the measured flow and storage properties, i.e., permeability and porosity, for the four samples. In Figure 8, we see a monotonically decreasing value of $Q$ with increasing permeability over three orders of magnitude. In Figure 9, we see a similar behavior of $Q$ versus porosity except for the distinct outlier for the high porosity but low
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Figure 6: Cross plot of resonance frequency and half-power bandwidth ($W_f$) for four different rocks. The traces correspond to sample position within the cavity. “x” marks the middle of the cavity, i.e., the pressure antinode, and “o” is the point far outside the cavity, i.e., “empty cavity.”

Figure 7: Cavity resonance frequency and quality factor for four different porous samples. $Q_1 = 285$ is the quality factor of the empty cavity. $Q_2$ is the quality factor for a sample at the center of the cavity.

Figure 8: Sample quality factor for the four rock samples as a function of permeability. The $Q$-value for Boise is calibrated to 200. Permeability is measured with a conventional method.

Figure 9: Sample “Q” or quality factors for the four rock samples as a function of porosity.

permeability Chalk sample. These results strongly indicate that the DARS “Q” can be used to qualitatively interpret permeability in porous media.

Conclusion

We have conducted laboratory measurements of attenuation on both synthetic rocks and real rock samples. These observations were made possible with the use of our newly developed Differential Acoustic Resonance Spectroscopy (DARS) system. The DARS technique operates on the mechanism that the introduction of an object into an acoustic resonator causes perturbation in the resonance modes. By analyzing the differential normal modes, we can estimate the acoustic properties of the object. Rock samples were measured using DARS at the frequency near 1 kHz that is close to the high-resolution field seismic frequency scope. The experimental results showed that both permeability and porosity have considerable effects on the attenuation property of porous media.

References

