A Streamline-Based 3D Field-Scale Compositional Reservoir Simulator

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We conclude with a qualitative discussion explaining the near-linear scaling of the streamline approach with the number of gridblocks and the cubic and higher scaling exhibited by one of the finite-difference codes.

Introduction

The use of streamlines and streamtubes to model convective displacements in heterogeneous media has been presented repeatedly since the early work by Muskat, Fay and Prats, and Higgins and Leighton. Important subsequent contributions are due to Parsons, Martin and Wegner, Bommer and Schechter, Lake et al., Mathews et al., Emanuel et al., Renard, and Hewett and Behrens. Recently, streamline methods have received renewed attention by several groups as a viable alternative to traditional finite-difference (FD) methods for large, heterogeneous, multowell, multiphase simulations, which are particularly difficult for FD simulators to model adequately. Large speed-up factors compared to traditional FD solutions, minimization of numerical diffusion and grid orientation effects, and the inherent simplicity of the approach offer unique opportunities for integration with modern reservoir characterization methods. Examples include ranking of equiprobable earth models, estimation of the uncertainty in production forecasts due to the uncertainty in the geological description, rapid assessment of production strategies such as infill drilling patterns and miscible gas injection. In addition, streamlines may offer an attractive alternative to well-known problems with upscaling of absolute and pseudorelative permeabilities by allowing larger geological models and requiring upscaling across a smaller range of scales.

Our streamline approach for reservoir simulation hinges on two important extensions to past streamline/streamtube methods: (1) the use of true 3D streamlines and (2) numerical solutions of the transport equations along periodically changing streamlines. With these extensions we have been able to simulate realistic fluid flow in detailed, heterogeneous, 3D reservoir models much more efficiently than FD methods.

We emphasize that reservoir simulation using streamlines is not a minor modification of current FD approaches, but instead represents a significant shift in methodology. By transporting fluids along periodically changing streamlines, the streamline
approach is equivalent to a dynamically adapting grid that is decoupled from the underlying, static, grid used to describe the reservoir geology. The 1D nature of a streamline is translated into a 1D transport problem that can be solved easily and efficiently. This is considerably different from FD methods that use the same grid to solve both for pressure and for saturation/composition, and are forced to move fluids only along grid directions.

The implementation of our approach uses five key ideas:

**Heterogeneity and Well Locations:** Streamlines represent the natural grid to capture high and low flow regions due to well placements and reservoir heterogeneity.\(^1\) Muskat first used streamline to estimate reservoir drainage volumes resulting from well placements. The use of streamlines to capture the impact of heterogeneity is more recent and is due to the work by Lake et al.\(^2\) and by Emanuel and co-workers.\(^3\)–\(^6\), \(40\), \(57\), \(64\)–\(66\)

**3D Streamlines:** Tracing 3D streamlines using a time-of-flight (TOF) approach as presented by Pollock\(^5\) and Datta-Gupta and King.\(^1\) Using a TOF formulation along streamlines is significantly easier than using a volumetric formulation along streamtubes, particularly for true 3D flow. It is straightforward to show that a volumetric coordinate along a streamtube is equivalent to a TOF coordinate along a streamline.\(^64\)

**1D Numerical Solutions:** Moving solutions forward in time numerically along 1D streamlines as proposed by Bommer and Schechter.\(^9\) By using a numerical FD solution along streamlines extends the approach to general initial conditions, changing boundary conditions, and any type of displacement or recovery mechanism which can be formulated in one dimension.

**Updating Streamlines:** Capturing the changing total velocity field due to problem nonlinearities as well as time-varying boundary conditions (wells coming online as well as shutting-in) by periodically recalculating the streamlines.\(^3\)–\(^6\), \(40\), \(57\), \(64\)–\(66\) Because streamlines represent the natural flow grid along which fluids want to move, far fewer streamline updates (global pressure solves) are required to move the fluids forward in time compared to FD methods. This results in a significant computational efficiency. Combining numerical 1D solutions (point 3 above) with periodically updated 3D streamlines is the centerpiece of our approach. It allows to capture nonlinear flow mechanism in 3D while retaining the speed advantages that made approximate 2D streamtube methods attractive in the past.

**Operator-Splitting:** Using operator-splitting to capture flow mechanisms that are not aligned with the total velocity field, such as gravity.\(^4\)\(^,\)\(^11\), \(28\) The inability of 2D streamtube methods in the past to account for gravity effects has been a long-standing criticism. By using operator splitting and 3D streamlines it is possible to account for any mechanism that is not aligned with the total velocity field, such as transverse diffusion, capillary crossflow, and gravity. Operator splitting is a well established mathematical concept and has been used in the past to solve convective-diffusive transport problems,\(^24\) transport problems on multiple grids,\(^56\) and transport problems with gravity.\(^4\), \(11\), \(29\)

**Compositional Displacements**

In this paper we consider the extension of the streamline approach to compositional displacements. We have described the application of the method to immiscible and miscible displacements in our previous work,\(^3\)–\(^5\), \(8\), \(64\), \(65\), \(67\) and the reader is referred to those publications for additional details.

Modeling mass transfer effects correctly becomes imperative when designing injection processes that will enhance the recovery of oil compared to traditional waterflooding methods. For example, under the right physical conditions gas or solvent injection can lead to very efficient recovery processes, as capillary forces that cause entrapment in immiscible displacements are reduced leading to much lower residual oil saturations. If the injected solvent is completely miscible, i.e. only a single phase is formed over the range of pressures and temperatures present in the reservoir, then the displacement is said to be first-contact-miscible (FCM) and the residual oil saturation in a swept zone is theoretically zero. In many cases however, miscibility is achieved in-situ after the solvent has contacted the oil repeatedly and mass transfer occurs between the injected solvent and the oil in place. These cases are referred to as multi-contact miscibility (MCM). If miscibility is never achieved for the range of pressures and compositions in a particular displacement, then the displacement is referred to as being immiscible, although mass transfer still occurs between the flowing phases. For a general overview on the subject the reader is referred to Stalkup’s monograph.\(^51\)

Although the residual oil saturations that can be achieved by gas injection processes can be very low, there is an offsetting effect caused by the injected fluids usually being significantly more mobile and of lower density than the resident oil. Higher mobility can lead to channeling and viscous fingering.\(^63\) A significant density contrast can lead to gravity override. These effects have been observed experimentally at the core scale by several investigators\(^12\), \(13\), \(26\), \(66\) and have also been confirmed numerically. Experimental work has also underscored the importance of crossflow mechanism due to capillary forces and concentration gradients, but is remains unclear to what extent these mechanisms act at the scale of typical field-scale reservoir simulation gridblock and how they affect the flow between gridblocks at that scale.

It is generally agreed though, that successful modeling of gas injection processes at the field scale must be able to capture the relevant interactions of the first-order transport mechanisms with the geological description of the reservoir. The successful study of these interactions has proven to be difficult in the specific case of compositional displacements. Problems usually encountered in traditional immiscible displacements such as numerical diffusion and upscaled of absolute and relative permeabilities\(^2\), \(15\) are exasperated in compositional displace-
ments because of the strong nonlinearities introduced by the thermodynamic equilibrium calculations.\textsuperscript{1,16} The impact of numerical diffusion is to mitigate the mobility and density contrasts of the fluids through mixing and underestimate bypassing and gravity override.\textsuperscript{17,55} Upscaling of fine-scale heterogeneities tends to modify local contact efficiencies and the multicontact miscibility process.\textsuperscript{1,23,46} An additional difficulty — which is unique to compositional modeling — is the need to reduce the large number of hydrocarbon components present in a real system to a number that can still represent the phase behavior of the system and is numerically feasible.\textsuperscript{47} Unfortunately, attempts to use finer grids with more geological detail and/or more pseudocomponents for a better representation of the phase behavior quickly lead to computational costs which are beyond current simulator capabilities, particularly for full-field development studies. As a result, most field-scale compositional simulation use grids with order 10\textsuperscript{3} to 10\textsuperscript{4} gridblocks.

Streamlines can offer an attractive alternative to FD methods because of the natural decomposition of a 3D, heterogeneous problem into a series of simpler 1D, homogeneous problems. The advantage comes from the fact that the 1D solutions along the streamlines are completely independent from the size of the original 3D problems as well as the magnitude of the absolute permeability. The computational requirements for large field-scale problems are thereby reduced significantly, and problems with numerical diffusion, upscaling, and equilibrium calculations are minimized leading to more accurate predictions.

The Streamline Formulation

Here we develop the streamline formulation under the special assumption that gravity, capillary, and diffusive forces are negligible. All of these effects can be included using operator splitting in the same way gravity is included in oil-water displacement.\textsuperscript{3,11}

The multidimensional mass conservation equations for a chemical species, \( i \), flowing under convective forces only can be written as

\[
\phi \frac{\partial C_i}{\partial t} + \vec{u} \cdot \nabla F_i = 0 \quad , \quad i = 1, \ldots, n_c \tag{1}
\]

where

\[
C_i = \sum_{j=1}^{n_p} x_{ij} \rho_j S_j \quad , \quad j = 1, \ldots, n_p \tag{2}
\]

are the local moles of component \( i \) and

\[
F_i = u_D \sum_{j=1}^{n_p} x_{ij} \rho_j f_j \quad , \quad j = 1, \ldots, n_p \tag{3}
\]

is the molar flux of component \( i \). \( n_p \) is the number of phases, \( n_c \) is the number of components, \( x_{ij} \) is the mole fraction of component \( i \) in phase \( j \), \( \rho_j \) is the density of phase \( j \), \( S_j \) is the saturation of phase \( j \), \( u_D \) is the total dimensionless volumetric velocity, \( \phi \) is the porosity, and \( f_j \) is the fractional flow of phase \( j \) given by

\[
f_j = \frac{k_{rj}/\mu_j}{\sum_{k=1}^{n_p} k_{rk}/\mu_k} \tag{4}
\]

Here \( k_{rj} \) is the relative permeability of phase \( j \) and \( \mu_j \) is the viscosity of phase \( j \).

The streamline approach centers on transforming Eq. 1 into a 1D problem along streamlines. To do that we determine the time-of-flight\textsuperscript{17,55} (TOF) along a streamline, which is given by

\[
\tau = \int_0^\infty \frac{\phi}{u_t} ds \quad , \tag{5}
\]

and leads to the definition

\[
\left| u_t \right| \frac{\partial}{\partial s} \equiv \vec{u} \cdot \nabla = \phi \frac{\partial}{\partial \tau} \tag{6}
\]

Using Eq. 6 it is possible to rewrite Eq. 1 as a one-dimensional formulation for each component

\[
\frac{\partial C_i}{\partial t} + \frac{\partial F_i}{\partial \tau} = 0 \quad , \quad i = 1, \ldots, n_c \tag{7}
\]

The total velocity field, \( u_t \), necessary to determine the TOF using Eq. 5 can be determined by solving the total mass balance equation

\[
\sum_{i=1}^{n_c} \left\{ \phi \frac{\partial C_i}{\partial t} + \vec{u} \cdot \nabla F_i = 0 \right\} , \tag{8}
\]

for pressure and then using Darcy’s law to determine the local velocity field. For the work presented here, we have approximated Eq. 8 by

\[
\nabla \cdot \vec{K} \cdot (\lambda_t \nabla P) = 0 \quad , \tag{9}
\]

where \( \vec{K} \) is the local permeability tensor and \( \lambda_t \) is the total mobility given by

\[
\lambda_t = \sum_{j=1}^{n_p} k_{rj}/\mu_j \tag{10}
\]

Eq. 9 assumes that the pressure distribution solved under the assumption of incompressibility is close the solution obtained assuming a compressible system. For systems with a small pressure drop compared to the average reservoir pressure between injectors and producers — which we consider here — this is probably not a bad assumption. We are currently pursuing the extension to a fully compressible system, in which Eq. 8 is used to find the total velocity field rather than Eq. 9. This involves only a minor modification in the tracing algorithm and is a straightforward extension.

Although Eq. 9 does not account for compressibility and volume-change-on-mixing, these effects are accounted for in the 1D solution, Eq. 7, by solving for the dimensionless total velocity, \( u_D \), which is no longer constant. Details of this are outlined in in the following section.

One-Dimensional Compositional Solver

Rewriting Eq. 1 as Eq. 7 decouples the global 3D, heterogeneous flow problem into a series of 1D, homogeneous problems along streamlines. The original permeability distribution is now reflected by the geometry and the TOF distribution of a
streamline. Using Eq. 7 allows to simulate any type of transport mechanism by simply using the appropriate 1D solver along streamlines. For this work we have written a 1D compositional solver for Eq. 7 which is described next.

We solve Eq. 7 for the total moles of component $i$ using an explicit first-order in time, second-order in space TVD scheme. The scheme can be improved to second-order time accuracy using a simple modification. Let

$$F_i = u_D \hat{F}_i$$

then we can write the finite-difference form of Eq. 7 as

$$C_{i,k}^{n+1} = C_{i,k}^n + \frac{\Delta t}{\Delta x} \left( \left[ u_D \hat{F}_i \right]_{k-\frac{1}{2}} - \left[ u_D \hat{F}_i \right]_{k+\frac{1}{2}} \right),$$

(12)

where the intercell fluxes, $\hat{F}_i$, are approximated by

$$\hat{F}_{i,k+\frac{1}{2}} = \frac{\Phi}{2} \left( \hat{F}_{i,k+1} - \hat{F}_{i,k} \right) \left( 1 - \frac{\Delta t}{\Delta x} \right).$$

(13)

$\Phi$ is the van Leer limiter and $r$ is a function of the ratio of adjacent flux differences

$$r_{i,k+\frac{1}{2}} = \frac{F_{i,k} - F_{i,k-1}}{F_{i,k+1} - F_{i,k}}.$$  

(15)

Here $k$ is the node counter of the discretized streamline, and $n$ is the local time level along each streamline. We then find the total mole fractions for each component using

$$z_i = \frac{C_i}{\sum_{i=1}^{n_e} C_i}.$$  

(16)

Compressibility effects due to volume-change-on-mixing and pressure gradients are accounted for by solving iteratively for the total dimensionless velocity, $u_D$, together with Eq. 12 using a total mass balance equation of the form

$$u_D^{n+1} = \frac{\sum_{i=1}^{n_e} \left( \frac{\Delta x}{\Delta x} \left( C_{i,k}^n - C_{i,k}^{n+1} \right) + \left( u_D \hat{F}_i \right)_{k-\frac{1}{2}} \right)}{\sum_{i=1}^{n_e} \hat{F}_{i,k+\frac{1}{2}}}.$$  

(17)

Phase compositions, densities, and saturations are determined from an equilibrium calculation using the Peng-Robinson equation of state, an accelerated successive substitution algorithm, and a negative flash algorithm. Liquid densities are corrected using volume translation. Phase viscosities are calculated using the Lohrenz-Bray-Clark correlation.

Fig. 1 shows a comparison of the one-dimensional solution obtained using 3DSL with a Courant-Friedrichs-Lewy (CFL) number of 0.5 with the solution obtained using UTCOMP for a 3 component displacement at 1600 psia and 160 °F. UTCOMP is an implicit-pressure, explicit-composition, isothermal, three-dimensional compositional simulator developed at the University of Texas at Austin with a third-order TVD scheme to control numerical diffusion. Except for a slight difference in the fastest front speed, the agreement is good.

**Two-Dimensional Solutions**

In this section we consider a three component (3C) condensing gas drive and a four component (4C) condensing-vaporizing gas drive through various heterogeneous cross sections. The aim of this section is two-fold: (1) to benchmark the solutions from the streamline approach with solutions obtained from UTCOMP and (2) to highlight the ability of the streamline approach to efficiently investigate the interaction of heterogeneity, numerical resolution (number of gridblocks), phase behavior and unstable flow. The second issue has been considered by many investigators. The recurrent difficulty faced by these studies was the limited size of the numerical problem that could be considered because of the large computational resources generally required. Stalkup attempted to overcome this limitation by extrapolating his results to an infinite number of gridblocks with some success. But if one-dimensional problems are any guide to the number of gridblocks required to
obtain converged solutions, it is reasonable to assume that most 2D and 3D solutions presented in the literature are probably not converged solutions – particularly for heterogeneous domains – but instead represent an intermediate solution in a sequence of refined grids. This observation is demonstrated through an example in the next section.

**Three Component Condensing Gas Drive** The injection of pure CO₂ into a 30/70 mole percent mixture of CH₄ and C₁₀ at 1600 psia and 160 °F is an example of a condensing gas drive. The corresponding 1D solution is shown in Fig. 1. For modeling 2D and 3D displacements, the expected nonlinearity (or instability) of the displacement can be estimated from the total mobility profile of the 1D solution. For the 3C case, Fig. 2 shows that the end-point mobility ratio of \( M = 10.3 \) is divided into a leading CO₂ front with a mobility ratio of \( M = 2.7 \), a rarefaction wave, and a trailing front with a mobility ratio of \( M = 2.2 \). Although the end-point mobility ratio is mitigated substantially by phase behavior effects alone, some nonlinearity can be expected due to the leading mobility ratio of \( M = 2.7 \). In addition, because the leading CO₂ shock is self-sharpening, numerical diffusion is expected to be less efficient in smearing out the mobility difference.

To investigate this displacement in 2D heterogeneous media, we considered an initial cross-section with \( N_x = 62 \) and \( N_z = 25 \) gridblocks (Fig. 3). Permeability varies over four orders of magnitude and has a correlation length of approximately 0.3 in the main direction of flow. A comparison of the spatial distribution of CO₂ is shown in Fig. 3, and the corresponding recoveries are shown in Fig. 4. Both figures, Fig. 3 and Fig. 4, demonstrate that the streamline solution is in agreement with the solution given by UTCOMP. The UTCOMP solutions were found using the third-order TVD option for controlling numerical diffusion. For this problem, the streamline code (3DSL) required 0.75 hrs to generate the recovery curve up to 2 pore volumes (PV) injected, whereas UTCOMP required 18 hrs – a speed-up of approximately 24.

To further verify the 2D solution, we refined the 62 × 25 grid to 124 × 50 and to 248 × 100 by maintaining the same permeability distribution as the original 62 × 25 grid. Since the leading CO₂ shock front has an unfavorable mobility ratio and is also self-sharpening, the increased numerical resolution of the local velocity field is expected to capture the instability of the problem. Fig. 5 shows the comparison between UTCOMP and 3DSL for the downscaled grids. The finer 248 × 100 grid could not be run with UTCOMP due to prohibitively high computational costs. Three important comments can be made about the results shown in Fig. 5:

1. As the grid is refined, the streamline solution shows signs of instability at the leading CO₂ front due to the unfavorable mobility ratio. This demonstrates that numerical diffusion in 3DSL is minimized to the point that we are able to see the onset of a viscous-fingering like behavior at the shock front and other small scale features of the displacement.

2. The UTCOMP solution for the 124 × 50 also has increased resolution compared to the 62 × 25 solution of Fig. 3. But unlike the streamline solution it appears as a sharpened image of the original 62 × 25 grid without noticeable changes in the small scale features of the CO₂ front. This points to the problem that FD-models can require grid refinement by several powers of 2 before any noticeable change in the flow behavior can be observed. This despite higher order numerical schemes.

3. We could not simulate the 248 × 100 grid with UTCOMP.
Figure 4: Cumulative recoveries of CH$_4$ plus C$_{10}$ for 3DSL and UTCOMP.

because the computational expense was simply too high. We believe that this is not because of specific inefficiencies in UTCOMP, but rather because the underlying finite-difference scheme combined with the strong nonlinearities of the equilibrium calculations make refinement studies of the sort presented here very expensive for FD methods. In many cases studies of this sort are not possible using FD.

The speed-up of 3DSL compared to UTCOMP for the 124 x 50 grid was 92 (92 hrs for UTCOMP vs. 1 hr for 3DSL) to reach 2 PV injected.

**Four Component Condensing-Vaporizing Gas Drive**

The displacement of a 3C mixture of CH$_4$/C$_8$/C$_{16}$ (20/40/40) by an enriched gas mixture of CH$_4$/C$_3$ (65/35) at 2,000 psia and 200 $^\circ$F exhibits a condensing behavior at the leading edge (the injected gas condenses into the resident oil) and a vaporizing behavior at the trailing edge (the injected gas volatilizes the resident oil). Because the injected composition is close to the minimum enrichment composition for miscibility, the displacement can be characterized as being near-miscible. Fig. 6 shows the 1D solution obtained using UTCOMP and 3DSL. The end-point mobility ratio for this 4C problem is $M \approx 8.4$. As a result of the near-miscible nature of the displacement, most of the mobility contrast ($M \approx 7.2$) occurs over a small region (Fig. 6), and thus one may expect a 2D solution to be more unstable than the 3C problem considered in the previous section. But a finer 1D solution with 500 gridblocks (Fig. 6) reveals that the mobility contrast of $M \approx 7.2$ in fact occurs over two rarefaction waves and a self-sharpening shock, with the mobility ratio across the shock being $M \approx 2$. Thus, although the two-phase region is significantly smaller than in the 3C component case, the mobility contrast is mitigate significantly by the spreading nature of the two rarefaction waves. The instability for this displacement then is expected to be primarily due to the self-sharpening shock following the rarefaction waves.

Figure 5: Spatial distribution of CO$_2$ for 3DSL at $t_D = 0.5$ for grids 124 x 50 and 248 x 100 compare to the UTCOMP solution for grid 124 x 50.

We considered this problem in our previous work with analytical solutions along streamtubes. Because mapping analytical solutions along streamtubes overestimates the mobility contrast of a displacement, we indeed found solutions that were characterized by a very large amount of instability (long, wispy channels) as shown in Fig. 7. In contrast, the UTCOMP solution appeared to be characterized by a large amount of mixing resulting in a negligible mobility contrast.

We revisit the problem here using numerical solutions along streamlines. Fig. 7 shows the gas saturation maps at $t_D = 0.4$ for the three displacements. The numerical solution obtained from 3DSL is clearly different from our first solution obtained using streamtubes in that it exhibits less channeling. But the 3DSL solution is also different from the UTCOMP solution, in that there appears to be more detail in the saturation distribution and signs of some channeling, particularly due to the self-sharpening shock following the rarefaction waves. The similarity between the 3DSL and UTCOMP solutions in Fig. 7 shows the ability of the streamline approach to capture the compositional nature of the problem correctly. It also suggests that for this displacement, the near-miscible condition does not pose a significant problem in terms of the instability of the displacement. UTCOMP required 86 hrs to generate the saturation map.
Sensitivity on Injected Composition. The speed and accuracy of the streamline approach clearly offers unique opportunities to revisit outstanding problems in predicting the effectiveness of enriched gas injection in heterogeneous media as a way to enhance production of hydrocarbons. As an example, we consider the sensitivity of the 4C condensing-vaporizing displacement on the level of enrichment of the injected mixture. This is a relevant question since increasing enrichment will lower the residual oil saturation and increase the local contact efficiency, but at the same time may cause bypassing and early breakthrough due to the unstable nature of the displacement. Finding the optimal level of enrichment, that will maximize oil recovery for a given description of the phase behavior and geological setting entails many forward simulations with sufficient geological and numerical resolution. We emphasize that the solution to such an exercise will depend strongly on the physical scale of the problem and the resulting interaction of viscous, gravity, capillary, and diffusive forces at that scale.\textsuperscript{12,13,60} Since we are neglecting gravity, capillary, and diffusive mechanisms here, the results are meant primarily to demonstrate the efficiency of the streamline approach, rather than to reveal any novel physical insight.

Fig. 8 shows the result of the sensitivity exercise through a 128 \times 64 permeability field with 5 orders of magnitude difference in permeability. The three injection composition mixtures of CH\textsubscript{4}/C\textsubscript{3} — (85/15), (65/35), and (45/55) mole percent — represent immiscible, MCM, and FCM displacements respectively of the initial oil composition. We also used a simple linear interpolation for the relative permeabilities to account for the fact that as the displacement goes from immiscibility to miscibility the relative permeabilities tend to straight lines and the residual saturations go to zero. For this particular example, Fig. 8 shows that injection of a CH\textsubscript{4}/C\textsubscript{3} mixture that achieves first-contact miscibility with the resident oil will be the most efficient displacement. We believe that the primary reason for this is that in particular this case reservoir heterogeneity has a first order effect compared to displacement instabilities. It is the geology that is dictating the primary channels of the solvent. The cumulative simulation time for the three cases on a standard workstation was 16 hrs.
the underlying geological grid, the streamline method is ideally suited for large, 3D, multiwell models dominated by heterogeneity. Because of grid CFL constraints, these problems are particularly difficult for conventional FD methods.

As a result, the streamline approach offers unique opportunities to model field-scale problems with an acceptable level of geological resolution. Although upscaling will still be necessary to go from the fine-scale earth models of $O(10^7 - 10^9)$ gridblocks to the reservoir simulation scale of $O(10^3 - 10^5)$, the level of upscaling required will be significantly less for simulations using streamlines.

As an example, we simulated the 4C problem discussed in the previous section on a $180 \times 180 \times 16 = 518,400$ heterogeneous Cartesian grid with a total of 36 wells for a period of 0.7 PVs injected. The average pressure gradient between injectors and producers was $\Delta P \approx 180$ psia. The simulation run required approximately 3 days on a standard workstation (DEC AlphaServer 2100 4/275 with 256MB of RAM). Although there are only 30 gridblocks between wells for this half-million gridblock example, it is a significant improvement in resolution compared to the limited number of gridblocks usually used in FD field-scale models. Fig. 9 shows recovery and breakthrough curves for $C_3$ obtained using UTCOMP and Eclipse 300 (E300). The upscaled $60 \times 60 \times 8$ model was generated from the fine-scale $180 \times 180 \times 16$ model using simple geometric averaging of the absolute permeabilities. More sophisticated upscaling techniques may have reduced the difference between fine-scale and coarse-scale results.

Fig. 9 is a reminder that the difference in the solution due to upscaling of the geological model, pseudization of the phase behavior, and the usually low numerical resolution between wells poses a challenging problem for traditional FD approaches for modeling field-scale compositional problems. Streamlines may offer a simple yet effective solution to increase the geological detail and resolution between wells using a fraction of the computational costs required by FD. As we show in the following section, the streamline approach may in fact allow to model problems that are beyond current hardware capabilities for FD methods. For this example, solution of the $60 \times 60 \times 8 = 28,800$ model with E300 required approximately the same CPU time as the $180 \times 180 \times 16 = 518,400$ model with 3DSL. We estimated
the runtime for the $180 \times 180 \times 16$ model with Eclipse in the tens of years (Fig. 10).

**Timing and Scaling**

In our previous work\textsuperscript{3,64} we found speed-up factors between 10 and 500 for waterflooding and FCM displacements in 2D and 3D heterogeneous domains and under a variety of flow conditions, geological descriptions, and boundary conditions. In most cases, the speed-up resulted from the reduced number of global pressure solves and large convective steps allowed by 3DSL.

Fig. 10 shows scaling results we found for the 3D, 4C problem described previously. We generated Fig. 10 by starting with the fine-scale model of $180 \times 180 \times 16$ and then considered upscaled models $60 \times 60 \times 8$, $45 \times 45 \times 8$, and $36 \times 36 \times 8$. We could only run the later three models with E300 using our available computational resources. For comparison, we also show the scaling of 3DSL on the same 3D problem for a waterflood. Although the results in Fig. 10 are specific to the example we studied, they show that the streamlines solutions scale near-linearly with the number of gridblocks, $N_{gb}$, whereas the FD solution obtained with Eclipse 300 scales approximately as $N_{gb}^{3.4}$.

The scaling behavior of each method can be explained qualitatively by considering the expression for the total run time, $T$, given by

$$T = N_{ts} \times \Delta t_{ts},$$

where $N_{ts}$ is the number of timesteps and $\Delta t_{ts}$ is the time required per timestep. $\Delta t_{ts}$ is the sum of the time required to invert the pressure matrix, $\Delta t_p$, and the time to move the saturation/concentration forward in time, $\Delta t_s$. It is reasonable to assume that for each method, $\Delta t_{ts}$, is roughly the same and with a near linear scaling. For example, our public-domain multigrid\textsuperscript{36} solver scales near-linearly, $\Delta t_p \propto N_{gb}^{1.3}$.

The efficiency of the streamline approach compared to FD is derived from the fact that the number of global timesteps, $N_{ts}$, is independent of the number of gridblocks of the system, because fluid transport is decoupled from the underlying grid. Instead, it is directly related to the nonlinearity of the problem (mobility contrast/gravity number) and the number of well-events (changing boundary conditions). A good example of this is tracer flow, where for constant boundary conditions, only a single global timestep is required, irrespective of the level of heterogeneity and the number of gridblocks. The reason that the magnitude of heterogeneity is irrelevant is because the 1D solution along the streamline is homogeneous, with all the heterogeneity buried in the TOF.

A finite-difference code, on the other hand, must take many timesteps that are directly dictated by the size of the gridblocks and the magnitude of the heterogeneity present. It is probably the changing level of heterogeneity (higher extremes of permeability with increasing grid resolution) that causes the cubic scaling exhibited by the FD code in Fig. 10. For homogeneous systems, the number of timesteps in FD are expected to scale linearly with $N_{gb}$. But as heterogeneity is added to the model, finer grids with higher permeability contrasts will cause the number of timesteps to scale with a power greater than one. Assuming a near-linear scaling for the pressure solution and the explicit timestepping, combined with a quadratic scaling of the number of timesteps would explain the $N_{gb}^{3.4}$ seen in Fig. 10 for the FD code.

**Conclusions**

We have extended the streamline approach to 2D and 3D compositional displacements by using a general 1D numerical solver along streamlines. The 1D solution accounts for compressibility and volume-change-on-mixing by solving for the total velocity. The advantages of using streamlines for two-phase immiscible as well as FCM displacements are amplified for compositional displacements:

1. By decoupling the 1D transport problem from the underlying geological grid, streamlines allow to retain the numerical accuracy required to model compositional displacements.

2. The streamline approach scales near-linearly with the number of gridblocks, which represents a considerable improvement over conventional FD methods.

3. The efficiency of the streamline approach offers a unique opportunity to revisit outstanding questions for designing field-scale enhanced-recovery processes.

4. Higher geological and numerical resolution reduces the number of scales that need to be crossed when going from the fine-scale earth model to the reservoir simulation model, thereby reducing the dependence on upscaling of absolute and relative phase permeabilities. This will lead to a significant improvement in the resolution of the transport problem and to more reliable predictions.

5. The increased numerical efficiency can be translated to a higher number of pseudocomponents for the description of the phase behavior, larger models, and better geological resolution.
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Nomenclature

\[ C_i = \text{total moles of component } i \]
\[ F_i = \text{molar flux of component } i \]
\[ f_j = \text{fractional flow of phase } j, \text{ dimensionless} \]
\[ K = \text{absolute permeability tensor, } L^2 \]
\[ k = \text{node index for finite-difference scheme} \]
\[ k_{rj} = \text{relative permeability of phase } j, \text{ dimensionless} \]
\[ M = \text{mobility ratio, dimensionless} \]
\[ n_c = \text{number of components} \]
\[ n_p = \text{number of phases} \]
\[ N_{gb} = \text{number of total gridblocks} \]
\[ N_{ts} = \text{number of global time steps} \]
\[ n = \text{time step level in finite-difference formulation} \]
\[ P = \text{Pressure } M/T^3 L \]
\[ PV = \text{pore volumes} \]
\[ r = \text{ratio of successive flux differences} \]
\[ S_j = \text{saturation of phase } j, \text{ dimensionless} \]
\[ s = \text{spatial distance coordinate along a streamline, } L \]
\[ t = \text{time, } T \]
\[ t_D = \text{dimensionless time} \]
\[ u_D = \text{dimensionless total velocity} \]
\[ x_{i,j} = \text{mole fraction of component } i \text{ in phase } j, \text{ dimensionless} \]
\[ z_i = \text{total mole fraction of component } i, \text{ dimensionless} \]
\[ \Delta t = \text{step size in 1D solution, } T \]
\[ \Delta t_p = \text{CPU time required per global pressure solution, } T \]
\[ \Delta t_s = \text{CPU time required per global convective step, } T \]
\[ \Delta t_{ts} = \text{CPU time required per global step, } T \]
\[ \phi = \text{van Leer limiter for TVD scheme} \]
\[ \rho = \text{porosity, dimensionless} \]
\[ \rho_j = \text{density of phase } j, \text{ } M/L^3 \]
\[ \zeta = \text{local streamline coordinate, } L \]
\[ \lambda_i = \text{total mobility, } L^3 T/M \]
\[ \tau = \text{time of flight, } T \]
\[ \mu_j = \text{viscosity of phase } j, \text{ } M/TL \]

References


