SPE 83470

A Decline Curve Analysis Model Based on Fluid Flow Mechanisms

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Abstract

Decline curve analysis models are frequently used but still have many limitations. Approaches of decline curve analysis used for naturally-fractured reservoirs developed by water flooding have been few. To this end, a decline analysis model derived based on fluid flow mechanisms was proposed and used to analyze the oil production data from naturally-fractured reservoirs developed by water flooding. Relative permeability and capillary pressure were included in this model. The model reveals a linear relationship between the oil production rate and the reciprocal of the oil recovery or the accumulated oil production. We applied the model to the oil production data from different types of reservoirs and found a linear relationship between the production rate and the reciprocal of the oil recovery as foreseen by the model, especially at the late period of production. The values of the maximum oil recovery for the example reservoirs were evaluated using the parameters determined from the linear relationship. The results demonstrated that the analytical decline analysis model is not only suitable for naturally-fractured reservoirs developed by water flooding but also for other types of water drive reservoirs. An analytical oil recovery model was also proposed. The results showed that the analytical model could match the oil production data satisfactorily. We also demonstrated that the frequently-used nonlinear type curves could be transformed to linear relationships in a log-log plot. This may facilitate the production decline analysis.

Introduction

Estimating reserves and predicting production in reservoirs has been a challenge for a long time. Many methods have been developed in the last several decades. One frequently-used technique is decline curve analysis approach. There have been a great number of papers on this subject[1-23].

Most of the existing decline curve analysis techniques are based on the empirical Arps equations[4]: exponential, hyperbolic, and harmonic equations. It is difficult to foresee which equation the reservoir will follow. On the other hand, each approach has some disadvantages. For example, the exponential decline curve tends to underestimate reserves and production rates; the harmonic decline curve has a trendency to overpredict the reservoir performance[2]. In some cases, production decline data do not follow any model but cross over the entire set of curves[8].

Fetkovich[15] combined the transient rate and the pseudosteady-state decline curves in a single graph. He also related the empirical equations of Arps[4] to the single-phase flow solutions and attempted to provide a theoretical basis for the Arps equations. This was realized by developing the connection between the material balance and the flow rate equations based on his previous papers[24, 25].

Many derivations[11, 13] were based on the assumption of single-phase oil flow in closed boundary systems. These solutions were only suitable for undersaturated (single-phase) oil flow. However many oilfields are developed by water flooding. Therefore two-phase fluid flow instead of single-phase flow occurs. In this case, Lefkovits *et al.*[19] derived the exponential decline form for gravity drainage reservoirs with a free surface by neglecting capillary pressure. Fetkovich *et al.*[18] included gas-oil relative permeability effects on oil production for solution gas drive through pressure ratio term, This assumes that the oil relative permeability is a function of pressure. It is known that gas-oil relative permeability is a function of fluid saturation which dependents on fluid/rock properties.

In water flooding, oil relative permeability can not be approximated as a function of pressure. The pressure during water flooding may increase, decrease, or remain unchanged. The oil production decline because of oil relative permeability reduction is associated with decrease in oil saturation instead of pressure in this case.

Masoner[21] correlated oil relative permeability to the Arps decline exponent by assuming a constant pressure potential and a pseudosingle-phase oil flow.

Many attempts have been made to interpret the empirical Arps equations or provide some theoretical basis in specific cases. New models with consolidated theoretical background have been few. As Raghavan[26] pointed out in 1993: "Until the 1970s, decline curve analysis was considered to be a convenient empirical procedure for analyzing performance; no particular significance was to be attributed to the values of D_i

and b. To an extent this is still true even today". This may be the case still, even though ten years have past.

Less attention has been paid to the production decline analysis in naturally-fractured reservoirs developed by water flooding. Aronofsky *et al.*[27] suggested an empirical model to match oil production by water injection in this type of reservoir. Baker *et al.*[6] used a similar model to infer the fracture spacing by matching production data from the Spraberry Trend naturally-fractured reservoir.

In this article, an analytical model developed by Li and Horne in previous papers[28-32] was used to conduct production decline analysis for naturally-fractured reservoirs developed by water flooding. The model was developed originally to characterize spontaneous water imbibition in reservoir rock and was confirmed both theoretically and experimentally[28-30]. Because spontaneous water imbibition is the main fluid flow mechanism that governs the oil production in naturally-fractured reservoirs developed by water injection, it may be reasonable for the model to be applicable in such reservoirs. However it was also found that the model is applicable in other types of reservoirs developed by water injection. Production decline data from different types of waterdrive reservoirs were analyzed as examples of using the new decline analysis model.

We would like to clarify that our study and the discussions in this article are limited to two-phase fluid flow.

Mathematics

The Arps decline curve analysis approach[4] was proposed nearly sixty years ago. However a great number of studies on production decline analysis are still based on this empirical method. Many published papers have tried to interpret the Arps[4] decline equation theoretically. The empirical Arps[4] decline equation represents the relationship between production rate and time for oil wells during pseudosteady-state period and is shown as follows:

$$q(t) = \frac{q_i}{(1 + bD_i t)^{1/b}} \tag{1}$$

where $q(t)$ is the oil production rate at time t and q_i is the initial oil production rate. b and D_i are two constants.

Eq. 1 can be reduced in two special cases: b=0 and b=1. b=0 represents an exponential decline in oil production, which is expressed as follows:

$$q(t) = q_i e^{D_i t} \tag{2}$$

b=1 represents a harmonic decline in oil production, which can be expressed as follows:

$$q(t) = \frac{q_i}{(1 + D_i t)} \tag{3}$$

Other values of b represent a hyperbolic decline in oil production.

The type curves based on the Arps equation are still used frequently for production decline analysis at the pseudosteady-state period. It was found that the nonlinear type curves shown in Fig. 1 could be transferred to linear relationships in a log-log plot, as shown in Fig. 2. The dimensionless rate q_{Dd} and the dimensionless time t_{Dd} used in Fig. 1 are defined as follows:

$$q_{Dd} = \frac{q(t)}{q_i} \tag{4}$$

$$t_{Dd} = D_i t \tag{5}$$

According to Eqs. 1, 4 and 5, one can obtain the following equation:

$$\frac{d \ln q_{Dd}}{dt_{Dd}} = (q_{Dd})^b \tag{6}$$

Eq. 6 demonstrates the linear relationships between $d\ln q_{Dd}/dt_{Dd}$ and q_{Dd} for different values of b in a log-log plot, as shown in Fig. 2. According to Eq. 6, the slope values of the straight lines in Fig. 2 are equal to the values of b. Compared to the frequently-used type curves shown in Fig. 1, the production decline analysis using the type curves (actually straight lines) shown in Fig. 2 has obvious advantages. For example, it is easier for reservoir engineers to judge which type line production data will match using straight lines than curves. Furthermore, Fig. 2 has better resolution than Fig. 1.

Li and Horne[30] developed an analytical model to predict oil production in core samples by spontaneous water imbibition. The model is expressed as follows:

$$q(t) = a_0 \frac{1}{R(t)} - b_0 \tag{7}$$

where *q(t)* is the oil production rate from core samples by spontaneous water imbibition. *R(t)* is the oil recovery at time t in the units of pore volume. a_0 and b_0 are two constants associated with capillary and gravity forces respectively. For experimental study in core samples, the constant a_0 is referred to as the imbibition index because a_0 is representative of the recovery rate by spontaneous imbibition. The greater the value of a_0, the faster the imbibition.

For investigation on decline analysis of oil production in reservoirs, the constant a_0 is referred to as production rate index.

One can see that the analytical model represented in Eq. 7 reveals a linear correlation between the oil production rate and the reciprocal of the oil recovery or the accumulated production.

The two constants, a_0 and b_0, in Eq. 7 are expressed as follows:

$$a_0 = \frac{AM_e^*(S_{wf} - S_{wi})}{L} P_c^* \qquad (8)$$

$$b_0 = AM_e^* \Delta\rho g \qquad (9)$$

where A and L are the cross-section area and the length of the core, S_{wf} is the water (or the wetting phase) saturation behind the imbibition front, S_{wi} is the initial water saturation in the core sample, $\Delta\rho$ is the density difference between water (the wetting) and oil (the nonwetting) phases ($=\rho_w - \rho_o$), g is the gravity constant, P_c^* is the capillary pressure at S_{wf}, and M_e^* is the global mobility. For cocurrent flow, M_e^* is expressed as follows:

$$M_e^* = \frac{kk_{re}^*}{\mu_e} = \frac{M_o^* M_w^*}{M_o^* - M_w^*} \qquad (10a)$$

while for countercurrent flow, M_e^* is expressed as follows:

$$M_e^* = \frac{M_o^* M_w^*}{M_o^* + M_w^*} \qquad (10b)$$

where k is the permeability of the rock sample, $\frac{kk_{re}^*}{\mu_e}$ is the global mobility of the two phases, M_o^* and M_w^* are the oil and water phase mobilities and expressed as follows:

$$M_o^* = \frac{kk_{ro}^*}{\mu_o} \qquad (11a)$$

$$M_w^* = \frac{kk_{rw}^*}{\mu_w} \qquad (11b)$$

here k_{ro}^* and k_{rw}^* are the oil and water relative permeabilities at a specific water saturation. μ_o and μ_w are the oil and water viscosities respectively.

Li and Horne demonstrated in previous papers that Eq. 7 is applicable for both gas-liquid[28-29] and oil-water systems[30] and the expressions of a_0 and b_0 can be reduced significantly in gas-liquid systems.

One can also see from Eq. 7 and the expressions of a_0 and b_0 that relative permeability and capillary pressure, the two important parameters in naturally-fractured reservoirs developed by water injection, are included in the model.

The values of the two constants, a_0 and b_0, can be determined from experimental data of spontaneous water imbibition tests in core samples.

It would be helpful for engineers to have the theoretical correlation between oil recovery and production time to infer reservoir rock and fluid properties. For this, Eq. 7 was solved in terms of production time and the solution is expressed as follows:

$$(1 - R^*)e^{R^*} = e^{-t_d} \qquad (12)$$

where R^* is the normalized oil recovery and $R^*=cR$. c is the ratio of the gravity force to the capillary force and $c=b_0/a_0$. t_d is the dimensionless time proposed in a previous paper by Li and Horne[30]. t_d is expressed as follows:

$$t_d = c^2 \frac{kk_{re}^*}{\phi} \frac{P_c^*}{\mu_e} \frac{S_{wf} - S_{wi}}{L_a^2} t \qquad (13)$$

where L_a is the characteristic length.

The oil production in highly fractured reservoirs developed by water injection may follow the model represented by Eq. 7. The considerations are discussed as follows. Firstly, the main development mechanism in such reservoirs is spontaneous water imbibition in which the dominant driving force for oil production is capillary pressure instead of the pressure drop between injection and production wells. This mechanism is confirmed to an extent by the practical observation that incremental oil production by infill drilling wells in naturally-fractured reservoirs is very little, as reported by Baker *et al.*[4] in Spraberry (a typical naturally-fractured reservoir developed by water injection). This will be discussed in more detail later. Secondly, Li and Horne showed in a previous paper[30] that the oil recovery in rock samples with different porosity, permeability, relative permeability, and different capillary pressure could be scaled satisfactorily by using the dimensionless time defined in Eq. 13. Thirdly, each matrix block surrounded by fractures in reservoirs may be treated as a rock unit.

According to Eq. 7, the maximum or ultimate oil recovery can be determined using the oil production history data. Set q=0, the inferred maximum oil recovery is equal to a_0/b_0. The values of a_0 and b_0 can be obtained by fitting the linear relationship between the oil production rate and the reciprocal of the oil recovery. Similarly, the oil recovery at an economic limit can also be evaluated once the economic limit of oil production rate is known or determined.

Note that compressibility of oil and water is not considered in the derivation of Eq. 7. However it can be included in the model readily.

The empirical equation suggested by Aronofsky *et al.*[27] is often used to predict oil production in naturally-fractured reservoirs developed by water injection. The equation represents an exponential relationship between oil recovery and the production time. The model is expressed as follows:

$$\eta = 1 - e^{-\lambda t} \tag{14}$$

where η is the oil recovery in terms of recoverable resident oil by water injection, λ is a constant giving the rate of convergence.

Later Schechter and Guo[33] proposed a similar model but used a dimensionless time instead of a production time. The model is expressed as follows:

$$\eta = 1 - e^{-\lambda t_D} \tag{15}$$

here t_D is the dimensionless time defined by Ma *et al*.[34]:

$$t_D = \sqrt{\frac{k}{\phi}} \frac{\sigma}{\mu_m L_a^2} t \tag{16}$$

where σ is the interfacial tension between oil and water phases, μ_m is the geometric mean of the viscosities of oil and water phases.

In a previous paper Li and Horne[29] demonstrated the limitations of utilizing the dimensionless time defined by Eq. 16. Relative permeability (instead of only the absolute permeability) and capillary pressure (instead of only the interfacial tension), including wettability, are the important parameters governing fluid flow in naturally-fractured reservoirs developed by water injection. However relative permeability and capillary pressure are not included in the dimensionless time defined in Eq. 16.

In this study, Eq. 7 was used to conduct oil production decline analysis for naturally-fractured reservoirs as well as other types of reservoirs developed by water injection.

Results

The oil production data from the E.T. O'Daniel lease in Spraberry[6] were used to test the decline model represented by Eq. 7. The oilfield is a naturally-fractured reservoir with a high density of fractures. Water flooding commenced in August 1959 with 21 wells on the 2160 acre lease. Fig. 3 depicts the oil production history since January 1950 (open circles) reported by Baker *et al*.[6] Water breakthrough occurred at production wells shortly after water injection began because of the high-density fractures. The oil recovery by water flooding in the oilfield is believed to be dominated by countercurrent water imbibition because of the early water breakthrough and the high-density fractures[6].

The oil production data shown in Fig. 3 were transformed into a relationship between the production rate and the reciprocal of oil recovery according to Eq. 7. The results are shown in Fig. 4. One can see that the relationship between the production rate and the reciprocal of oil recovery is linear after a specific period of early production, as predicted by the decline model represented in Eq. 7. Note that Li and Horne obtained the same linear correlation in different fluid/rock systems experimentally.[28-31]

The estimated values of a_0 and b_0 for the oil field were 0.003 and 0.0056 OOIP (oil recovery original in place)/year respectively. The maximum oil recovery predicted using the decline model was about 0.536 OOIP. The ultimate oil recovery estimated by Baker *et al*.[6] using Eq. 15 was about 0.38 OOIP at an economic limit of 48 STB/D for the lease.

Oil production data can be matched using the oil recovery model represented in Eq. 12 once the values of a_0 and b_0 are available. The results of the model match to the oil production data from the E.T. O'Daniel lease in Spraberry are shown in Fig. 3 (solid curve). It can be seen that the oil recovery data at the later production period was matched satisfactorily. However the model could not match the early oil production data after water flooding began. As pointed out by Baker *et al*.[6], the reason might be because of sweep efficiency effects. It is reasonable that the model better represents the observed oil production history after the entire fracture network has been filled with injected water. Note that there was a time shift because of sweep efficiency effects when the matching process was conducted.

It is interesting that Baker *et al*.[6] also reported a match to the oil production shown in Fig. 3 using the modified model (see Eq. 15) proposed by Aronofsky *et al.*[27]. As we pointed out in a previous paper[29], the model represented by Eq. 7 encompasses both the model proposed by Aronofsky *et al*.[27] and the Handy model[35].

According to Eqs. 8 and 9, it is necessary to know the values of oil-water relative permeability (instead of only absolute permeability) and capillary pressure (instead of only interfacial tension) at a specific water saturation in order to calculate fracture spacing from oil production data by water injection.

The expressions of a_0 and b_0 vary with the units of oil production rate and oil recovery. For example, when the units of oil production rate and oil recovery are OOIP/year and OOIP, a_0 and b_0 are expressed as follows:

$$a_0 = \frac{M_e^*}{\phi L^2} \frac{(S_{wf} - S_{wi})}{(1 - S_{wi})^2} P_c^* \tag{17}$$

$$b_0 = \frac{M_e^*}{\phi L(1 - S_{wi})} \Delta\rho g \tag{18}$$

Published oil production data in naturally-fractured reservoirs developed by water injection have been scarce in the literature. In order to further test the decline model represented by Eq. 7, the oil production data in an offshore waterdrive field reported by Dake[36] (p.417) were used. This oil field is not a naturally-fractured reservoir but has a large permeability contrast between layers, as shown in Fig. 5. Water breakthrough happened at the early injection period because of the deltaic depositional environment and the large permeability contrast. The high permeability layers in between low permeability layers may function as fractures. The oil recovery data are plotted in Fig. 6 (open circles).

The oil recovery data shown in Fig. 6 were plotted as the production rate versus the reciprocal of oil recovery to further test the decline model (see Eq. 7). The results are shown in Fig. 7. The relationship between the production rate and the reciprocal of oil recovery in the offshore oil field is also linear, as foreseen by Eq. 7.

The values of a_0 and b_0 determined from regression analysis using Eq. 7 for the oil field were 0.0169 and 0.0306 OOIP/year respectively. The maximum oil recovery estimated by the decline model was about 0.552 OOIP. The maximum oil recovery estimated by Dake[36] using a different technique was about 0.554 OOIP.

The oil recovery data calculated using the values of a_0 and b_0 with Eq. 12 are shown in Fig. 6 (solid curve). One can see that the oil recovery data after a short production period was matched well with the theoretical oil recovery model represented by Eq. 12.

Dake[36] (p.443) also reported oil production data (see Fig. 8) after water breakthrough in another similar North Sea oil field developed by water flooding. This is an isolated fault block of an extremely complex oil field. It is of the delta top depositional environment with similar permeability feature as shown in Fig. 5. Dake[36] reported that attempts to history match the field's performance using numerical simulation modeling failed to produce a reliable tool.

Fig. 9 demonstrates the relationship between the oil production rate and the reciprocal of oil recovery in the oil field. One can see that the relationship could be matched using the linear decline model (see Eq. 7) satisfactorily.

The two constants, a_0 and b_0, were also evaluated using Eq. 7 for the oil field and the values were 0.0399 and 0.1666 OOIP/year respectively. The maximum oil recovery estimated by the decline model was about 0.233 OOIP. The extremely low maximum oil recovery might be because of the rare complexity of the fault oil field. The complexity might also bring about the failure of numerical simulation as a prediction tool in the oil field.

Using the values of a_0 and b_0 obtained by a fit to the linear relationship shown in Fig. 9, the oil recovery data were calculated using Eq. 12 and the results are shown in Fig. 8 (solid curve). The values of oil recovery calculated using the model proposed in this article were consistent with the production data after water breakthrough (see Fig. 8).

Another example decline analysis was conducted for the East Texas field, the largest known oil field in the United States. The oil is produced from the Woodbine sand, Upper Cretaceous age, which is overlain by the Austin chalk except for a small area on the western side where the Eagleford shale is present[37]. The field was discovered in 1930 and has been undergoing water injection for pressure maintenance since 1938. Water breakthrough occurred at the early period of production in 1932 because of the massive natural water influx into the field (Dake[36], p.450). The oil production data since 1930 are plotted in Fig. 10. There is an obvious change of oil recovery in trend around 1965, which was caused by a large amount of infill drilling and other well remedial activity in the field at that time. The effect of infill drilling and other remedial activity on oil production can be seen more clearly in Fig. 11.

The oil production data shown in Figs. 10 and 11 were transformed according to Eq. 7. The calculated results are shown in Fig. 12. There are two linear sections on the relationship between the production rate and the reciprocal of oil recovery. One linear section occurs before the infill drilling and other remedial activity commenced and another one occurs after. The interesting observation is that the linear sections have almost the same values of slope (b). The inferred maximum oil recovery before the infill drilling and other remedial activity commenced was about 0.651 OOIP. The inferred maximum oil recovery (OOIP) after the infill drilling was about 0.966 OOIP. This value seems to be too high. It is speculated that the estimated reserve of 7034 MMstb[36] may be smaller than the true value. The infill drilling and other remedial activity might access reserves that have not be discovered and estimated.

Note that the East Texas field is not a naturally-fractured reservoir. Although the relationship between the production rate and the reciprocal of oil recovery is linear, the significances of the two constants, a_0 and b_0, may not be the same as represented by Eqs. 17 and 18.

For the East Texas field, the oil recovery data were also calculated using the two sets of values of a_0 and b_0 with Eq. 12. The comparison of the calculated model oil recovery data to the production data is shown in Fig. 10. The calculated oil recovery data are consistent with the production data both before and after infill drilling.

Discussion

As stated previously, one unique feature of naturally-fractured reservoirs developed by water injection is that incremental oil production by infill drilling wells is very little. This practical observation implies that the dominant driving force in water flooding naturally-fractured reservoirs is capillary pressure. The effect of pressure drop between injection and production wells on oil production may not be significant. The explanation to this is discussed as follows.

The fracture permeability and relative permeability are usually much greater than those of matrix. On the other hand, the well spacing may be far greater than the fracture spacing even after infill drilling (the fracture spacing in Spraberry estimated by Baker *et al.*[6] was about 3 feet). Therefore the pressure drop applied on each matrix is small even though the pressure drop between injection and production wells is great. Water injected in injection wells will be produced at the production wells very fast through high permeability fractures. Based on the description, one can see that the effect of pressure drop between injection and production wells on oil production may be small.

In order to enhance oil production in naturally-fractured reservoirs, one should take measures to increase the value of the production rate index, a_0, based on Eq. 7. These measures include decreasing fracture spacing, modifying wettability of reservoir rock/fluid systems, changing interfacial tension, and modifying oil-water relative permeability. Increase in interfacial tension may or may not enhance oil production, which was found experimentally by Schechter *et al.*[38] This is mainly because capillary pressure and global mobility are not independent parameters. Increase in capillary pressure because of increase in interfacial tension may result in decrease in global mobility, as analyzed by Li and Horne[30].

Eq. 7 has other applications except for production decline analysis. For example, we demonstrated in a previous paper[31] that it could be used to investigate the effect of initial water saturation in reservoirs on recovery. The equation can also be used to extract capillary pressure data in some cases[32].

The results showed that the decline analysis model proposed in this article is also applicable to water drive reservoirs with great permeability contrast or serious heterogeneity although the model was originally developed for naturally-fractured reservoirs. Actually we also found that the model is suitable for single-phase fluid flow during pseudosteady-state production period as well as for geothermal reservoirs. However the physical significances of the two constants a_0 and b_0 in the model may be different from Eqs. 17 and 18 in these cases (heterogeneous reservoirs, geothermal reservoirs, and single-phase fluid flow).

Reservoir production decline behavior may depend on the fluid flow mechanisms attributed to development approaches, reservoir types, and rock/fluid properties. It may be difficult to represent all the fluid flow mechanisms in different reservoirs developed by different techniques using only one mathematical equation.

Conclusions

Based on the present study, the following conclusions may be drawn:

1. The proposed decline analysis model derived from fluid flow mechanisms works satisfactorily in naturally-fractured reservoirs as well as in other types of reservoirs developed by water flooding.
2. The production data of oil recovery in reservoirs studied (naturally-fractured or highly heterogeneous) could be matched adequately using the analytical model of oil recovery.
3. The frequently-used nonlinear type curves based on the empirical Arps equation could be transferred to linear relationships in a log-log plot.

Acknowledgements

This research was conducted with financial support to the Stanford Geothermal Program from the US Department of Energy under grant DE-FG07-99ID13763, the contribution of which is gratefully acknowledged.

Nomenclature

a_0 = coefficient associated with capillary forces, m/t
A = cross-section area of the core or reservoir, L^2
b = constant
b_0 = coefficient associated with gravity, m/t
c = constant, the ratio of the gravity force to the capillary force.
D_i = constant
g = gravity constant, L/t^2
k = absolute permeability, L^2
k_{re}^* = relative permeability pseudofunction of the two phases at a specific water saturation
k_{ro}^* = relative permeability of oil phase at a specific water saturation
k_{rw}^* = relative permeability of water phase at a specific water saturation
L = core length, L
L_a = characteristic length, L
M_e = global mobility of the two phases, mL/t
M_e^* = global mobility of the two phases at a specific water saturation, mL/t
M_o^* = mobility of oil phase at a specific water saturation, mL/t
M_w^* = mobility of water phase at a specific water saturation, mL/t
P_c^* = capillary pressure at a specific water saturation, m/Lt^2
q = oil production rate, L^3/t
q_{Dd} = dimensionless oil production rate.
q_i = initial oil production rate, L^3/t
R = oil recovery in the units of pore volume
R^* = normalized oil recovery
S_{wf} = water saturation behind imbibition front
S_{wi} = initial water saturation
t = production time, t
t_d = dimensionless time
t_D = dimensionless time defined by Eq. 16
t_{Dd} = dimensionless time defined by Eq. 5
V_p = pore volume, L^3
μ_e = the effective viscosity of the two phases, m/Lt
μ_m = geometric mean of oil and water viscosities, m/Lt
μ_o = viscosity of oil phase, m/Lt
μ_w = viscosity of water, m/Lt
ϕ = porosity
η = recoverable oil recovery
λ = constant
ρ_o = density of oil phase, m/L^3
ρ_w = density of water phase, m/L^3
$\Delta\rho$ = density difference between water and oil phases, m/L^3

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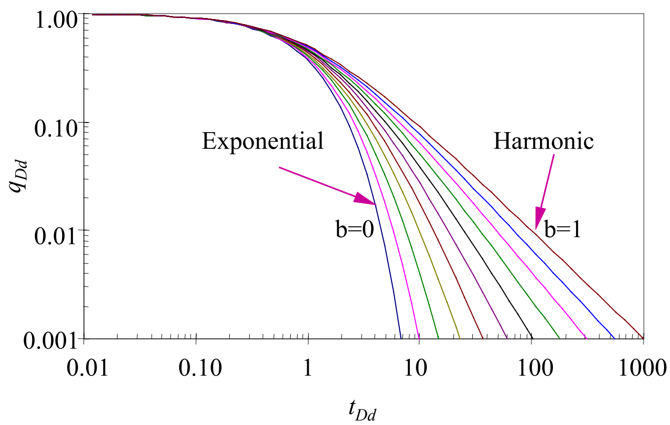

Fig. 1: Type curves based on the empirical Arps equation for decline curve analysis.

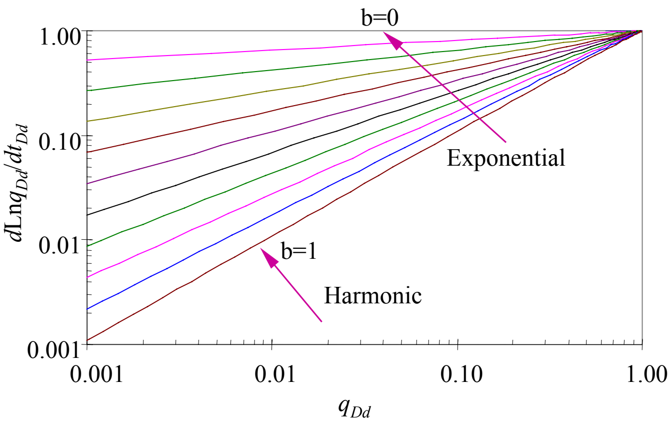

Fig. 2: Linear type lines inferred from the empirical Arps equation.

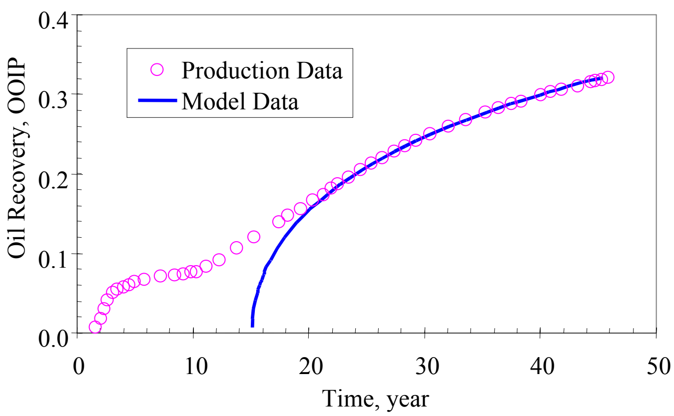

Fig. 3: Oil recovery history of the E.T. O'Daniel lease in Spraberry (naturally fractured reservoir)[6].

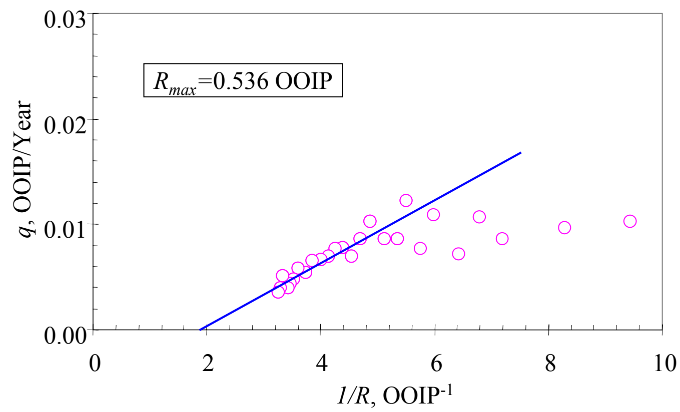

Fig. 4: Relationship between oil production rate and the reciprocal of oil recovery in a naturally fractured reservoir (Spraberry).

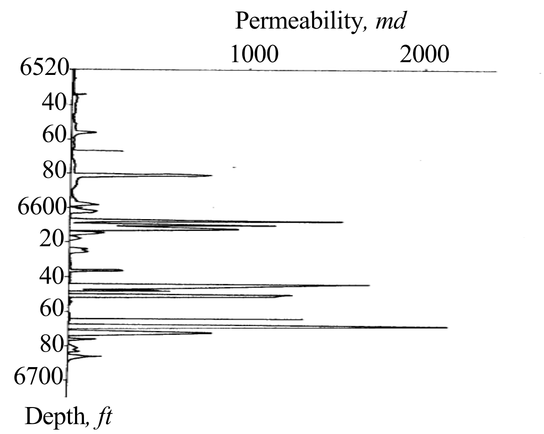

Fig. 5: Permeability distribution across the deltaic sand section in an offshore oil field[36].

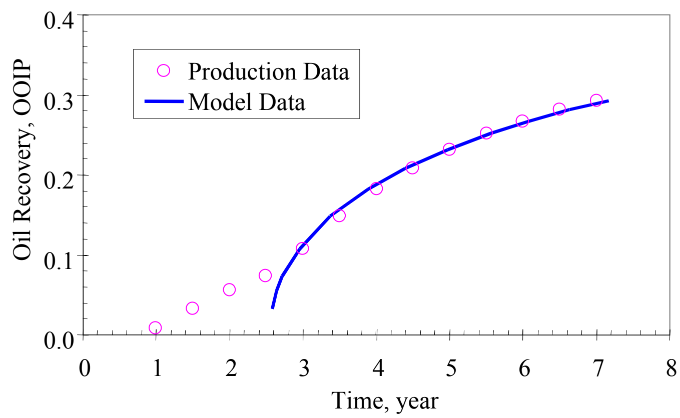

Fig. 6: Oil recovery data of an offshore oil field developed by water flooding[36].

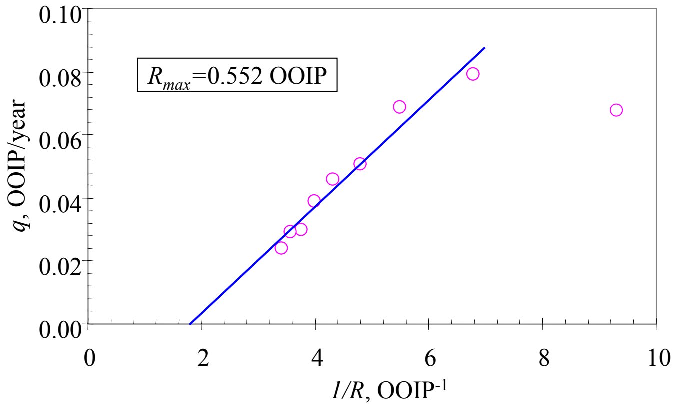

Fig. 7: Relationship between oil production rate and the reciprocal of oil recovery in an offshore oil field.

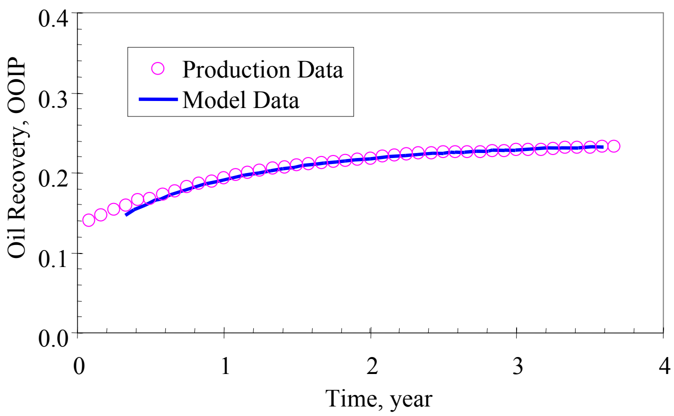

Fig. 8: Oil recovery after water breakthrough in a North Sea fault oil field developed by water flooding[36].

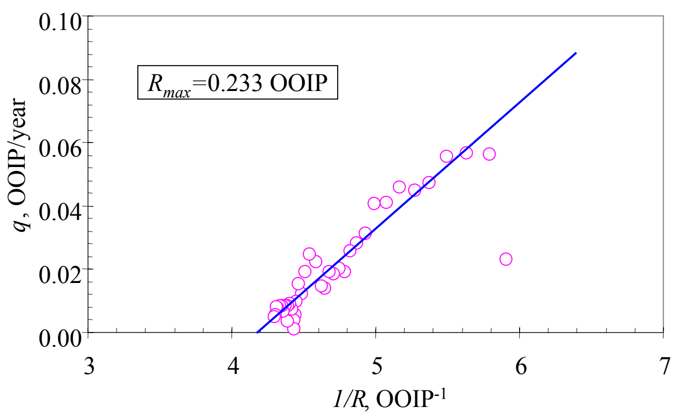

Fig. 9: Relationship between oil production rate and the reciprocal of oil recovery in a North Sea fault oilfield.

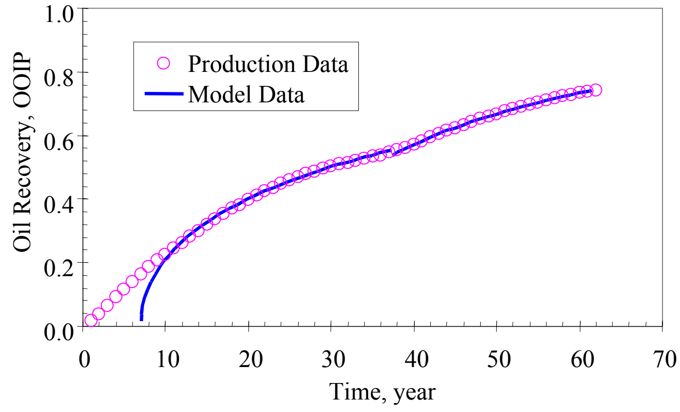

Fig. 10: Oil production data in the East Taxis oil field developed by water flooding[36].

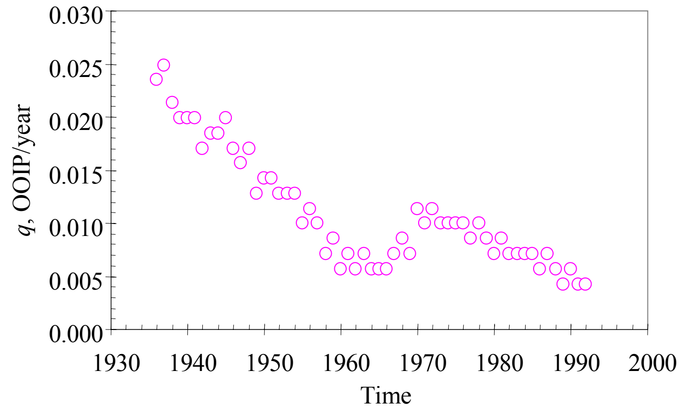

Fig. 11: Oil production rate in the East Taxis oil field developed by water flooding[36].

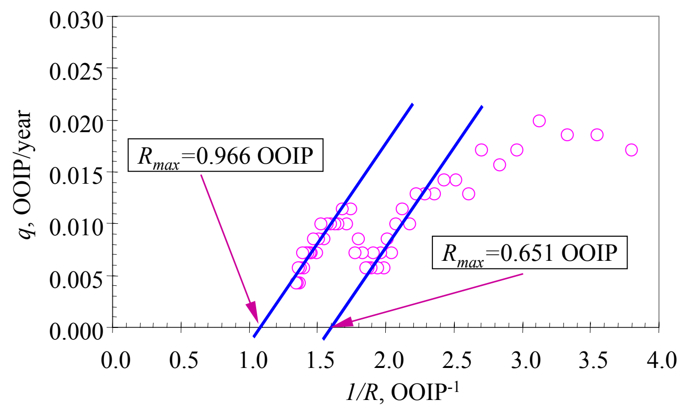

Fig. 12: Relationship between oil production rate and the reciprocal of oil recovery in the East Taxis oil field.