Phenomenological Modeling of Critical Condensate Saturation and Relative Permeabilities in Gas Condensate Systems

Kewen Li and Abbas Firoozabadi
Reservoir Engineering Research Institute (RERI)

Abstract
The effects of gravity, viscous forces, interfacial tension, and wettability on the critical condensate saturation and relative permeability of gas condensate systems are studied using a phenomenological simple network model. The results from the simple model show that wettability significantly affects both critical condensate saturation and gas phase relative permeability. Gas phase relative permeability at some saturations may increase ten times as contact angle is altered from 0° (strongly liquid-wet) to 85° (intermediately gas-wet). The results suggest that gas well deliverability in condensate reservoirs can be enhanced by wettability alteration near the wellbore.

Introduction
Much attention has been paid to the study of in-situ liquid formation and fluid flow mechanisms in gas condensate systems in recent years. When gas condensate reservoirs are developed by pressure depletion, gas well deliverability is affected by the amount and the distribution of the condensate formed around the wellbore. The understanding of the parameters that affect the distribution and the amount of condensate saturation ($S_c$) in the wellbore and the effect of these parameters on gas and liquid flow are important in the development of the methods for increased gas deliverability.

There are numerous field examples of gas condensate reservoirs that experience a sharp drop in gas well deliverability at high pressures due to condensation near the wellbore. There are examples that show that the condensation may even kill gas production. The sharp reduction in gas deliverability may be due to the shape of gas phase relative permeability ($k_{rg}$); the mechanisms of gas productivity impairment are not yet clear. Two main parameters affect condensate recovery and gas well deliverability. These two parameters are critical condensate saturation ($S_{cc}$, which affects liquid recovery) and gas phase relative permeability ($k_{rg}$, which affects well deliverability).

Various authors have measured $S_{cc}$ and a wide range of observations has been reported. The measured values of $S_{cc}$ are generally in the range of 10 to 50%. Gravier et al. (1986) measured $S_{cc}$ in low-permeability carbonate cores with permeability in the range of 0.5 to 40 md and interfacial tension ($\sigma$) variations from 0.5 to 1.5 dynes/cm. $S_{cc}$ varied from 24.5 to 50.5% with an average of 35.0%. These measurements were conducted in the presence of connate water. Morel et al. (1992) reported a critical condensate saturation of less than 1% for $\sigma = 0.05$ dynes/cm and connate water saturation of 20%. The critical condensate saturation was measured in a vertical dolomite core. The measured $S_{cc}$ is apparently related to various effects including gravity and interfacial tension.

Some authors have accounted for the effect of gravity by conducting flow experiments both in horizontal and vertical directions. Danesh et al. (1991) studied the effect of gravity on $S_{cc}$ in water-wet micromodels and sandstone cores. They found that $S_{cc}$ in a
vertical core was lower than that in a horizontal core; their tests showed that gravity reduced the critical condensate saturation. These authors also examined the dependence of critical condensate saturation on $\sigma$. As expected, $S_{cc}$ increases with an increase in $\sigma$. Ali et al.\textsuperscript{11} (1993) observed an improved condensate recovery of 17.2\% in a vertical gas injection compared with that in a horizontal injection. Their experiments were conducted with a high interfacial tension for gas condensates ($\sigma = 0.92$ dynes/cm). Sandstone cores and a synthetic six-component fluid were used by these authors. When $\sigma$ was decreased to 0.04 dynes/cm, condensate recovery improved to 27.5\% with vertical injection. Henderson et al.\textsuperscript{12} (1993) also reported a significant reduction of the critical condensate saturation in long vertical core samples in comparison with those in horizontal cores. Munkerud\textsuperscript{6} (1989), however, did not find significant difference in condensate recovery and saturation between horizontal and vertical pressure depletion. He used Berea cores and a synthetic six-component fluid.

A number of authors have studied gas phase relative permeability ($k_{rg}$) of gas condensate systems. Gravier et al.\textsuperscript{4} (1986) found an abrupt decrease in $k_{rg}$ at the saturation close to $S_c$. Chen et al.\textsuperscript{5} (1995) measured $k_{rg}$ at reservoir pressures with reservoir rock and reservoir fluids from two North Sea gas condensate reservoirs. The interfacial tension $\sigma$ varied from 0.01 to 0.42 dynes/cm. The results showed that $k_{rg}$ reduced about ten-fold due to condensation when $S_c$ reached 20\% for the core and fluid from Reservoir A. Reduction of $k_{rg}$ with the increase of $S_c$ for the core and fluid from Reservoir B is less pronounced but an abrupt decrease of $k_{rg}$ at the saturation close to $S_{cc}$ was also observed. Chen et al.\textsuperscript{5} (1995) showed that $k_{rg}$ is rate dependent; it increases with the increase of flow rate. Henderson et al.\textsuperscript{13} measured gas-condensate relative permeabilities using a long Berea core and a 5-component synthetic gas condensate fluid; $\sigma$ varied from 0.05 to 0.40 dynes/cm. They found that the relative permeabilities of both gas and liquid phases decreased with the increase of $\sigma$ and increased with the increase of flow rate. Gas relative permeability was more sensitive to $\sigma$ and flow rate than liquid relative permeability. The results from the work of Munkerud\textsuperscript{6} (1989) and Haniff et al.\textsuperscript{14} (1990) differ from the other authors. They did not observe the abrupt decrease of $k_{rg}$.

The brief review above points out that there are many unresolved issues in the study of critical condensate saturation and gas-condensate relative permeability. The sharp decrease in gas relative permeability at saturation close to $S_{cc}$ that was observed experimentally has not yet been studied theoretically. The dependence of $S_{cc}$, $k_{rg}$, and $k_{rc}$ on interfacial tension, viscous force, and gravity are not clear from the experimental data. The question remains how to measure $S_{cc}$, $k_{rg}$, and $k_{rc}$ in the laboratory and how to scale them to field conditions. The conclusions drawn from the experimental results are sometimes not consistent. A theoretical understanding of the effect of $\sigma$, gravity, viscous force, and wettability on $S_{cc}$, $k_{rg}$, and $k_{rc}$ provides insight into the recovery of liquid condensate from rich gas condensate reservoirs, and gas well deliverability from both rich and lean gas condensate reservoirs.

Theoretical work on the critical condensate saturation and relative permeabilities of gas condensate systems is very limited. Mohammadi et al.\textsuperscript{15} found that the relative permeabilities in gas condensate systems follow the conventional displacement process. These authors used percolation theory in their work for horizontal porous media. The
effect of interfacial tension, viscous force, gravity, and wettability on critical condensate saturation and gas-condensate relative permeability was not included in the work of Mohammadi et al.\textsuperscript{15}. Fang et al.\textsuperscript{16} (1996) developed a phenomenological network model for critical condensate saturation. The main conclusion from this model is that critical condensate saturation is a function of interfacial tension and contact angle in gas-oil-rock systems. The model of Fang et al.\textsuperscript{16} is based on the assumption of negligible viscous forces and large tube length.

In this work, we first modify the $S_{cc}$ model of Fang et al.\textsuperscript{16} for predicting critical condensate saturation to include the effect of viscous forces and to relax the assumption of large tube length. We then develop a relative permeability model ($k_r$ model) for gas condensate systems from a renormalization technique. Using the model, the effects of interfacial tension, flow rate, and contact angle hysteresis on the critical condensate saturation and gas and condensate relative permeabilities are investigated. The results from the modified $S_{cc}$ model and the $k_r$ model show that $S_{cc}$ and $k_r$ are significantly affected by interfacial tension, gravity, flow rate, and wettability. Wettability may have the most pronounced effect on both $S_{cc}$ and $k_r$.

**Network Model for Condensation in Porous Media**

The porous medium used in this study is represented by a simple 2D pore network with circular capillary tubes of length $L$ and cubic intersections of side width $b$; we assume $b$ to be equal to the maximum diameter of the capillary tube in the network. There are four tubes connected to each intersection; therefore, the coordination number is four. A log-normal size distribution is assumed with a standard deviation of 5.74 and an average radius of 23 $\mu$m for the capillary tubes. The size distribution of the network with $20 \times 20$ intersections is sketched in **Fig. 1**. In this sketch, the tube diameter is proportional to the thickness. The total pore volume of the network with a tube length of 6000 $\mu$m is about 8.93 mm$^3$; the total volume of the intersections with a side width of around 38.23 $\mu$m in the network is about 0.03 mm$^3$ and the estimated permeability of the network is about 1 darcy. The value of threshold radius $r_p$ used in this study is 20 $\mu$m (from condensation, liquid will first form in tubes with $r \leq r_p$). Note that the simple network described is selected for some sensitivity studies.

**Critical Liquid Length and Critical Condensate Saturation in a Single Capillary Tube**

Fang et al.\textsuperscript{16} developed a phenomenological model for the critical condensate saturation in a porous medium represented by a network. Their model is based on two assumptions: viscous forces are negligible and the critical height ($h_{cc}$) of liquid formed in-situ in the vertical capillary tubes is less than the tube length. The critical height of liquid in a vertical capillary tube is the maximum length of the liquid column before it becomes mobile. In this work, we extend the model of Fang et al.\textsuperscript{16} to include the effect of viscous forces, and provide the option for $h_{cc} > L$. When a liquid is first formed in a capillary tube, we assume that it appears in its middle in the form of a liquid bridge. During growth, interfacial tension, gravity, contact angle hysteresis, viscous force and the size of capillary tubes are the main factors that affect the stability of the new liquid phase.
The critical height $h_{cc}$ of liquid formed in-situ in a vertical circular tube of radius $r$ is expressed as (see Fig. 2a):

$$h_{cc} = \frac{2\sigma}{\Delta \rho g r} (\cos \theta_R - \cos \theta_A) - \frac{p_R - p_A}{\Delta \rho g}, \quad (1a)$$

where $p_R$ is the pressure at the top (or the receding side) and $p_A$ the pressure at the bottom (or the advancing side) of the liquid column; $\theta_R$ is the receding contact angle and $\theta_A$ the advancing contact angle; $\Delta \rho$ is the density difference between gas and liquid phase. The equation above is derived from the balance between capillary pressure, viscous force and gravity in a vertical capillary tube. In this study, for simplicity it is assumed that the gas density $\rho_g$ is equal to zero (that is, $\Delta \rho = \rho_c$) and $\sigma$ does not change with the pressure. The effect of pressure on $\sigma$ can be readily considered in our model.

In a vertical capillary tube, the critical length of liquid is indirectly calculated from

$$\Delta \rho g \cos \theta_R \cos \theta_A = -\frac{2 \sigma}{\Delta \rho g r} (\cos \theta_R - \cos \theta_A). \quad (1b)$$

The above equation implies that without viscous forces (that is, when $p_R - p_A = 0$) liquid cannot be moved, unless $\theta_R = \theta_A$. In a vertical tube, liquid will flow downward to the bottom of the tube when the height of the liquid column exceeds $h_{cc}$. The advancing contact angle will then change to $90^\circ$. At this time, the critical height of the liquid column will increase and can be calculated from Eq.1a. Once $h_{cc}$ is determined, the critical condensate saturation in a single vertical capillary tube is calculated as $h_{cc}/L$. The calculation of $S_{cc}$ in a horizontal capillary tube is relatively simple. $S_{cc}$ is equal to zero when the viscous force $(p_R - p_A)$ is greater than the interfacial force $2\sigma (\cos \theta_R - \cos \theta_A)$. Otherwise, $S_{cc}$ is equal to one or less, according to Eq. 1b. $S_{cc}$ in a single horizontal capillary tube may be less than one because of limited condensation.

**Process of Condensate Formation, Growth and Flow for $h_{cc}>L$**

Liquid condensation may take place in capillary tubes of smaller diameters based on the thermodynamics of curved interfaces. The condensation process in a network is represented by discrete steps. When pressure decreases below dewpoint pressure, the gas in those tubes with radius $r \leq r_p$ is allowed to condense. The amount of condensation per step in a liquid bridge is assumed to be 200 $\mu$m$^3$. There is no initial condensation in pores with $r > r_p$. Presented in Fig. 3 is the process of in-situ condensate formation, growth and flow when $h_{cc} > L$ in vertical tubes and the viscous force is greater than the interfacial force in horizontal tubes. It is known from our modeling results that critical condensate saturation and relative permeabilities are affected by the choice of $r_p$ – the larger the value of $r_p$, the greater the critical condensate saturation. But, the trend of the effects of gravity, wettability, and viscous forces on critical condensate saturation and relative permeabilities at various values of $r_p$ is similar. Therefore, only the results for $r_p=20$ $\mu$m are presented in this paper. For the study of viscous effect, it is assumed that gas is injected at the top of the network.
As the pressure in the system decreases below the dewpoint pressure, gas is assumed to condense in the middle of tube 3 \((r_3 < r_p)\) in the form of a liquid bridge, as shown in step 1. The top receding contact angle \(\theta_r\) (measured through the liquid phase) will be less than the bottom advancing contact angle \(\theta_a\) due to gravity and viscous effects.

Continuous condensation increases the liquid height in tube 3. Assuming the critical liquid height \(h_{3cc} > L\), the advancing contact angle will increase and reach 90°. This is shown in step 2. With further condensation, the advancing contact angle will exceed 90°. The receding contact angle at the top will also increase and reach 90° (see step 3).

If both the advancing and the receding contact angles reach 90° at the same time, the liquid column becomes unstable and will flow downward. The advancing contact angle will then exceed 90°. The receding and advancing contact angles will continue to increase with further condensation in tube 3 (see step 4). Given \(r_3 < \text{Min} (r_4, r_5, r_6)\), the condensate formed in-situ will contact the corner point “A” shown in step 5. The top interface has the maximum radius. At this stage, the condensate becomes unstable and forms two new interfaces, one in tube 2 and another in the intersection as shown in step 6. Because the radius of the interface in tube 2 is smaller than that of the intersection, the liquid pressure in the intersection is higher than that in tube 2. The liquid will be pulled into tube 2 as a result. The new configuration is shown in step 7.

Now, the liquid interface in tube 2 serves as a new condensation site. After tube 2 is filled by the condensate, the contact angle at the left end of tube 2 will increase until it reaches 90° (i.e., flat interface); then there will be no further condensation. However, condensation will continue at the right end of tube 2; as a result, a new liquid interface in the intersection will develop. The liquid interface will touch the corner point “D” with further condensation (see step 8).

The liquid interface will then break into two new liquid interfaces, one in tube 1 and another in tube 4, as shown in step 9. Given that \(r_1\) is less than \(r_4\), the liquid in tube 4 will be pulled into tube 1. The condensation will continue in tube 1 until it reaches the critical height \(h_{13cc}\). The liquid formed later in tube 1 will flow downward into tube 4. The liquid height in tube 1 remains the same. The critical height \(h_{13cc}\) is calculated from:

\[
h_{13cc} = \frac{2\sigma}{\Delta \rho g} \left( \frac{\cos \theta_r}{r_1} - \frac{\cos \theta_a}{r_3} \right) - \frac{p_R - p_A}{\Delta \rho g}.
\]  

(2)

After tube 4 is filled, the contact angle at the right end of tube 4 will increase until it reaches 90° (dashed line shown in step 9). Then, the contact angle at the left end of tube 2 will start to increase. Because \(r_4\) is larger than \(r_2\), the condensate will flow into the intersection connecting tubes 4, 8, 9, 10 (see step 10). The contact angle at the left end of tube 2 will exceed 90° with further condensation.

When in-situ condensation continues (see step 10), if \(r_8\) is less than \(r_{10}\), the liquid interface in the intersection will touch the corner point “F”’. The liquid interface will also break into two new interfaces, one in tube 8 and another still in the intersection after the interface contacts point “F’’. The configuration of the liquid interface in the intersection near point “F’’ will change and adjust itself as shown in step 11. At the same time, the contact angle at the left end of tube 2 will decrease. Because the radius of the liquid interface in the intersection is larger than that in tube 8 and the liquid pressure in the
intersection is higher than that in tube 8, the liquid will be pulled into it. After the force balance is established between the capillary force and the gravity in the liquid column in tube 8, the liquid in this tube will reach the critical height. With further condensation, the liquid formed in tubes 1 and 8 will flow downward and the liquid in the intersection will touch the corner point “G”.

The liquid interface in the intersection will break after it contacts the corner point “G”. If \( r_9 < r_{10} \), the liquid formed in tubes 1 and 8 will be pulled into tube 9 until it is filled (see step 12). With further condensation, the liquid formed in tubes 1 and 8 will flow downward into tube 10. Assuming the critical liquid height \( h_{10cc} \) in tube 10 is less than the tube length, the liquid will flow downward when the liquid height \( > h_{10cc} \). At the same time, the liquid in tube 8 will also flow downward into tube 10. The configuration developed is shown in step 13. With further condensation, the liquid formed in tube 1 will flow downward through tube 4 to tube 10 and then to the next element. When the liquid interface in the intersection touches point “G”, the liquid will first enter tube 10 instead of tube 9, if \( r_9 > r_{10} \). When the liquid height in tube 10 is equal to or larger than \( h_{10cc} \), the liquid in tube 10 flows downward (see step 14).

If \( r_9 > r_{10} \), the liquid interface will form two new interfaces after contacting the corner point “G”, one in tube 10 and another in the intersection. Because the radius of the liquid interface in the intersection is larger than in tube 10, the liquid will be pulled into tube 10. The liquid formed later will also enter tube 10 instead of tube 8. Assuming \( h_{10cc} < L \), the configuration shown in step 15 is established after the liquid height in tube 10 reaches critical height. With further condensation in this element, liquid will flow downward through tube 4 to tube 10 and then to the next element.

As stated previously, the process of in-situ condensate formation, growth and flow when \( h_{cc} < L \) is described in Ref. 17.

**Calculation of \( S_{cc} \)**

The first step for the estimation of \( S_{cc} \) is to determine the differential pressure in each tube. The pressure distribution in the entire network is calculated as follows. At every intersection one can write:

\[
\sum_{i=1}^{4} q_i = 0, \quad (3)
\]

where \( q_i \) is the flow rate of fluid in tube \( i \). The flow rate in a circular capillary tube can be calculated from

\[
q_i = \frac{\pi r_i^4}{8\mu} \left( \frac{\Delta p_i}{L} + \rho g \right), \quad (4)
\]

provided that the fluid is incompressible. In the above equation, \( \mu \) is fluid viscosity, \( \rho \) is fluid density, and \( \Delta p_i \) and \( r_i \) are differential pressure (inlet-outlet) and radius of tube \( i \), respectively.

Once pressures at inlet and outlet ends of the network are provided, Eqs.3 and 4 can be used to obtain the pressure distribution in the network. The Gauss iteration method can
be used to solve the set of linear equations. Note that for every tube $i$ with $h_c < h_{cc}$, $q_i = 0$, as we will discuss later. $S_{cc}$ in each tube can be calculated with the known pressure distribution according to Eqs. 1a, 1b and 2. Then the value of the total $S_{cc}$ in the network is readily calculated.

**Calculation of Fluid Conductance**

Consider single-phase flow in a capillary tube of radius $r_i$ and length $L$. According to Darcy’s law, the flow rate across the tube is $q_i = (k_i/\mu_i)\pi r_i^2(\Delta p_i + \rho g L)$, where $k_i$ is the permeability of tube $i$. An alternative form of the Darcy equation is $q_i = G_i(\Delta p_i + \rho g L)$, where $G_i$ is the conductance of tube $i$. Comparison of the Darcy equation with Eq. 4 provides the expression for the absolute fluid conductance of tube $i$:

$$G_{ij} = \frac{\pi r_i^4}{8\mu_j L}, \quad (S_j = 1), \quad (5)$$

where $G_{ij}$ and $S_j$ are the absolute conductance and the saturation of fluid $j$ in tube $i$, respectively; $\mu_j$ is the viscosity of fluid $j$. Prior to $S_{cc}$, when there is some liquid saturation in the tube, gas may not flow, and therefore, $q_i = 0$. The effective conductance of fluid $j$ in tube $i$, which is related to the effective permeability of phase $j$, can be calculated using the following equation when $S_c \leq S_{cc}$:

$$G_{ij} = 0, \quad (S_c \leq S_{cc}). \quad (6)$$

Now consider a tube with some liquid saturation $S_c$ (see Fig. 2b) which is greater than $S_{cc}$. In this case, liquid is located at the bottom of the tube and both liquid and gas may flow. The equation for calculating the effective gas conductance $G_{ig}$ of tube $i$ in 2-phase flow is:

$$G_{ig} = \frac{\pi r_i^4}{8\mu_g L} [1 - S_c + \frac{\mu_c}{\mu_g} S_{ic}]^{-1}, (S_c > S_{cc}), \quad (7)$$

where $\mu_g$ and $\mu_c$ are the gas and liquid condensate viscosity, respectively. $S_{ic}$ is the liquid condensate saturation in tube $i$. The derivation of Eq. 7 is provided in Appendix A. Alternatively, Eq. 7 can be obtained from Eq. 5, using the weighted average viscosity $\mu = S_g \mu_g + S_c \mu_c$. Similarly, the effective liquid conductance of tube $i$ in 2-phase flow can be calculated as:

$$G_{ic} = \frac{\pi r_i^4}{8\mu_c L} [S_{ic} + \frac{\mu_g}{\mu_c} (1 - S_{ic})^{-1}, (S_c > S_{cc}), \quad (8)$$

where $G_{ic}$ is the conductance of the liquid phase for case $S_c > S_{cc}$. The reason for presenting Eqs. 7 and 8 separately is to show that the effective conductance of each phase in two-phase state can be a function of its absolute conductance in single-phase state. On
the other hand, effective gas and liquid conductance may not be identical for tube shapes other than circular.

The next step is to calculate the effective conductance of both gas and liquid phases in the entire network based on conductances of individual tubes. Once the effective conductance of both gas and liquid phases in the network is calculated, gas and condensate relative permeabilities can be readily calculated. In the following, we will use the renormalization method\textsuperscript{18} to calculate the effective conductance of the network from conductances of individual tubes.

**Renormalization Method and Relative Permeabilities**

A useful technique to study transport in disordered systems is real-space renormalization group method, which has arisen in condensed-matter physics\textsuperscript{18}. King\textsuperscript{19-20} (1989,1993) has applied this technique to single- and two-phase fluid flows. Filippi and Toledo\textsuperscript{21} (1993) also used the method for heterogeneous and highly anisotropic systems. The renormalization method is accurate and computationally cost-effective in comparison with direct numerical simulation.

For a given phase and a known saturation distribution, the effective phase conductance is calculated in small regions, the so-called renormalization cells, and then in larger scales. Fig. 4 shows the grid units with different renormalization cell number $n_b$ for a 2D network. In this figure, renormalization cell number varies from 2 to 5. One unit cell with a renormalization cell number of $n_b$ is a unit consisting of $n_b^2$ nodes and $2n_b^2$ tubes. The effective conductance of one phase in the whole system can be obtained from the repeated application of the renormalization procedure.

Fig. 5 shows the renormalization procedure for $n_b = 2$. The primary network is comprised of four unit cells of $n_b = 2$. After the first step of renormalization, the network reduces to one unit cell of $n_b = 2$. In the last step, the primary network is converted to two conductors, one in the horizontal and the other in the vertical direction. Let us consider a unit cell with eight tubes (that is, $n_b = 2$), sketched in Fig. 6a. The effective networks in the vertical and horizontal directions are shown in Figs. 6b and 6c, respectively. The effective conductance, $G_z$, in the vertical direction and the effective conductance in the horizontal direction, $G_x$, for $n_b = 2$ are calculated from

$$
(9)
$$

and

$$
G_x = \frac{G_4G_5G_2 + G_4G_5G_3 + G_4G_5G_4 + G_4G_5G_5 + G_4G_5G_6 + G_4G_5G_7 + G_4G_5G_8 + G_4G_5G_9 + G_4G_5G_{10}}{G_2G_1 + G_4G_2 + G_6G_4 + G_4G_1 + G_4G_4 + G_4G_1 + G_4G_3 + G_4G_4}, \\
(10)
$$

respectively, where $G_1, G_2, \ldots, G_8$ represent the phase conductance of each tube (see Fig. 6). The derivation of Eq.9 is provided in Appendix B. Relative permeabilities for the network presented in Fig. 1 are calculated according to the following procedure: (1) Calculate the absolute gas conductance in the vertical direction $G_{abs}$ for the whole network first saturated completely with gas. This is realized by repeating the renormalization steps
at different scales as shown in Fig. 5; (2) Calculate the effective conductance of gas $G_{zg}$ or liquid phase $G_z$ at a given condensate saturation $S_c$ in the vertical direction when liquid is formed in the network due to the pressure decrease below the dewpoint pressure; (3) Calculate gas relative permeability $k_{rg}$ from $G_{zg}/G_{abs}$ and liquid relative permeability $k_{rc}$ from $(\mu_c G_{zc})/(\mu_g G_{abs})$. Calculations at different values of $S_c$ provide relative permeabilities.

**Results**

The model described above is used to calculate $S_{cc}$, $k_{rg}$, and $k_{rc}$. The results and the sensitivity of the calculation to various parameters are described in the following.

**Critical Condensate Saturation.** Critical condensate saturation is calculated using the modified $S_{cc}$ model in which viscous forces are considered and the critical height ($h_{cc}$) of liquid formed in-situ in vertical capillary tubes can be less than the tube length. The results show that critical condensate saturation is a function of interfacial tension, flow rate, gravity, and contact angle hysteresis.

**Effect of Interfacial Tension and Gravity.** The effect of $\sigma$ and gravity on $S_{cc}$ is shown in Fig. 7 for two different values of the viscous force $\Delta \rho$ across the network. The results are based on a tube length of 6000 $\mu$m, $\theta_k = 0^\circ$ and $0^\circ < \theta_A \leq 90^\circ$. Fig. 7 shows that $S_{cc}$ increases as $\sigma$ increases ($0 < \sigma < 4.0$ dynes/cm). Fang et al.\textsuperscript{16} (1996) made a similar observation for $\sigma$ in the range of 0 to 0.45 dynes/cm. The figure shows that $S_{cc}$ decreases with an increase of $\Delta \rho$. When $\sigma$ is large enough, $S_{cc}$ reaches the same maximum value for different $\Delta \rho$ values, which implies that the effect of gravity on $S_{cc}$ becomes negligible at high values of $\sigma$. The results in Fig. 7 are consistent with the experimental data by Danesh et al.\textsuperscript{10} (1991), Chen et al.\textsuperscript{5} (1995), Ali et al.\textsuperscript{11} (1993) and Henderson et al.\textsuperscript{12} (1993). These authors observed that $S_{cc}$ increases as $\sigma$ increases and the increase in gravity lowers $S_{cc}$. Various calculations from the model show that as long as $\theta_A$ is not zero ($\theta_A > 0$), the critical condensate saturation is not affected by the magnitude of $\theta_A$.

A comparison of Figs. 7a and 7b reveals that $S_{cc}$ decreases when $\Delta \rho$ increases. The effect of gravity on $S_{cc}$ becomes negligible at high $\Delta \rho$ when $\sigma$ varies in the range of 0 to 2 dynes/cm (see Fig. 7b). The results presented in Fig. 7 demonstrate that gravity may or may not affect $S_{cc}$. When $\Delta \rho$ is small (that is, at low flow rates), $S_{cc}$ is affected by gravity, whereas for high $\Delta \rho$, the effect of gravity is less pronounced. The results in Fig. 7 explain the effect of gravity on $S_{cc}$ and reveal that there may be no inconsistency in the experimental data reported by various authors. The effect of viscous forces is discussed in more detail next.

**Effect of Flow Rate.** The effect of viscous force (that is, flow rate) on $S_{cc}$ is shown in Fig. 8 for $\Delta \rho = 0.1$ and 0.5 g/ml. Other parameters are the same as those used previously. $\Delta \rho$ varies in the range of 0.05 to 0.4 psi in Fig. 8. This figure shows that the critical condensate saturation decreases with the increase of the viscous force (that is, increase in pressure drop). A comparison of Figs. 8a and 8b reveals that the effect of $\Delta \rho$ on $S_{cc}$ is
more pronounced at lower gravity when $\sigma$ varies in the range of 0 to 2 dynes/cm (see Fig. 8a). The effect of $\Delta p$ on $S_{cc}$ reduces at high $\sigma$ ($\sigma > 3$ dynes/cm). The calculated results shown in Fig. 8 are consistent with the experimental data of Chen et al. who found that $S_{cc}$ decreases as flow rate increases.

**Effect of Contact Angle Hysteresis.** In a gas-oil-rock system, oil phase is considered as the wetting phase and gas is the non-wetting phase. It is often assumed that the contact angle through the liquid phase is zero. The assumption of liquid-wetting in a gas-liquid system is valid but the assumption of $\theta = 0^\circ$ may be invalid. If the wettability of the fluid-rock system in a gas condensate reservoir near wellbore could be altered to intermediate gas-wetting by some chemicals, $S_{cc}$ and $k_{rg}$ may also change. Fig. 9a presents $S_{cc}$ vs. $\sigma$ for $\theta_R = 85^\circ$ and $85^\circ < \theta_A \leq 90^\circ$. Other parameters are the same as those used in Fig. 7. This figure shows that $S_{cc}$ becomes very small and variations of $\sigma$ or $\Delta \rho$ have very little effect on $S_{cc}$ when the receding contact angle increases to $85^\circ$. Fig. 9b shows the effect of receding contact angle; $S_{cc}$ decreases as the receding contact angle increases from 0 to $85^\circ$ at a fixed $\sigma$. This figure demonstrates that wettability alteration to preferential gas-wetting has the most pronounced effect on $S_{cc}$.

The observation that there may be no effect of $\sigma$ on $S_{cc}$ for intermediate gas-wettability (see Fig. 9) is interesting. The critical condensate saturation is sensitive to $\sigma$ if the fluid-rock system is strongly liquid-wet as was shown in Fig. 7. However, for an intermediately gas-wet system, the critical condensate saturation becomes small and does not increase much even at very high $\sigma$, as shown in Fig. 9.

**Effect of Network Size.** We calculated $S_{cc}$ with different network sizes in order to study the sensitivity of our $S_{cc}$ model to the network size. The results showed that for $\sigma = 0.01$ dynes/cm with or without viscous force, $S_{cc}$ becomes approximately constant when the network size is larger than $15 \times 15$. This demonstrates that the network size chosen for this study, $20 \times 20$, is appropriate.

**Relative Permeabilities.** In the following we will present the results for the study of relative permeability from the model.

**Features of Gas Relative Permeability.** Fig. 10a depicts a sketch of the measured gas-phase relative permeability curve for low-permeability carbonate rocks in gas condensate systems. Fig. 10b presents the calculated gas phase relative permeability curve from our model. This figure shows that the model reproduces the features of the experimental data. Both the model and experimental data show an abrupt drop in gas phase relative permeability at condensate saturation close to $S_{cc}$. When gas phase relative permeability decreases abruptly, the gas well deliverability reduces sharply.

**Effect of Interfacial Tension.** The effect of $\sigma$ on gas-condensate relative permeabilities is plotted in Fig. 11. The saturation interval is selected to be for $S_c > S_{cc}$ to compare model results with experimental data. $\sigma$ varies in the range of 0.01 to 0.50 dynes/cm. Fig. 11
shows that both gas and liquid phase relative permeabilities decrease with the increase of \( \sigma \). Liquid phase relative permeability is less sensitive to \( \sigma \) than gas phase relative permeability. The model results are in agreement with the experimental data of Henderson et al.\(^{13} \). These authors found that the relative permeabilities of both phases decrease with the increase of \( \sigma \). The gas relative permeability reduction was, however, more pronounced.

**Effect of Flow Rate.** The effect of flow rate on gas and condensate relative permeabilities is graphed in Fig. 12. This figure shows that both \( k_{rg} \) and \( k_{rc} \) increase with the increase of \( \Delta p \). Our results are in agreement with the experimental work by Chen et al.\(^{5} \) who found that gas relative permeability increased as flow rate increased. The liquid phase relative permeabilities shown in Fig. 12 are much less sensitive to viscous forces than the gas phase relative permeability. The model results of relative permeabilities are also consistent with the experimental observation of Henderson et al.\(^{9} \). These authors found that the relative permeability of both phases increased with the increase of flow rate but that liquid relative permeability increased less than gas relative permeability.

**Effect of Contact Angle Hysteresis.** If the wettability of a fluid-rock system in a gas condensate reservoir near wellbore can be altered from strongly liquid-wet to intermediately gas-wet, the gas and condensate relative permeabilities behavior may also change. Fig. 13 depicts model results for \( k_{rg} \) and \( k_{rc} \) at different receding contact angles. This figure shows that both \( k_{rg} \) and \( k_{rc} \) increase as the wettability to liquid decreases (\( \theta_{R} = 0, 40, 80^o \) and \( \theta_{R} < \theta_{A} = 90^o, \sigma = 0.4 \text{ dynes/cm} \)). There is significant increase in the gas phase relative permeability over the whole saturation range when receding contact angle increases from zero (strongly liquid-wet) to 80° (intermediately gas-wet). These results suggest that if the rock wettability around wellbore can be altered from liquid-wet to intermediately or preferentially gas-wet, gas well deliverability may increase significantly. Gas phase relative permeability, \( k_{rg} \), reduces very rapidly for the strongly liquid-wet condition (\( \theta_{R} = 0^o \)) and approaches zero when \( S \) increases to 22%. Fig. 13 gives a clear indication of the significance of wettability and contact angle hysteresis on gas relative permeability.

We also studied the effect of network size on relative permeabilities and found it was very small for a network size up to 35×35. Although the network size used in this study is small, the relative permeabilities from the model could indeed represent some critical features of experimental results, as indicated in Fig. 10b.

To our knowledge, the effect of wettability on relative permeabilities in gas condensate systems has not yet been experimentally studied. We have embarked on an experimental research program in order to measure the relative permeabilities of gas condensate systems under varying wettability conditions. Some preliminary data on relative permeabilities are presented in Ref. 22.

**Blocking Effect Due to In-situ Condensation.** In some gas reservoirs, a well may be blocked completely and the gas production may stop at high pressures\(^{4} \). This phenomenon is a result of in-situ liquid formation near the wellbore as reservoir pressure drops below
the dewpoint. The liquid formed newly in the porous medium blocks both small and large pores and reduces the effective permeability of gas phase significantly.

The distribution of the liquid condensate formed in-situ at different stages of condensation in the pore network is presented in **Fig. 14.** **Fig. 14a** shows the condensate distribution at $S_c = 5.0\%$. The black represents the liquid condensate and the white the gas phase. The condensate is first formed in the middle of smaller tubes in the form of liquid bridges. The capillary tubes that contain liquid bridges may be blocked for gas-phase flow. As a result, the number of tubes for gas flow is reduced and the effective gas permeability through the whole network is decreased. The gas production will, therefore, reduce when pressure drops below the dewpoint. With further pressure decrease, more liquid drops out and condensate saturation $S_c$ increases. **Fig. 14b** shows the liquid distribution at $S_c = 15.0\%$. The liquid occupies some large capillary tubes. The number of capillary tubes available for gas flow is further reduced. **Fig. 14c** shows the liquid distribution at the critical condensate saturation, $S_{cc} = 58.0\%$; there are almost no continuous paths available for gas flow. The effective gas permeability is zero; when this occurs, gas production may stop completely.

**Discussion**

There are a number of simplifying assumptions that we have made in the simple network model for critical condensate saturation and relative permeabilities in gas condensate systems. These include 1) keeping $\Delta \rho$ and $\sigma$ constant with pressure decline, 2) neglecting the effect of water-film, and connate water, 3) simultaneous formation of liquid bridges in all the tubes with $r \leq r_p$, and 4) two-dimensional model representation of porous media. Assumptions 1 and 3 could be relaxed without much difficulty, but the extension of the model from two to three dimensions complicates the bookkeeping. Extension of the model to include the effect of initial water may not be straightforward. The model may also be extended to include tube shapes other than circular. The main goal of the study was to find out the most sensitive parameter that can influence $S_{cc}$ and $k_{rg}$. The results presented above have clearly shown that wettability alteration from strongly liquid condensate-wetting to weak liquid condensate-wetting is the key parameter. The experimental work presented in Ref. 22 and work in progress are in line with the model results. Interestingly, both with and without initial water saturation, the alteration of wettability to preferential gas wetting affects $S_{cc}$ and $k_{rg}$ the most.

**Conclusions**

The major conclusions drawn from the work are:

1. The predictions from our simple model show that gravity generally reduces critical condensate saturation; but as interfacial tension increases, the gravity effect becomes less pronounced. The effect of gravity is pronounced when viscous forces are small. These results are in agreement with experimental data.

2. Viscous forces may have a pronounced effect on $S_{r}$, especially at low interfacial tension. However, the most important parameter is wettability expressed in terms of the receding contact angle; wettability alteration to intermediate gas-wetting reduces
critical condensate saturation dramatically, irrespective of gravity and interfacial tension.

3. The model predicts a significant effect of interfacial tension and viscous forces on gas phase relative permeability. These predictions are in line with experimental data.

4. The model results show that there is a very significant effect of gas-wetting on gas relative permeability; gas phase relative permeability may increase orders of magnitude as the receding contact angle increases from 0 to 80°. The results imply that the most effective method for increasing gas well deliverability may be the alteration of wettability around the wellbore.

Nomenclature

- $G_{abs}$ = absolute gas conductance of the network at single-phase state
- $G_{ic}$ = effective conductance of liquid condensate in tube $i$ at two-phase state
- $G_{ij}$ = absolute conductance of the fluid $j$ in tube $i$ at single-phase state
- $G_{ig}$ = effective conductance of gas in tube $i$ at two-phase state
- $G_{zc}$ = effective gas conductance of the network at two-phase state
- $G_{zc}$ = effective liquid conductance of the network at two-phase state
- $h_c$ = height of liquid column
- $h_{cc}$ = critical height of liquid column
- $I$ = total electrical current through the network in the vertical direction
- $I_i$ = electric current through the conductance $G_i$ ($i = 3, 4, 6, 7, 8$)
- $k_{rg}$ = gas-phase relative permeability
- $k_{rc}$ = liquid-phase relative permeability
- $L$ = capillary tube length
- $p_A$ = pressure at the bottom
- $p_R$ = pressure at the top
- $P_c$ = capillary pressure between gas and condensate
- $\Delta p$ = total pressure differential
- $\Delta p_c$ = pressure differential across condensate phase
- $\Delta p_g$ = pressure differential across gas phase
- $\Delta p_i$ = pressure differential between two ends of the capillary tube $i$
- $q_g$ = flow rate of gas
- $q_i$ = flow rate of fluid through the capillary tube $i$
- $q_c$ = flow rate of condensate
- $r$ = radius of a capillary tube
- $r_i$ = radius of the capillary tube $i$
- $r_p$ = threshold radius
- $S_c$ = condensate saturation
- $S_{cc}$ = critical condensate saturation
- $V_o$ = the voltage at the top of the effective network
- $V_1$ = voltage at point $A$
- $V_2$ = voltage at point $B$
- $V_t$ = voltage at the bottom of the effective network
- $\theta_A$ = advancing contact angle
\( \theta_h \) = receding contact angle
\( \rho_c \) = condensate density
\( \rho_g \) = gas density
\( \Delta \rho \) = density difference between condensate and gas
\( \sigma \) = interfacial tension
\( \mu \) = fluid viscosity
\( \mu_g \) = gas viscosity
\( \mu_c \) = condensate viscosity

**Acknowledgement**

Dr. P.G. Toledo initiated work on renormalization calculation of gas phase relative permeability at the Reservoir Engineering Research Institute (RERI). We have benefited from his work considerably. This work was supported by the Technology Research Center of the Japan National Oil Corporation (JNOC), the US DOE grant DE-FG22-96BC14850 and member companies of RERI. Their support is greatly appreciated.

**References**

Appendix A: Gas Conductance in a Single Circular Tube

When both gas and liquid phases flow in a circular tube (see Fig. 2b), the gas flow rate can be calculated from the Poiseuille equation:

\[ q_g = \frac{\pi \Delta p_g}{8 \mu_g (L - h_t)} , \]  

(A-1)

where \( \Delta p_g \) is the differential pressure across gas phase, and \( h_t \) is the liquid height (or length). The effect of gravity and compressibility are both neglected. The expression for the liquid condensate flow rate is:

\[ q_c = \frac{\pi \Delta p_c}{8 \mu_c \left( \frac{\Delta p_c}{h_t} + \rho_c g \right) } , \]  

(A-2)

where \( \mu_c \) is the condensate viscosity and \( \Delta p_c \) the differential pressure across the condensate phase. The differential pressure \( \Delta p \) between the two ends of a tube is given by:

\[ \Delta p = \Delta p_g + \Delta p_c + P_c , \]  

(A-3)
where \( P_c \) is the capillary pressure between gas and condensate liquid phase. The gas flow rate is equal to the liquid flow rate when the fluid compressibility is neglected. That is:

\[
q_g = q_c. \tag{A-4}
\]

Combining the above equations (Eqs. A-1 to A-4), the gas phase differential pressure can be obtained:

\[
\Delta p_g = \frac{1}{1 + \frac{\mu_c}{\mu_g} \frac{h_c}{L - h_c}} (\Delta p - P_c + \rho_g g h_c). \tag{A-5}
\]

Substituting Eq. (A-5) into Eq. (A-1):

\[
q_g = \frac{\pi d^4}{8 \mu_g (L - h_c)} \frac{1}{1 + \frac{\mu_c}{\mu_g} \frac{h_c}{L - h_c}} (\Delta p - P_c + \rho_g g h_c). \tag{A-6}
\]

The gas conductance is then determined as:

\[
G_{gas} = \frac{\pi d^4}{8 \mu_g (L - h_c)} \frac{1}{1 + \frac{\mu_c}{\mu_g} \frac{h_c}{L - h_c}}. \tag{A-7}
\]

For tube \( i \) in the network, the condensate saturation is given by:

\[ S_i = h_c / L. \tag{A-8} \]

Equation (7) in the text can be obtained by substituting Eq. (A-8) into Eq. (A-7).

**Appendix B: Effective Conductance in Vertical Direction for \( n_b = 2 \)**

The effective network in vertical direction for \( n_b = 2 \) is shown in Fig. 6c. One can write the following equations based on the theory of electric circuits:

\[
I_3 = G_3(V_2 - V_1), \tag{B-1}
\]
\[
I_4 = G_4(V_2 - V_1), \tag{B-2}
\]
\[
I_6 = G_6(V_0 - V_2), \tag{B-3}
\]
\[
I_7 = G_7(V_0 - V_1), \tag{B-4}
\]
\[
I_8 = G_8(V_1 - V_1). \tag{B-5}
\]

At points A and B
\[ \sum_i I_i = 0, \text{ (at point A: } i=4,7,8) \]  \hspace{1cm} (B-6)

\[ \sum_i I_i = 0, \text{ (at point B: } i=3,4,6). \]  \hspace{1cm} (B-7)

The total electric current through the system, \( I = I_6 + I_7 \), is given by

\[ I = G_z (V_0 - V_i). \]  \hspace{1cm} (B-8)

In the above equations, \( I_i \) (\( i = 3, 4, 6, 7, 8 \)) is the electric current through the conductance \( G_i \) (\( i = 3, 4, 6, 7, 8 \)); \( V_o \) and \( V_t \) are the voltages at the top and the bottom of the effective network, respectively; \( V_1 \) and \( V_2 \) are the voltages at points A and B, respectively. \( G_z \) is effective conductance of the network in the vertical direction.

Combining Eqs.(B-1) – (B-7):

\[
V_1 = \frac{G_4(G_6V_0 + G_3V_i) + (G_3 + G_4 + G_6)(G_2V_0 + G_8V_i)}{(G_4 + G_7 + G_6)(G_3 + G_4 + G_6) - G_4^2} \hspace{1cm} (B-9)
\]

\[
V_2 = \frac{G_4(G_3V_0 + G_6V_i) + (G_4 + G_7 + G_8)(G_6V_0 + G_3V_i)}{(G_4 + G_7 + G_6)(G_3 + G_4 + G_6) - G_4^2}. \hspace{1cm} (B-10)
\]

Substituting Eqs.(B-3) and (B-4) into Eq.(B-9):

\[
G_z = \frac{(G_6 + G_7)V_0 - (G_6V_2 + G_7V_1)}{V_0 - V_t}. \hspace{1cm} (B-11)
\]

Equation (9) in the text can be obtained by substituting Eqs.(B-9) and (B-10) into Eq.(B-11).

Fig. 1 Size distribution of the 20X20 network
(a) Liquid bridge in a circular tube after formation

(b) Liquid bridge at the bottom of a circular tube

Fig. 2 Liquid configuration in a single circular tube
Fig. 3 Condensation in a network, $h_{cc} > L$
Step 10

Step 11

Step 12

Step 13

Step 14

Step 15

Fig. 3  Condensation in a network, $h_{cc} > L$ (Continued)
Fig. 4 Schematic of renormalization for different cell numbers

Fig. 5 Schematic of the renormalization procedures ($n_b=2$)
Fig. 6  Effective conductance in the horizontal and vertical directions ($n_b = 2$)

(a) Conductance network

(b) Effective conductance in the horizontal direction

(c) Effective conductance in the vertical direction

Fig. 6  Effective conductance in the horizontal and vertical directions ($n_b = 2$)
Fig. 7  Effect of $\sigma$ and gravity on $S_{cc}$ ($\theta_R = 0^\circ$, $0^\circ < \theta_A < 90^\circ$)

(a) $\Delta \rho = 0.05$ psi

(b) $\Delta \rho = 0.40$ psi
Fig. 8  Effect of $\sigma$ and viscous force on $S_{cc}$ ($\theta_R = 0^\circ$, $0^\circ < \theta_A < 90^\circ$)
Fig. 9  Effect of $\sigma$, gravity, and wettability on $S_{cc}$

(a) Effect of $\sigma$ and gravity on $S_{cc}$
($\Delta \rho =$ 0.2 psi, $\theta_R = 85^\circ$, $85^\circ < \theta_A < 90^\circ$)

(b) Effect of $\sigma$ and wettability on $S_{cc}$
($\Delta \rho =$ 0.3 g/ml, $\theta_R =$ 0 - 85$^\circ$, $\theta_R < \theta_A < 90^\circ$)

Fig. 9  Effect of $\sigma$, gravity, and wettability on $S_{cc}$
Fig. 10  Gas-phase relative permeability in gas-condensate systems

(a) Sketch of experimental data (Gravier, et al.⁴)

(b) Model result

\( \sigma = 0.5 \text{ dynes/cm}, \Delta p = 0.2 \text{ psi}, \Delta \rho = 0.5 \text{ g/ml}, \theta_R = 0^\circ, 0^\circ < \theta_A < 90^\circ \)

Fig.10  Gas-phase relative permeability in gas-condensate systems
Fig. 11 Effect of $\sigma$ on relative permeability
($\Delta p = 0.3$ psi, $\Delta \rho = 0.5$ g/ml, $\theta_R = 0^o$, $0^o < \theta_A < 90^o$)

Fig. 12 Effect of viscous force on relative permeability
($\sigma = 0.5$ dynes/cm, $\Delta \rho = 0.5$ g/ml, $\theta_R = 0^o$, $0^o < \theta_A < 90^o$)
Fig. 13  Effect of $\theta_R$ on relative permeability
($\sigma=0.4$ dynes/cm, $\Delta p=0.2$ psi, $\Delta \rho=0.5$ g/ml, $\theta_R < \theta_A < 90^\circ$)

Fig. 14a  Saturation distribution at $S_c = 5\%$
($\sigma=0.5$ dynes/cm, $\Delta p=0.02$ psi, $\Delta \rho=0.1$ g/ml, $\theta_R = 0^\circ$, $0^\circ < \theta_A < 90^\circ$)
Fig. 14b Saturation distribution at $S_c = 15\%$
($\sigma = 0.5$ dynes/cm, $\Delta p = 0.02$ psi, $\Delta \rho = 0.1$ g/ml, $\theta_R = 0^\circ$, $0^\circ < \theta_A < 90^\circ$)

Fig. 14c Saturation distribution at $S_c = 58\%$
($\sigma = 0.5$ dynes/cm, $\Delta p = 0.02$ psi, $\Delta \rho = 0.1$ g/ml, $\theta_R = 0^\circ$, $0^\circ < \theta_A < 90^\circ$)