A Universal Capillary Pressure Model: Verification and Application

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Abstract
A universal capillary pressure model developed from fractal modeling of a porous medium was verified using the experimental data of capillary pressure curves of rock samples from The Geysers geothermal field. The capillary pressure curves were measured using a mercury intrusion technique. The results showed that the universal model could represent the capillary pressure curves of The Geysers rock satisfactorily while the frequently-used Brooks-Corey capillary pressure model could not. The values of fractal dimension, a representation of rock heterogeneity, were inferred from the match of the universal capillary pressure model to the experimental data.

Introduction
Capillary pressure plays an important role in geothermal reservoirs. As an example, Tsypkin and Calore (1999) developed a mathematical model of steam-water phase transition with capillary forces included. They investigated the main characteristics of the vaporization process and found that capillary pressure can play a stabilizing role for the vaporization front, causing a sharp front to develop. Urmeneta et al. (1998) also studied the role of capillary forces in the natural state of fractured geothermal reservoirs and found that capillary pressure tended to keep the vapor phase in the fractures and the liquid phase in the matrix. The numerical results from Urmeneta et al. (1998) showed that capillary forces control the transfer of fluids between fractures and matrix, the stability of the liquid-dominated two-phase zone, and the distribution of steam and water in geothermal reservoirs. This shows that the value of capillary pressure will influence the estimation of the energy reserves and production performance.

It is essential to represent capillary pressure curves properly because of the important role that capillary pressure plays in geothermal reservoirs. However Li and Horne (2003) found that the frequently-used Brooks-Corey model (1964) could not represent the capillary pressure curves of The Geysers rock samples satisfactorily. The main reason may be that The Geysers rock has a lot of microfractures. Later Li (2004) developed a universal capillary pressure model theoretically from fractal modeling of a porous medium. The main purpose of the current study was to verify the universal capillary pressure model using experimental data of capillary pressures in The Geysers rocks.

Methodology
A brief description of the universal capillary pressure model developed by Li (2004) is discussed in this section. The model is expressed as follows:
\[ P_e = \left[ p_{\text{max}} - \left( p_{\text{max}} - p_e \right) S_w^* \right]^{\frac{1}{\lambda}} \]  \hspace{1cm} (1)

where \( p_{\text{max}} \) is the maximum capillary pressure at \( S_{\text{Hg,max}} \), which is the maximum mercury saturation; \( p_e \) is the entry capillary pressure; \( \lambda \) is the pore size distribution index \((\lambda = 2 - D_f)\); \( D_f \) is the fractal dimension; \( S_w^* \) is the normalized wetting-phase saturation, defined as follows:

\[ S_w^* = \frac{S_w - S_{wr}}{1 - S_{wr}} \]  \hspace{1cm} (2)

where \( S_w \) is the wetting-phase saturation and \( S_{wr} \) is the residual saturation of the wetting-phase. Air is the wetting-phase and mercury is the nonwetting-phase in the case of measuring capillary pressure using a mercury intrusion technique.

Eq. 1 can be reduced as follows:

\[ P_e = p_{\text{max}} \left( 1 - b S_w^* \right)^{-\frac{1}{\lambda}} \]  \hspace{1cm} (3)

where \( b \) is a constant and expressed as follows:

\[ b = 1 - \left( \frac{p_e}{p_{\text{max}}} \right)^{-\lambda} \]  \hspace{1cm} (4)

For \( D_f < 2 \), if \( p_{\text{max}} \) approaches infinity, then Eq. 1 can be reduced to:

\[ P_e = p_e \left( S_w^* \right)^{-\frac{1}{\lambda}} \]  \hspace{1cm} (5)

Eq. 5 is the frequently-used Brooks-Corey model, which was proposed empirically by Brooks and Corey (1964).

According to this derivation, one can see that the Brooks-Corey capillary pressure model has a solid theoretical basis. This may be why the Brooks-Corey model has been seen to be a good fit to capillary pressure curves of many rock samples.

In the case in which \( b = 1 \), Eq. 1 can be reduced to:

\[ P_e = p_{\text{max}} \left( 1 - S_w^* \right)^{-\frac{1}{\lambda}} \]  \hspace{1cm} (6)
Eq. 6 is the imbibition capillary pressure model proposed by Li and Horne (2001) empirically (for $D_o > 2$).

In the case in which $b = 0$, Eq. 1 can be reduced to: $P_c = P_{max}$. This equation may be considered a capillary pressure model for a single capillary tube.

One can see that Eq. 1, as a general capillary pressure model, could be applied in both complicated porous media and in a single capillary tube as well as in both drainage and imbibition cases.

**Experimental Measurements**

The six core samples used in this study were the same as those used by Li and Horne (2003). The six core samples were from different wells at The Geysers geothermal field. The samples were irregular and too small to drill a plug for permeability measurements. The measured porosity of the core samples ranged from 0.1 to 4.0%. Capillary pressure curves of the six samples from The Geysers geothermal field were measured using the mercury-injection technique.

The surface tension of air/mercury is 480 mN/m and the contact angle through the mercury phase is $140^\circ$ (Purcell, 1949).

**Results**

The experimental data from the six Geysers samples were used to verify the universal capillary pressure model (Eq. 3). The results are presented and discussed in this section.

Fig. 1 shows a comparison between the experimental data and the universal capillary pressure model for the core sample SB15D_1. The solid diamond symbols represent the experimental data and the solid line represents the results matched using the universal capillary pressure model (Eq. 3). Firstly, one can see that the capillary pressure curve is not a straight line on a log-log plot, which implies that the frequently-used Brooks-Corey capillary pressure model can not be applied in such a geothermal rock. Secondly, the results show that the universal capillary pressure model (Eq. 3) can match the experimental data of satisfactorily.
Comparisons between the experimental data and the universal capillary pressure model for the other core samples are shown in Figs. 2-6. One can see that the universal capillary pressure model can match the experimental data suitably for all six of the core samples studied.

Figure 1: Model fit to the experimental capillary pressure curve of The Geysers rock (No. SB15D_1).

Figure 2: Model fit to the experimental capillary pressure curve of The Geysers rock (No. MLM_3).
Figure 3: Model fit to the experimental capillary pressure curve of The Geysers rock (No. $P_c_{92}$).

Figure 4: Model fit to the experimental capillary pressure curve of The Geysers rock (No. $SB15D_2$).
Figure 5: Model fit to the experimental capillary pressure curve of The Geysers rock (No. PRATI_5).

Figure 6: Model fit to the experimental capillary pressure curve of The Geysers rock (No. CA1862_4).

The values of the three parameters, $p_c$, $p_{\text{max}}$, and $D_f$, were also inferred from the model match for all of the core samples and the results are listed in Table 1. Note that the values of the entry capillary pressure ($p_c$) are small. This may be because of the effect of the fractures in the rock.

The values of fractal dimension listed in Table 1 are different from those calculated using another fractal model (Li and Horne, 2003) for some of the core samples. However the results are consistent with the visual observation of the heterogeneity shown in Fig. 7. The greater the fractal dimension, the greater the heterogeneity. The values of fractal dimension shown in Table 1 demonstrate that the heterogeneity of the core samples decreases from left to right. This
phenomenon can also be observed visually in Fig. 7 based on the curvatures of capillary pressure curves.

Table 1: Parameters inferred from model match

<table>
<thead>
<tr>
<th>Sample</th>
<th>SB15D_1</th>
<th>MLM_3</th>
<th>Pc_92</th>
<th>SB15D_2</th>
<th>PRATI_5</th>
<th>CA1862_4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$ (%)</td>
<td>4.0</td>
<td>2.5</td>
<td>0.4</td>
<td>0.1</td>
<td>1.1</td>
<td>0.8</td>
</tr>
<tr>
<td>$p_e$ (MPa)</td>
<td>0.007</td>
<td>0.06</td>
<td>0.046</td>
<td>0.034</td>
<td>0.244</td>
<td>0.046</td>
</tr>
<tr>
<td>$p_{\text{max}}$ (MPa)</td>
<td>208.2</td>
<td>148.6</td>
<td>139.4</td>
<td>183.7</td>
<td>209.9</td>
<td>147.1</td>
</tr>
<tr>
<td>$D_f$</td>
<td>3.120</td>
<td>3.069</td>
<td>2.957</td>
<td>2.483</td>
<td>2.224</td>
<td>2.188</td>
</tr>
</tbody>
</table>

One can see from Table 1 that there is no clear relationship between fractal dimension and porosity, or with the other parameters.

Figure 7: Capillary pressure curves of The Geysers rock samples.

Conclusions

Based on the present work, the following conclusions may be drawn:

1. The universal capillary pressure model developed from fractal modeling of a porous medium can be reduced to the frequently-used Brooks-Corey model and the Li-Horne model in specific cases.

2. The universal capillary pressure model can match the experimental data form The Geysers satisfactorily in all the cases studied. However the Brooks-Corey model cannot match the experimental data properly.

3. Fractal dimension, entry capillary pressure, and maximum capillary pressure can be evaluated from the match of the universal capillary pressure model to experimental data.

4. The inferred values of fractal dimension can be used to represent the heterogeneity of different rock samples quantitatively.
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Nomenclature

- $b$ = constant
- $D_r$ = fractal dimension
- $P_c$ = capillary pressure
- $p_e$ = entry capillary pressure
- $p_{max}$ = maximum capillary pressure at $S_{Hg, max}$
- $S_w$ = wetting-phase saturation
- $S_w^*$ = normalized wetting-phase saturation
- $S_{wr}$ = residual saturation of the wetting-phase
- $\lambda$ = pore size distribution index

References


