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# Correction to "A Stochastic-Conceptual Analysis of Rainfall-Runoff Processes on a Hillslope" by R. Allan Freeze

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In the paper "A stochastic-conceptual analysis of rainfall-runoff processes on a hillslope" by R. A. Freeze (*Water Resources Research*, *16*(2), 391–408, 1980), *Freeze* [1980] used a stochastic-conceptual mathematical model to investigate rainfall-runoff processes. Recently, several of the experiments performed in the earlier study were repeated by the senior author of this note, as part of a project related to the one reported in the work by *Loague and Freeze* [1985]. The purpose of this comment is to correct an error in the results reported by *Freeze* [1980]. In particular, we reexamine the influence that the autocorrelation properties of surface saturated hydraulic conductivity have in controlling runoff.

In the work by *Freeze* [1980], the notation $Y = \log K^S$ was used, where $K^S$ is the saturated hydraulic conductivity. The hydraulic conductivity distribution on a hillslope is viewed as a spatial stochastic process, with its properties represented by the three parameters $\mu_Y$, $\sigma_Y$, and $\alpha_Y$, where $\mu_Y$ is the mean of the log-transformed data, $\sigma_Y$ is the standard deviation, and $\alpha_Y$ is an autocorrelation parameter. *Freeze* [1980] showed that each of these parameters exerts an important influence on the statistical properties of runoff events arising from a hillslope under a given climatic regime. He concluded that the mean value is the most important parameter, the standard deviation is quite important, and the autocorrelation parameter is the least important.

On the basis of this analysis, *Freeze* [1980] also reported that a reduction in the degree of autocorrelation had a similar effect on simulated rainfall-runoff processes to a reduction in standard deviation. This result seems counterintuitive and is, in fact, incorrect. It will be shown here that with regard to saturated hydraulic conductivity fields, a reduction in the degree of autocorrelation has an effect similar to an increase in the standard deviation, at least for the synthetic hydrologic systems studied here.

In this exercise, two-dimensional autocorrelated surfaces of saturated hydraulic conductivity were generated using the same method described in the original study. The statistical parameters used to generate six different saturated hydraulic conductivity fields, for the hillslope illustrated in Figure 1, are listed in Table 1. The remaining parameters used in the rainfall-runoff simulations are listed in Table 1 of *Freeze* [1980]. Horton overland flow is the dominant streamflow generation mechanism for each of the six cases. The reader is directed to the original paper for the details of the experiments. Figure 2 shows the distribution of saturated hydraulic conductivity for case A. The reader should be forewarned that this conductivity surface (as well as other parameter and variable surfaces) differs from the one(s) reported in the original paper due to the separate random number generations. However, this nonuniqueness of stochastically generated surfaces does not corrupt the methods or findings of this study.

TABLE 1. Comparison of Saturated Hydraulic Conductivity Input Parameters for Stochastic-Conceptual Analysis

| Case | | Saturated Hydraulic Conductivity Parameters* | | |
|---|---|---|---|---|
| This Study | Freeze [1980] | $\mu_Y$ | $\sigma_Y$ | $\alpha_Y$† |
| A | A | −5.0 | 0.8 | 0.3 |
| E | E | −5.0 | 0.1 | 0.3 |
| F | F | −5.0 | 0.1‡ | 1.0 |
| H | | −5.0 | 0.8 | 0.9 |
| I | | −5.0 | 0.8 | 0.6 |
| J | | −5.0 | 1.6 | 0.3 |

*The saturated hydraulic conductivity $K^s$ was taken as lognormally distributed; therefore a parameter $Y$ was defined such that $Y = \log K^s$. The values of $Y_{ij}$ were generated from the distribution $N(\mu_Y, \sigma_Y, \alpha_Y)$; the values of $K_{ij}{}^s$ were determined from $K_{ij}{}^s = \exp(2.3\ Y_{ij})$.

†The spatial autocorrelation structure is described by the relation $\rho_Y(l) = e^{-\alpha_Y|l|}$, where $\rho_Y(l)$ is the autocorrelation function at lag $l$. An increase in the degree of autocorrelation corresponds to a decrease in the autocorrelation parameter (see Figure 6 of *Freeze* [1980]).

‡This value was given incorrectly as 0.8 by *Freeze* [1980].

TABLE 2. Comparison of Cases Showing Effects of Saturated Hydraulic Conductivity Distribution Parameters on Runoff Generation

| | Overland Flow: Number of Elements in Percentage Range | | | | Streamflow | |
|---|---|---|---|---|---|---|
| Case | 0–25 | 26–50 | 51–75 | 76–100 | $N^0$ | $\bar{Q}_{PK}$, L/s |
| A | 44 | 58 | 81 | 17 | 11 | 91 |
| | (50)* | (47) | (45) | (58) | (4) | (94)† |
| E | 0 | 87 | 113 | 0 | 25 | 84 |
| | (0) | (95) | (103) | (2) | (15) | (91)† |
| F | 0 | 83 | 116 | 1 | 19 | 87 |
| | (0) | (115) | (81) | (4) | (12) | (87)† |
| H | 52 | 52 | 74 | 22 | 6 | 75 |
| I | 55 | 43 | 75 | 27 | 5 | 77 |
| J | 67 | 32 | 43 | 58 | 1 | 96 |

*The values in brackets for cases A, E, and F are results from the original analysis [*Freeze*, 1980].

†These results were not reported by *Freeze* [1980].

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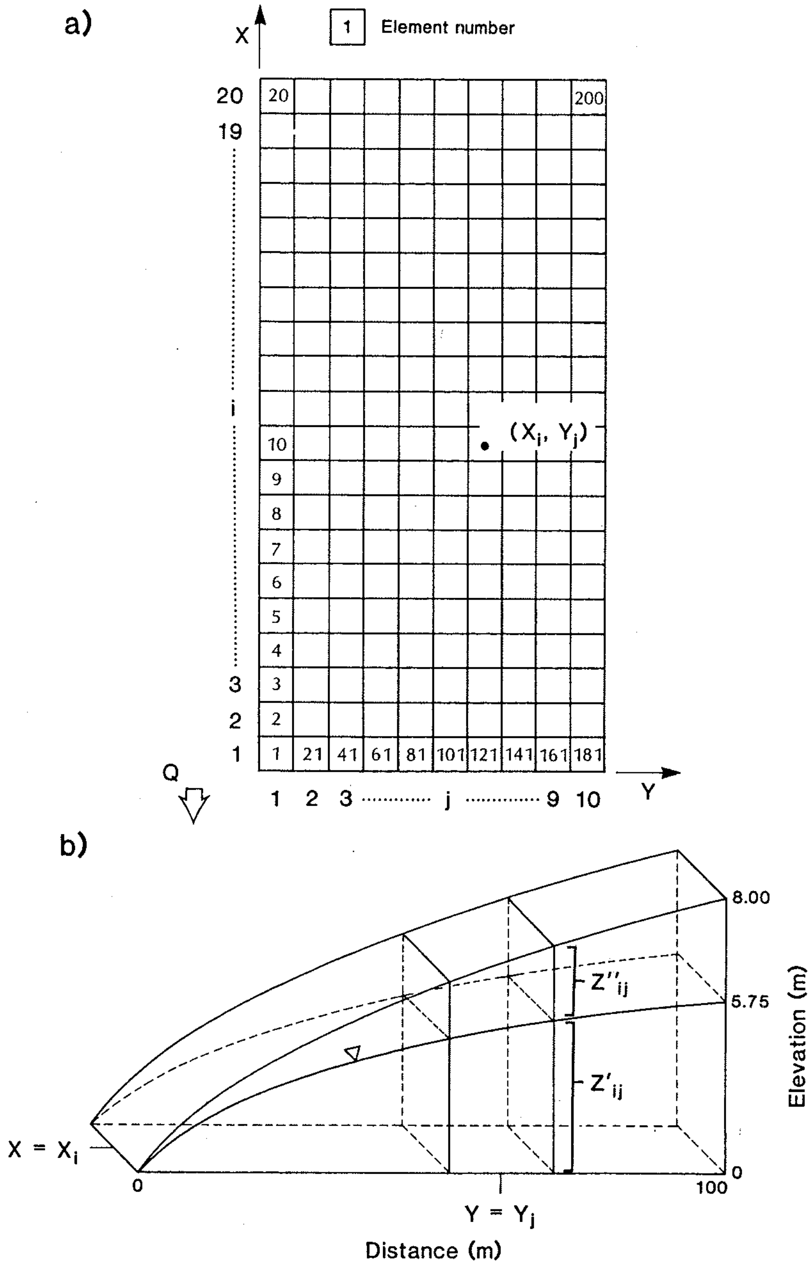

Fig. 1. (*a*) Two-dimensional grid of 200 elements with $\Delta x = \Delta y = 10$ m. (*b*) Vertical section through the hillslope. $Z_{ij}''$ refers to the unsaturated zone and $Z_{ij}'$ to the saturated zone [adapted from *Freeze*, 1980].

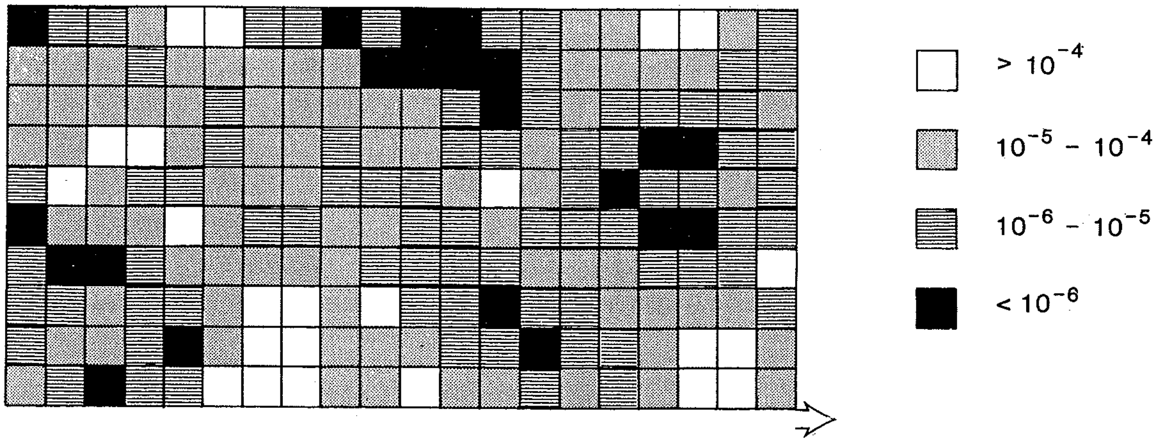

Fig. 2. Distribution of saturated hydraulic conductivity for case A. Units of $K^s$ are in meters per second.

Each of the six cases listed in Table 1 constitutes an ensemble of 100 rainfall-runoff events as described by *Freeze* [1980]. The results from these Monte Carlo experiments are presented in Table 2. The output is summarized in terms of the distribution of overland flow sources and the mean peak flow $\bar{Q}_{PK}$ for each case. The parameter $N^0$ listed in Table 2 refers to the number of events in each experiment for which no runoff was generated. For case A, for example, 44 of the 200 elements that make up the discretized hillslope produced overland flow in less than 25% of the 100 events, while 17 did so in more than 75% of the events; 11 events produced no runoff at all; the mean peak outflow was 91 L/s.

The influence of the standard deviation in hydraulic conductivity $\alpha_Y$ can be seen by comparing cases E and J with case A. In the first of these comparisons (case E and case A), the reduced range of hydraulic conductivity results in a more uniform response of overland flow sources. It also results in a reduced mean peak flow and a greater number of events that do not generate runoff. In general, these are the same observations made by *Freeze* [1980]. In the second comparison (case J and case A), the increased range results in a less uniform response of overland flow sources, an increased mean peak flow, and fewer events that do not generate runoff.

The influence of the autocorrelation parameters $\alpha_Y$ is indicated by comparison of cases H and I with case A. A reduction in the degree of autocorrelation (indicated by an increase in $\alpha_Y$) has a similar effect on the system to an increase in $\sigma_Y$. This observation is just the opposite of the one made by *Freeze* [1980]. The decrease in autocorrelation also results in reduced mean peak flows.

The combined influence of the standard deviation and autocorrelation parameters is illustrated by comparing case F with case A. The reduction in $\sigma_Y$ is offset by the increase in $\alpha_Y$. The uniform response in the overland flow sources is similar to that seen for case E. The closer relationship between cases E and F suggests that $\sigma_Y$ is more important than $\alpha_Y$, as was suggested in the earlier paper [*Freeze*, 1980].

In the original paper, the value for $\sigma_Y$ for case F was mistakenly reported to be 0.8, when, in fact, it was 0.1. This explains the erroneous interpretation for reduced autocorrelation in the original paper [*Freeze*, 1980]. The effect of the reduction in autocorrelation in the earlier paper was overshadowed by the effect of the reduction in the standard deviation.

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